Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.2-Cosine/83-4.2.10-c+d-x-^m-a+b-cos-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [189]. This is test number [83].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (189)	0.00 (0)
Mathematica	100.00 (189)	0.00 (0)
Maxima	74.07 (140)	25.93 (49)
Fricas	72.49 (137)	27.51 (52)
Maple	71.43 (135)	28.57 (54)
Giac	59.26 (112)	40.74 (77)
Mupad	39.15 (74)	60.85 (115)
Sympy	29.10 (55)	70.90 (134)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

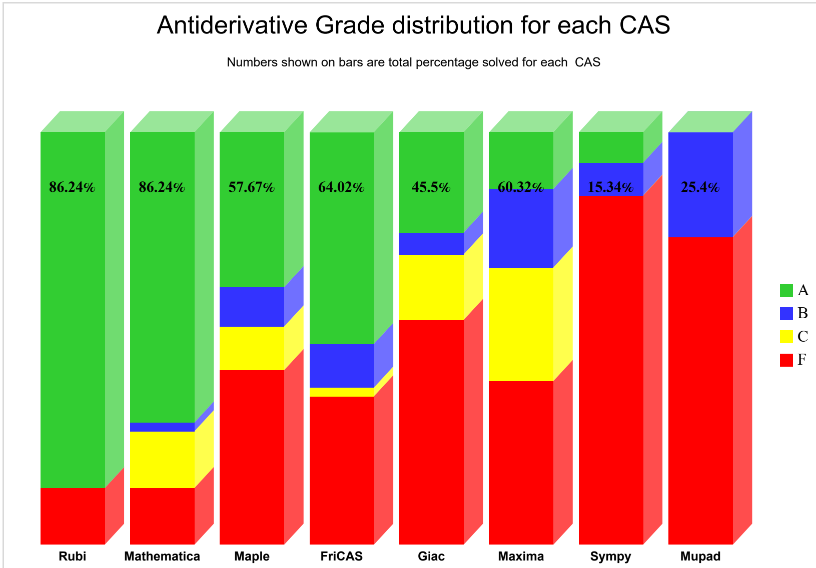
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

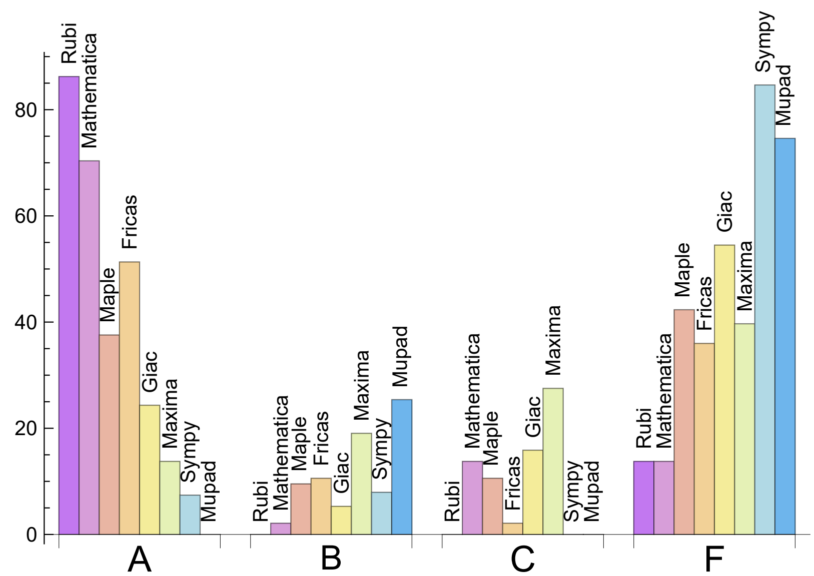
System	% A grade	% B grade	% C grade	% F grade
Rubi	86.243	0.000	0.000	13.757
Mathematica	70.370	2.116	13.757	13.757
Fricas	51.323	10.582	2.116	35.979
Maple	37.566	9.524	10.582	42.328
Giac	24.339	5.291	15.873	54.497
Maxima	13.757	19.048	27.513	39.683
Sympy	7.407	7.937	0.000	84.656
Mupad	0.000	25.397	0.000	74.603

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	52	17.31	0.00	82.69
Maxima	49	91.84	0.00	8.16
Maple	54	100.00	0.00	0.00
Giac	77	100.00	0.00	0.00
Mupad	115	0.00	100.00	0.00
Sympy	134	97.01	2.99	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.25
Giac	0.41
Rubi	0.48
Maxima	0.65
Maple	0.90
Mathematica	1.17
Sympy	6.46
Mupad	11.37

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	77.66	1.25	34.50	1.10
Mathematica	105.86	0.98	76.00	0.93
Rubi	112.00	1.01	86.00	1.00
Sympy	138.16	1.82	60.00	1.55
Maple	165.08	1.62	108.00	1.12
Fricas	201.34	1.60	114.00	1.18
Maxima	326.14	5.15	163.50	1.49
Giac	1602.83	11.15	80.50	1.14

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

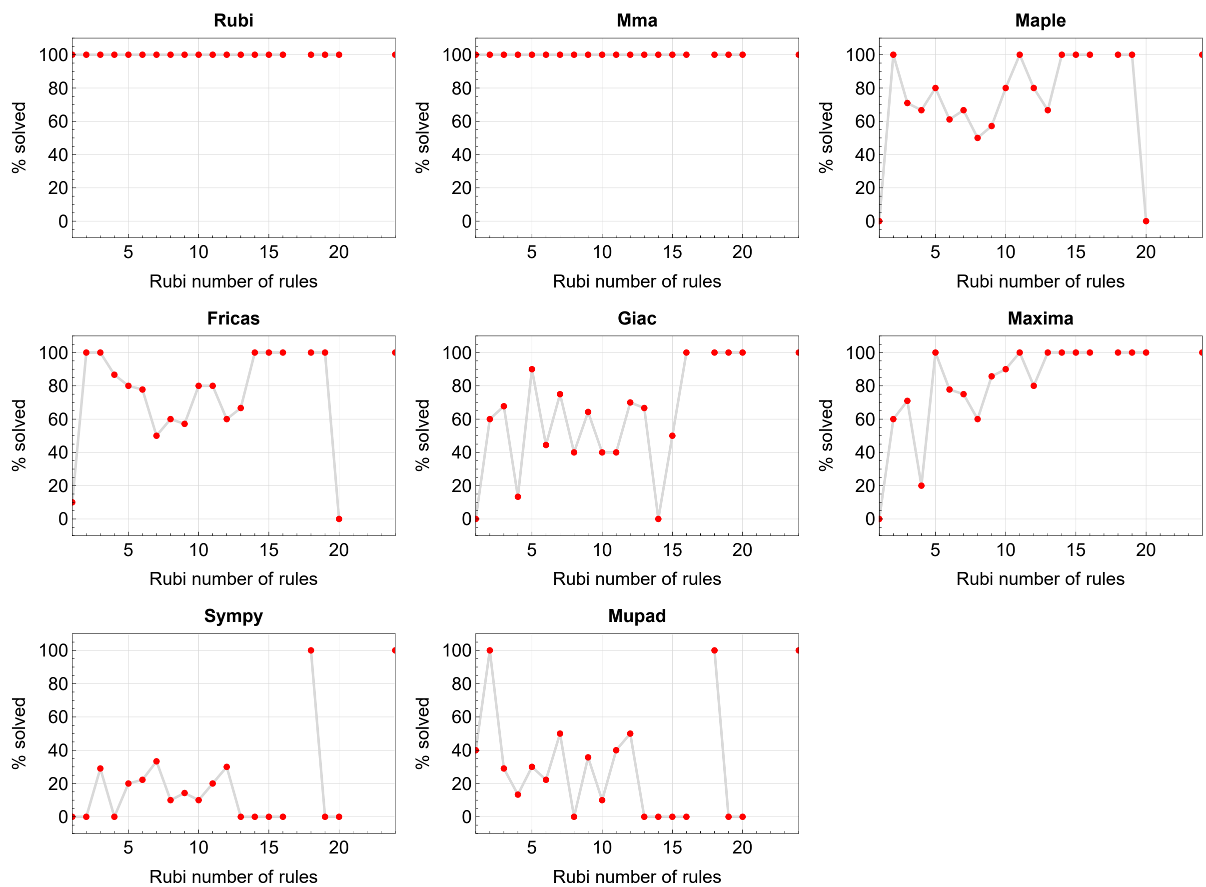


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

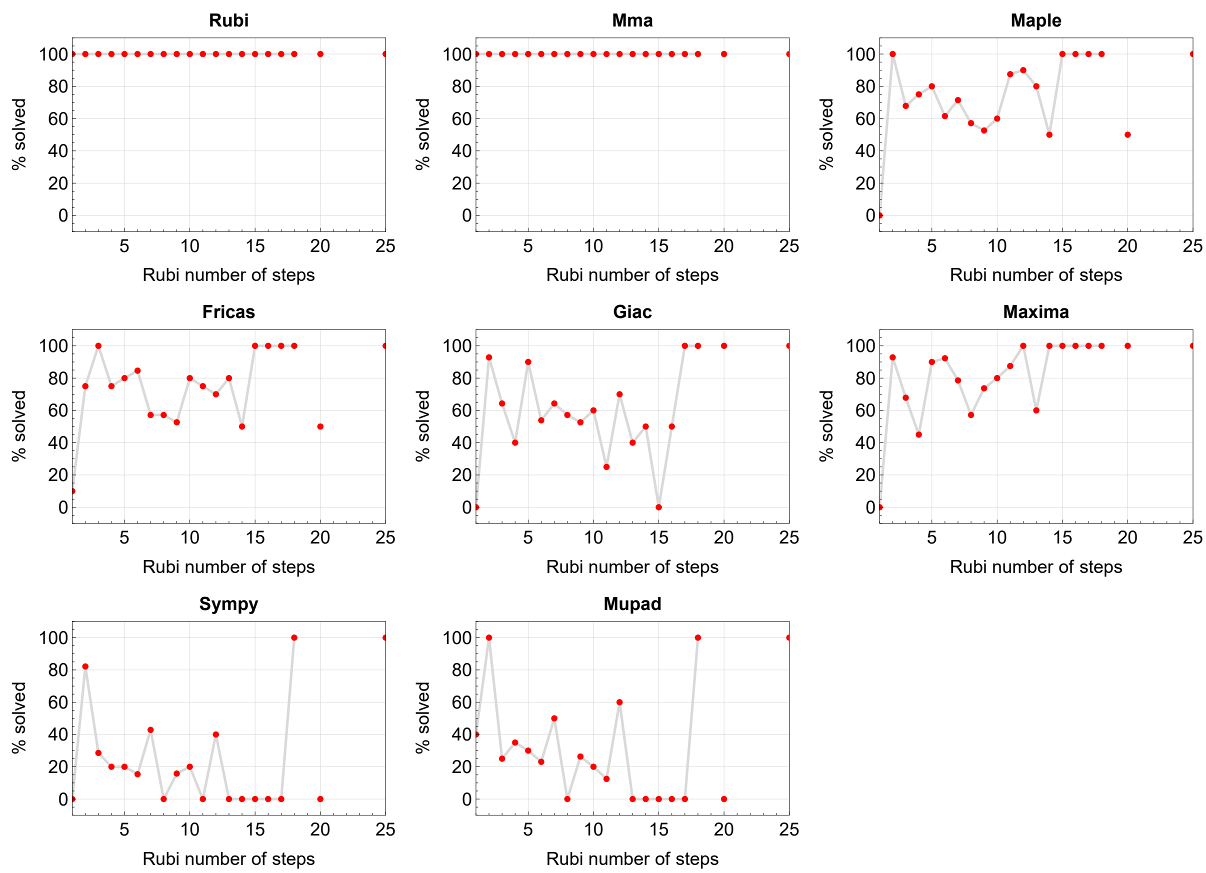


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

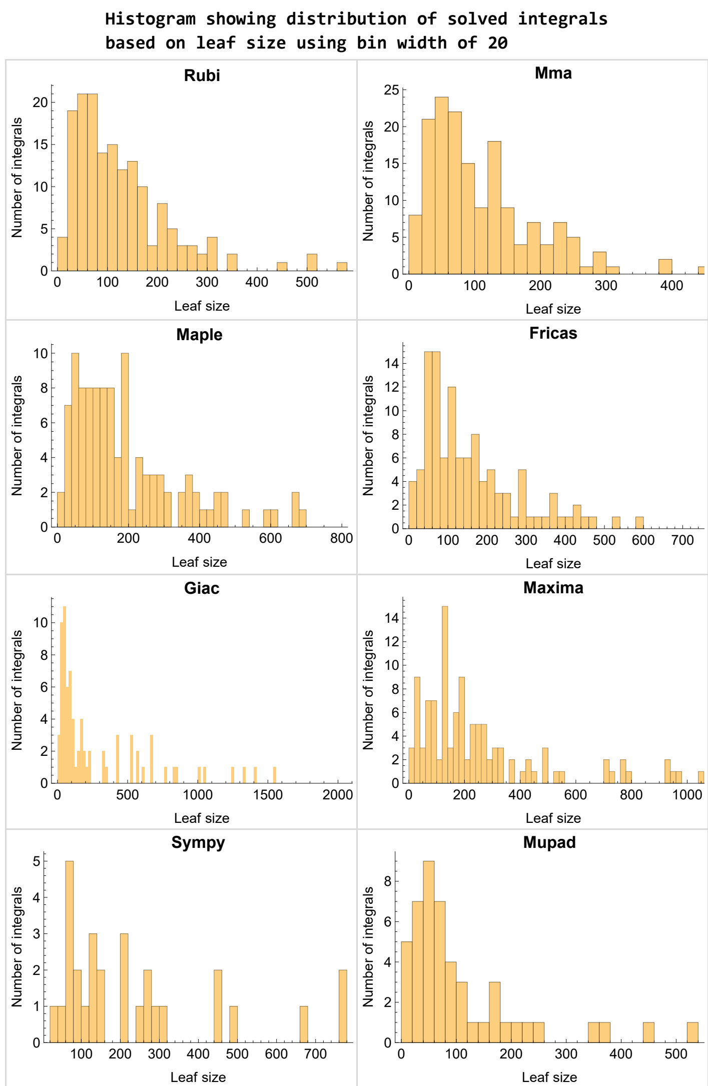


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

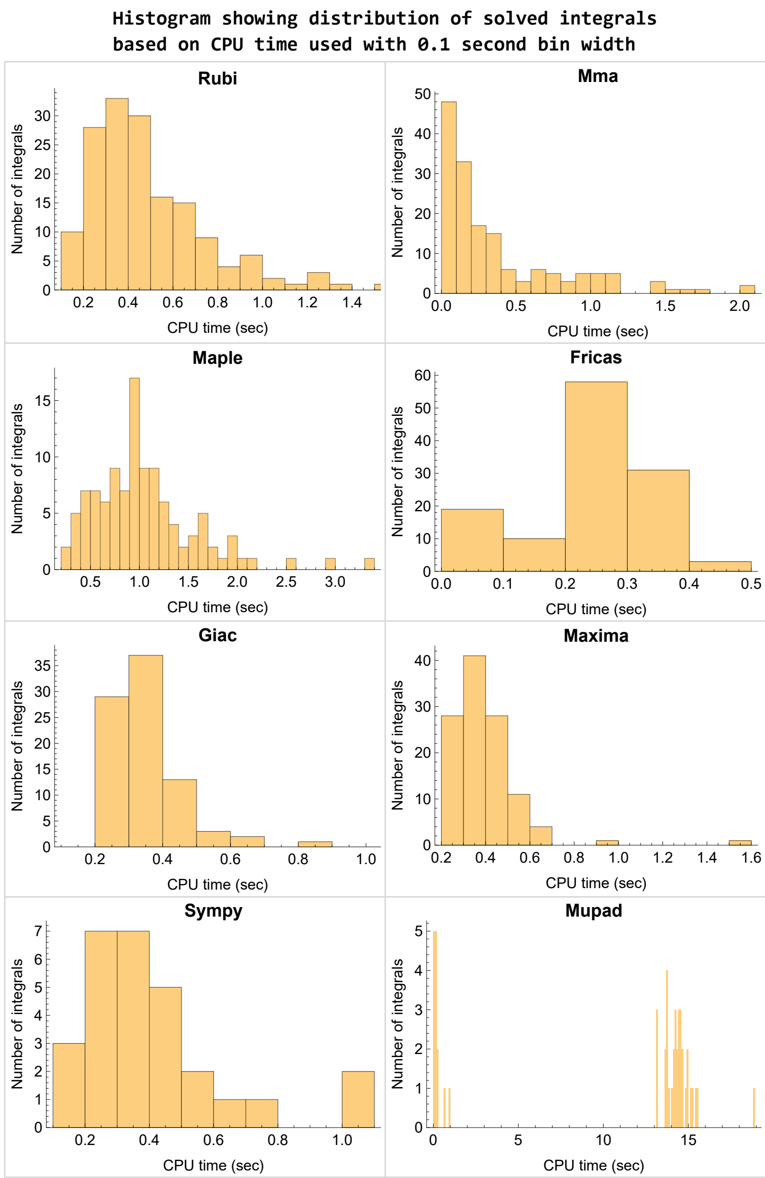


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

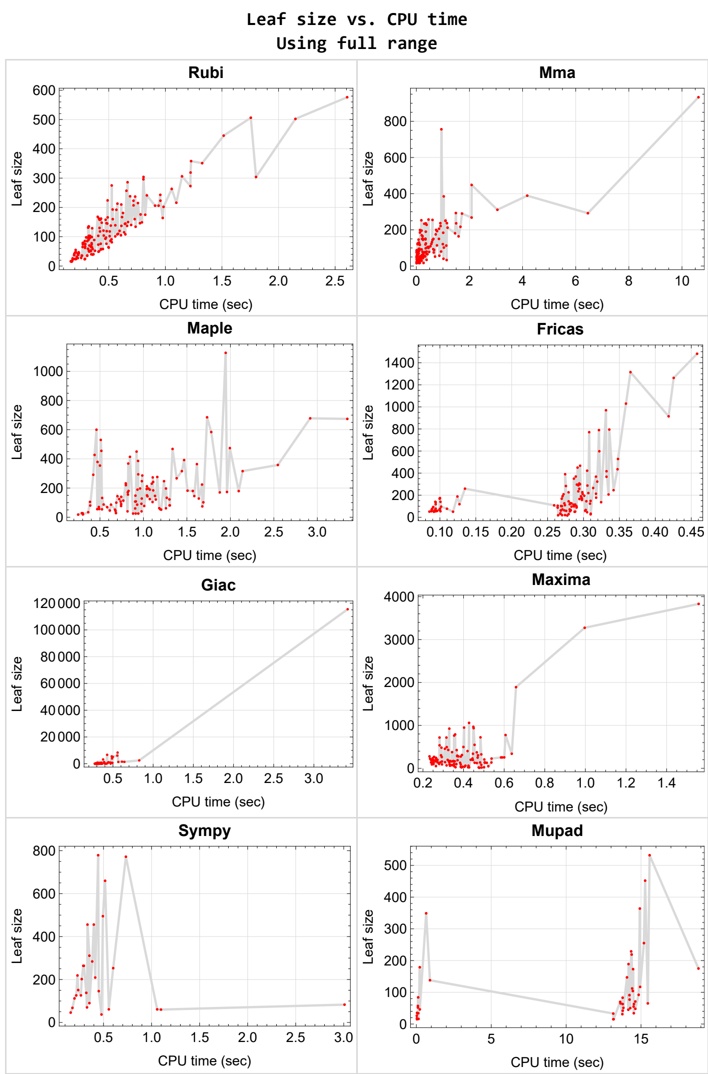


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{32, 36, 40, 75, 77, 78, 80, 82, 83, 85, 86, 88, 98, 102, 103, 131, 132, 136, 137, 141, 142, 174, 179, 183, 184, 188}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {189}

Maple {173}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

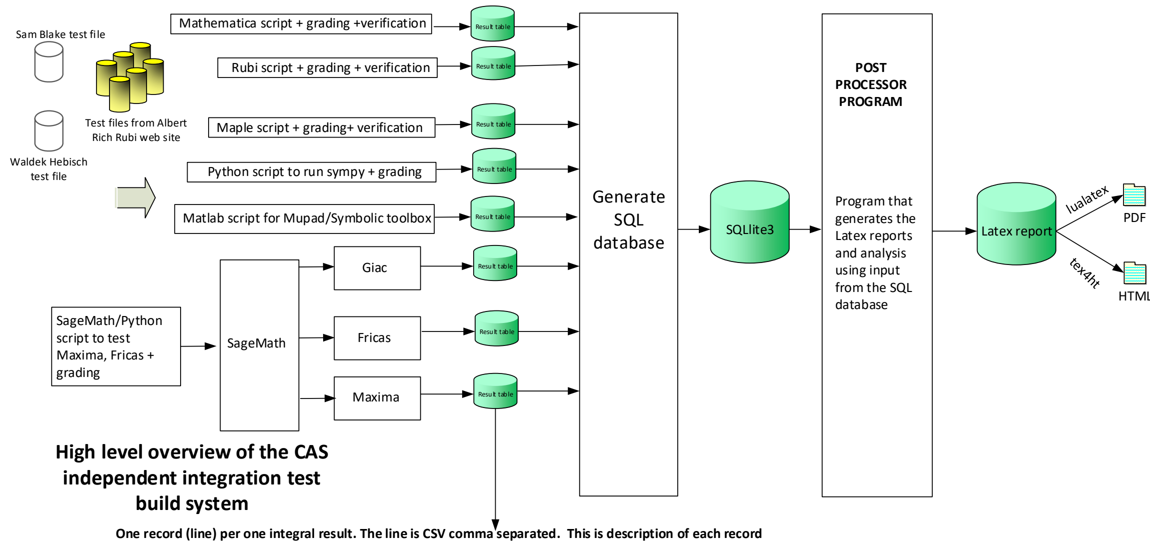
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 138, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186 }

B grade { 39, 139, 187, 189 }

C grade { 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 135, 140, 146, 153, 160, 178 }

B grade { 29, 30, 33, 34, 37, 38, 39, 76, 79, 87, 128, 129, 133, 134, 138, 139, 187, 189 }

C grade { 13, 14, 84, 104, 105, 106, 107, 108, 109, 110, 143, 144, 145, 150, 151, 152, 157, 158, 159, 173 }

F normal fail { 67, 68, 69, 70, 71, 72, 73, 74, 81, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 111, 112, 113, 114, 115, 116, 117, 147, 148, 149, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 35, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 91, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 135, 140, 146, 153, 160, 173, 178 }

B grade { 8, 29, 30, 31, 33, 34, 37, 38, 39, 55, 128, 129, 133, 134, 138, 139, 185, 186, 187, 189 }

C grade { 76, 79, 84, 87 }

F normal fail { 170, 171, 172, 175, 176, 177, 180, 181, 182 }

F(-1) timedout fail { }

F(-2) exception fail { 75, 77, 78, 80, 81, 82, 83, 85, 86, 88, 89, 90, 92, 93, 94, 95, 96, 97, 143, 144, 145, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 184 }

2.1.5 Maxima

A grade { 4, 12, 19, 23, 24, 25, 67, 68, 69, 70, 71, 72, 73, 74, 125, 144, 145, 146, 150, 151, 152, 153, 160, 164, 165, 166 }

B grade { 1, 2, 3, 9, 10, 11, 16, 17, 18, 29, 30, 33, 34, 35, 37, 38, 118, 119, 120, 123, 124, 128, 129, 130, 133, 134, 135, 138, 139, 140, 143, 157, 158, 159, 173, 178 }

C grade { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 26, 27, 28, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 121, 122, 126, 127, 147, 148, 149, 154, 155, 156, 167, 168, 169 }

F normal fail { 31, 39, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 161, 162, 163, 170, 171, 172, 175, 176, 177, 180, 181, 182 }

F(-1) timedout fail { }

F(-2) exception fail { 185, 186, 187, 189 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 9, 10, 11, 12, 16, 17, 18, 19, 23, 24, 25, 118, 119, 120, 123, 124, 125, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 178 }

B grade { 6, 14, 21, 35, 122, 127, 130, 135, 140, 173 }

C grade { 5, 7, 8, 13, 15, 20, 22, 26, 27, 28, 41, 42, 43, 44, 48, 49, 50, 51, 56, 57, 58, 59, 63, 64, 65, 121, 126, 147, 148, 149 }

F normal fail { 29, 30, 31, 33, 34, 37, 38, 39, 45, 46, 47, 52, 53, 54, 55, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 128, 129, 133, 134, 138, 139, 170, 171, 172, 175, 176, 177, 180, 181, 182, 185, 186, 187, 189 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 9, 10, 11, 12, 16, 17, 18, 19, 23, 24, 25, 35, 64, 65, 76, 79, 84, 87, 89, 90, 91, 92, 118, 119, 120, 123, 124, 125, 130, 135, 140, 143, 144, 145, 146, 150, 151, 152, 153, 157, 158, 159, 160, 173 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 26, 27, 28, 29, 30, 31, 33, 34, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 66, 67, 68, 69, 70, 71, 72, 73, 74, 81, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 126, 127, 128, 129, 133, 134, 138, 139, 147, 148, 149, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187, 189 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 4, 19, 23, 24, 25, 26, 63, 64, 65, 66, 120, 125, 130, 135 }

B grade { 1, 2, 3, 9, 10, 11, 12, 16, 17, 18, 118, 119, 123, 124, 140 }

C grade { }

F normal fail { 5, 6, 7, 8, 13, 14, 15, 20, 21, 22, 27, 28, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 76, 79, 81, 84, 87, 89, 90, 91, 93, 94, 95, 96, 97, 99, 100, 101, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 126, 127, 128, 129, 133, 134, 138, 139, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 180, 181, 182, 185, 186, 187 }

F(-1) timedout fail { 55, 56, 92, 189 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	101	76	143	481	169	311	170	219
N.S.	1	1.11	0.84	1.57	5.29	1.86	3.42	1.87	2.41
time (sec)	N/A	0.551	0.314	1.046	0.291	0.298	0.353	0.293	14.378

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	61	107	278	109	202	110	147
N.S.	1	1.09	0.87	1.53	3.97	1.56	2.89	1.57	2.10
time (sec)	N/A	0.435	0.210	0.905	0.232	0.274	0.273	0.295	14.064

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	52	44	60	136	62	112	64	84
N.S.	1	1.06	0.90	1.22	2.78	1.27	2.29	1.31	1.71
time (sec)	N/A	0.330	0.172	0.905	0.330	0.273	0.198	0.313	0.122

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	28	50	28	46	30	34
N.S.	1	1.00	0.96	1.04	1.85	1.04	1.70	1.11	1.26
time (sec)	N/A	0.229	0.113	0.684	0.264	0.279	0.158	0.324	0.093

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	77	142	62	0	577	0
N.S.	1	1.00	0.96	1.48	2.73	1.19	0.00	11.10	0.00
time (sec)	N/A	0.385	0.113	0.743	0.309	0.268	0.000	0.342	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	76	65	114	164	96	0	523	0
N.S.	1	1.04	0.89	1.56	2.25	1.32	0.00	7.16	0.00
time (sec)	N/A	0.479	0.417	0.769	0.327	0.283	0.000	0.314	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	105	89	148	199	165	0	5518	0
N.S.	1	1.01	0.86	1.42	1.91	1.59	0.00	53.06	0.00
time (sec)	N/A	0.593	0.668	0.908	0.435	0.294	0.000	0.484	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	134	144	184	249	235	0	8378	0
N.S.	1	1.06	1.13	1.45	1.96	1.85	0.00	65.97	0.00
time (sec)	N/A	0.731	0.611	1.113	0.469	0.293	0.000	0.558	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	166	132	145	717	287	660	222	349
N.S.	1	1.03	0.82	0.90	4.45	1.78	4.10	1.38	2.17
time (sec)	N/A	0.475	0.585	1.585	0.282	0.305	0.517	0.332	0.656

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	129	106	121	428	190	456	153	229
N.S.	1	0.96	0.79	0.90	3.19	1.42	3.40	1.14	1.71
time (sec)	N/A	0.346	0.447	1.142	0.325	0.271	0.399	0.301	14.343

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	97	77	81	222	113	264	94	179
N.S.	1	1.02	0.81	0.85	2.34	1.19	2.78	0.99	1.88
time (sec)	N/A	0.306	0.326	1.304	0.235	0.286	0.290	0.307	0.229

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	50	46	90	53	126	48	57
N.S.	1	1.00	0.91	0.84	1.64	0.96	2.29	0.87	1.04
time (sec)	N/A	0.210	0.269	0.613	0.244	0.281	0.217	0.292	0.103

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	107	163	71	0	610	0
N.S.	1	1.00	0.83	1.37	2.09	0.91	0.00	7.82	0.00
time (sec)	N/A	0.347	0.264	0.738	0.357	0.281	0.000	0.352	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	86	75	155	171	100	0	534	0
N.S.	1	1.04	0.90	1.87	2.06	1.20	0.00	6.43	0.00
time (sec)	N/A	0.512	0.654	0.822	0.350	0.270	0.000	0.356	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	146	102	193	204	176	0	5136	0
N.S.	1	1.30	0.91	1.72	1.82	1.57	0.00	45.86	0.00
time (sec)	N/A	0.489	1.037	1.037	0.443	0.297	0.000	0.499	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	304	385	173	925	350	772	351	532
N.S.	1	1.35	1.71	0.77	4.11	1.56	3.43	1.56	2.36
time (sec)	N/A	1.837	1.032	1.961	0.330	0.287	0.733	0.324	15.574

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	216	121	224	535	227	495	231	364
N.S.	1	1.23	0.69	1.28	3.06	1.30	2.83	1.32	2.08
time (sec)	N/A	1.104	0.996	1.674	0.282	0.298	0.494	0.311	14.923

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	137	93	128	267	128	284	137	173
N.S.	1	1.11	0.76	1.04	2.17	1.04	2.31	1.11	1.41
time (sec)	N/A	0.607	0.634	1.637	0.255	0.282	0.382	0.309	14.472

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	74	52	61	103	60	126	69	77
N.S.	1	0.99	0.69	0.81	1.37	0.80	1.68	0.92	1.03
time (sec)	N/A	0.344	0.227	1.259	0.279	0.268	0.265	0.299	14.235

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	103	171	278	123	0	6075	0
N.S.	1	1.00	0.85	1.41	2.30	1.02	0.00	50.21	0.00
time (sec)	N/A	0.450	0.409	0.820	0.374	0.309	0.000	0.554	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	148	200	247	304	177	0	1000	0
N.S.	1	1.02	1.38	1.70	2.10	1.22	0.00	6.90	0.00
time (sec)	N/A	0.438	0.744	0.982	0.471	0.318	0.000	0.440	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	243	221	316	339	289	0	115446	0
N.S.	1	1.32	1.20	1.72	1.84	1.57	0.00	627.42	0.00
time (sec)	N/A	0.962	0.913	1.442	0.637	0.293	0.000	3.420	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	237	100	102	303	115	253	108	138
N.S.	1	1.38	0.58	0.59	1.76	0.67	1.47	0.63	0.80
time (sec)	N/A	0.750	0.457	1.689	0.289	0.288	0.601	0.354	0.908

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	181	92	81	188	88	209	84	104
N.S.	1	1.35	0.69	0.60	1.40	0.66	1.56	0.63	0.78
time (sec)	N/A	0.615	0.197	1.300	0.248	0.290	0.414	0.398	14.456

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	85	53	62	98	63	138	64	63
N.S.	1	1.06	0.66	0.78	1.22	0.79	1.72	0.80	0.79
time (sec)	N/A	0.287	0.135	1.182	0.252	0.282	0.321	0.412	14.527

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	52	91	49	60	428	0
N.S.	1	1.00	0.88	0.88	1.54	0.83	1.02	7.25	0.00
time (sec)	N/A	0.316	0.158	0.673	0.331	0.288	1.098	0.433	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	70	79	90	717	66	0	3220	0
N.S.	1	1.06	1.20	1.36	10.86	1.00	0.00	48.79	0.00
time (sec)	N/A	0.311	0.230	0.744	0.316	0.303	0.000	0.377	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	138	119	124	790	100	0	3920	0
N.S.	1	1.53	1.32	1.38	8.78	1.11	0.00	43.56	0.00
time (sec)	N/A	0.493	0.342	0.721	0.359	0.290	0.000	0.489	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	223	196	685	722	970	0	0	0
N.S.	1	1.09	0.96	3.34	3.52	4.73	0.00	0.00	0.00
time (sec)	N/A	0.724	0.198	1.732	0.483	0.331	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	145	130	392	402	598	0	0	0
N.S.	1	1.06	0.95	2.86	2.93	4.36	0.00	0.00	0.00
time (sec)	N/A	0.483	0.101	1.468	0.420	0.322	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	87	128	0	306	0	0	0
N.S.	1	1.00	1.16	1.71	0.00	4.08	0.00	0.00	0.00
time (sec)	N/A	0.287	0.010	0.981	0.000	0.294	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.29
time (sec)	N/A	0.194	4.359	0.528	0.577	0.289	0.379	0.499	13.675

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	137	109	316	1059	790	0	0	0
N.S.	1	1.20	0.96	2.77	9.29	6.93	0.00	0.00	0.00
time (sec)	N/A	0.656	0.529	2.141	0.427	0.322	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	97	75	170	324	450	0	0	0
N.S.	1	1.18	0.91	2.07	3.95	5.49	0.00	0.00	0.00
time (sec)	N/A	0.476	0.264	1.878	0.487	0.292	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	36	52	159	45	0	1404	55
N.S.	1	1.00	1.29	1.86	5.68	1.61	0.00	50.14	1.96
time (sec)	N/A	0.250	0.017	1.204	0.352	0.305	0.000	0.642	14.505

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	290	18	14	18	18
N.S.	1	1.00	1.12	1.00	18.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.209	6.061	0.534	0.547	0.293	0.409	0.381	13.698

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	358	311	1127	3831	1315	0	0	0
N.S.	1	1.06	0.92	3.34	11.37	3.90	0.00	0.00	0.00
time (sec)	N/A	1.222	3.050	1.946	1.558	0.365	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	207	184	584	1891	795	0	0	0
N.S.	1	1.07	0.95	3.03	9.80	4.12	0.00	0.00	0.00
time (sec)	N/A	0.754	1.011	1.780	0.658	0.336	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	118	389	267	0	435	0	0	0
N.S.	1	1.01	3.32	2.28	0.00	3.72	0.00	0.00	0.00
time (sec)	N/A	0.405	4.174	1.382	0.000	0.347	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	290	18	14	18	18
N.S.	1	1.00	1.12	1.00	18.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.205	1.726	0.023	0.579	0.322	0.403	0.309	0.003

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	202	124	232	263	190	0	1246	0
N.S.	1	1.04	0.64	1.20	1.36	0.98	0.00	6.42	0.00
time (sec)	N/A	0.987	0.063	0.797	0.293	0.297	0.000	0.390	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	175	122	189	242	156	0	779	0
N.S.	1	1.04	0.72	1.12	1.43	0.92	0.00	4.61	0.00
time (sec)	N/A	0.812	0.094	0.931	0.264	0.298	0.000	0.357	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	145	122	144	196	126	0	426	0
N.S.	1	1.02	0.86	1.01	1.38	0.89	0.00	3.00	0.00
time (sec)	N/A	0.663	0.044	0.701	0.240	0.273	0.000	0.340	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	124	100	159	108	0	168	0
N.S.	1	1.00	1.05	0.85	1.35	0.92	0.00	1.42	0.00
time (sec)	N/A	0.513	0.061	0.688	0.294	0.259	0.000	0.322	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	144	147	140	129	144	0	0	0
N.S.	1	1.04	1.06	1.01	0.93	1.04	0.00	0.00	0.00
time (sec)	N/A	0.642	0.344	0.744	0.506	0.279	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	176	190	180	129	208	0	0	0
N.S.	1	1.05	1.13	1.07	0.77	1.24	0.00	0.00	0.00
time (sec)	N/A	0.793	0.355	0.829	0.515	0.272	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	206	228	220	129	296	0	0	0
N.S.	1	1.07	1.18	1.14	0.67	1.53	0.00	0.00	0.00
time (sec)	N/A	0.955	0.386	0.798	0.455	0.291	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	238	150	242	295	258	0	1331	0
N.S.	1	1.03	0.65	1.05	1.28	1.12	0.00	5.76	0.00
time (sec)	N/A	0.687	0.851	1.112	0.369	0.294	0.000	0.567	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	210	150	197	274	195	0	821	0
N.S.	1	1.03	0.74	0.97	1.35	0.96	0.00	4.04	0.00
time (sec)	N/A	0.600	0.643	0.961	0.386	0.301	0.000	0.472	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	150	150	229	148	0	436	0
N.S.	1	1.00	0.95	0.95	1.45	0.94	0.00	2.76	0.00
time (sec)	N/A	0.471	0.248	1.041	0.444	0.310	0.000	0.395	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	145	108	187	114	0	167	0
N.S.	1	1.00	1.12	0.83	1.44	0.88	0.00	1.28	0.00
time (sec)	N/A	0.399	0.250	0.913	0.371	0.286	0.000	0.328	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	140	175	146	135	136	0	0	0
N.S.	1	1.04	1.30	1.08	1.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.692	0.729	1.133	0.537	0.325	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	213	181	189	136	206	0	0	0
N.S.	1	1.25	1.06	1.11	0.80	1.21	0.00	0.00	0.00
time (sec)	N/A	0.566	1.458	1.073	0.481	0.335	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	223	237	230	136	323	0	0	0
N.S.	1	1.03	1.10	1.06	0.63	1.50	0.00	0.00	0.00
time (sec)	N/A	0.950	1.150	1.096	0.506	0.319	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	296	237	273	136	417	0	0	0
N.S.	1	1.20	0.96	1.11	0.55	1.69	0.00	0.00	0.00
time (sec)	N/A	0.795	0.904	1.107	0.504	0.332	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	576	236	474	547	368	0	2480	0
N.S.	1	1.40	0.58	1.16	1.33	0.90	0.00	6.05	0.00
time (sec)	N/A	2.632	1.499	1.994	0.454	0.317	0.000	0.827	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	502	251	386	497	299	0	1549	0
N.S.	1	1.42	0.71	1.09	1.40	0.84	0.00	4.38	0.00
time (sec)	N/A	2.174	1.100	0.938	0.399	0.295	0.000	0.612	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	234	294	424	245	0	848	0
N.S.	1	1.00	0.77	0.97	1.39	0.81	0.00	2.79	0.00
time (sec)	N/A	0.827	0.298	0.940	0.366	0.342	0.000	0.467	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	236	212	377	213	0	332	0
N.S.	1	1.00	0.92	0.82	1.47	0.83	0.00	1.29	0.00
time (sec)	N/A	0.653	0.356	1.040	0.344	0.317	0.000	0.330	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	286	293	286	253	265	0	0	0
N.S.	1	1.06	1.08	1.06	0.93	0.98	0.00	0.00	0.00
time (sec)	N/A	0.676	1.488	0.987	0.591	0.313	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	445	268	368	253	367	0	0	0
N.S.	1	1.52	0.92	1.26	0.87	1.26	0.00	0.00	0.00
time (sec)	N/A	1.550	2.080	0.825	0.585	0.332	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	506	448	450	254	528	0	0	0
N.S.	1	1.42	1.26	1.26	0.71	1.48	0.00	0.00	0.00
time (sec)	N/A	1.774	2.082	0.921	0.601	0.348	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	A	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	55	34	74	35	83	69	0
N.S.	1	1.00	1.12	0.69	1.51	0.71	1.69	1.41	0.00
time (sec)	N/A	0.331	0.016	0.365	0.337	0.268	3.008	0.295	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	48	27	67	26	61	53	26
N.S.	1	1.00	1.33	0.75	1.86	0.72	1.69	1.47	0.72
time (sec)	N/A	0.256	0.009	0.302	0.316	0.310	0.555	0.286	0.031

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	51	19	60	18	37	35	18
N.S.	1	1.00	2.12	0.79	2.50	0.75	1.54	1.46	0.75
time (sec)	N/A	0.202	0.009	0.304	0.333	0.265	0.479	0.300	0.037

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	C	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	63	28	21	31	61	0	0
N.S.	1	1.00	1.80	0.80	0.60	0.89	1.74	0.00	0.00
time (sec)	N/A	0.259	0.051	0.292	0.428	0.268	1.059	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	192	122	0	235	174	0	0	0
N.S.	1	1.05	0.67	0.00	1.28	0.95	0.00	0.00	0.00
time (sec)	N/A	0.571	0.114	0.000	0.248	0.100	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	159	122	0	186	140	0	0	0
N.S.	1	1.05	0.80	0.00	1.22	0.92	0.00	0.00	0.00
time (sec)	N/A	0.430	0.049	0.000	0.262	0.101	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	159	122	0	186	140	0	0	0
N.S.	1	1.05	0.80	0.00	1.22	0.92	0.00	0.00	0.00
time (sec)	N/A	0.421	0.053	0.000	0.253	0.094	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	124	0	137	118	0	0	0
N.S.	1	1.00	0.92	0.00	1.01	0.87	0.00	0.00	0.00
time (sec)	N/A	0.311	0.063	0.000	0.253	0.127	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	124	0	138	118	0	0	0
N.S.	1	1.00	0.92	0.00	1.02	0.87	0.00	0.00	0.00
time (sec)	N/A	0.326	0.066	0.000	0.242	0.094	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	158	121	0	138	169	0	0	0
N.S.	1	1.05	0.80	0.00	0.91	1.12	0.00	0.00	0.00
time (sec)	N/A	0.416	0.055	0.000	0.273	0.100	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	162	121	0	138	169	0	0	0
N.S.	1	1.06	0.79	0.00	0.90	1.10	0.00	0.00	0.00
time (sec)	N/A	0.429	0.056	0.000	0.285	0.100	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	193	125	0	137	259	0	0	0
N.S.	1	1.06	0.69	0.00	0.75	1.42	0.00	0.00	0.00
time (sec)	N/A	0.534	0.053	0.000	0.240	0.135	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.174	26.255	0.119	0.701	0.000	6.357	0.329	13.311

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	133	0	57	0	0	15
N.S.	1	1.00	1.00	8.31	0.00	3.56	0.00	0.00	0.94
time (sec)	N/A	0.165	0.009	1.268	0.000	0.096	0.000	0.000	0.042

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	12	14	14
N.S.	1	1.00	1.14	0.86	1.00	0.00	0.86	1.00	1.00
time (sec)	N/A	0.182	0.808	0.151	0.574	0.000	2.994	0.344	13.504

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.264	2.291	0.131	0.696	0.000	87.663	0.368	13.726

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	36	179	0	68	0	0	35
N.S.	1	1.00	0.86	4.26	0.00	1.62	0.00	0.00	0.83
time (sec)	N/A	0.223	0.029	2.096	0.000	0.099	0.000	0.000	0.047

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	12	14	14
N.S.	1	1.00	1.14	0.86	1.00	0.00	0.86	1.00	1.00
time (sec)	N/A	0.190	7.418	0.146	0.599	0.000	37.920	0.328	13.556

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.201	1.020	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	10	0	10	10	10
N.S.	1	1.00	1.20	0.80	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.293	6.110	0.195	0.414	0.000	92.969	0.323	13.057

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.181	0.278	0.178	0.644	0.000	3.071	0.345	13.326

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	0	51	0	0	15
N.S.	1	1.00	1.00	1.12	0.00	3.19	0.00	0.00	0.94
time (sec)	N/A	0.164	0.009	0.249	0.000	0.085	0.000	0.000	13.172

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.186	0.453	0.118	0.577	0.000	10.382	0.326	13.056

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	12	0	12	12	12
N.S.	1	1.00	1.17	0.83	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.267	2.672	0.130	0.662	0.000	14.099	0.357	13.610

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	182	0	93	0	0	42
N.S.	1	1.00	1.00	4.79	0.00	2.45	0.00	0.00	1.11
time (sec)	N/A	0.224	0.051	1.507	0.000	0.091	0.000	0.000	13.781

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	0	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.182	10.729	0.130	0.630	0.000	36.925	0.352	13.676

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	33	0	0	0	0	0	51
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	1.34
time (sec)	N/A	0.190	1.141	0.000	0.000	0.000	0.000	0.000	13.825

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	15
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	0.75
time (sec)	N/A	0.179	0.132	0.000	0.000	0.000	0.000	0.000	13.152

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	15	0	0	16
N.S.	1	1.00	0.71	0.00	0.00	0.62	0.00	0.00	0.67
time (sec)	N/A	0.177	0.111	0.000	0.000	0.277	0.000	0.000	0.147

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0	31
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	0.66
time (sec)	N/A	0.205	0.184	0.000	0.000	0.000	0.000	0.000	13.756

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	29	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.196	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.209	0.270	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.238	0.134	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	51	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.11
time (sec)	N/A	0.214	1.111	0.345	0.672	0.260	8.741	1.195	14.057

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	253	0	0	188	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.540	0.188	0.000	0.000	0.124	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	150	0	0	136	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.417	0.215	0.000	0.000	0.094	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	122	0	0	96	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.313	0.052	0.000	0.000	0.102	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.29
time (sec)	N/A	0.193	9.537	0.372	0.479	0.286	2.123	0.434	14.366

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.212	0.858	0.326	0.532	0.259	5.245	0.365	13.861

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	455	0	54	0	0	0
N.S.	1	1.00	1.00	6.07	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.270	0.022	0.516	0.000	0.091	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	354	0	54	0	0	0
N.S.	1	1.00	1.00	4.48	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.279	0.020	0.500	0.000	0.088	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	291	0	54	0	0	0
N.S.	1	1.00	1.00	3.88	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.268	0.018	0.423	0.000	0.101	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	379	0	50	0	0	0
N.S.	1	1.00	1.00	4.80	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.271	0.017	0.472	0.000	0.118	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	427	0	50	0	0	0
N.S.	1	1.00	0.95	6.57	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.265	0.029	0.438	0.000	0.092	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	530	0	54	0	0	0
N.S.	1	1.00	1.00	7.07	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.271	0.019	0.509	0.000	0.094	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	600	0	54	0	0	0
N.S.	1	1.00	1.00	8.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.270	0.018	0.460	0.000	0.094	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	92	0	0	77	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.78	0.00	0.00	0.00
time (sec)	N/A	0.350	0.115	0.000	0.000	0.094	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	96	0	0	77	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.335	0.114	0.000	0.000	0.110	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	97	90	0	0	77	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.333	0.105	0.000	0.000	0.100	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	90	0	0	69	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.322	0.125	0.000	0.000	0.089	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	64	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.309	0.065	0.000	0.000	0.093	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	91	0	0	77	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.325	0.118	0.000	0.000	0.094	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	95	0	0	77	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.331	0.095	0.000	0.000	0.089	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	122	104	456	168	264	154	189
N.S.	1	1.00	1.37	1.17	5.12	1.89	2.97	1.73	2.12
time (sec)	N/A	0.325	0.564	1.105	0.308	0.270	0.296	0.282	14.166

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	80	77	235	102	151	92	112
N.S.	1	1.00	1.19	1.15	3.51	1.52	2.25	1.37	1.67
time (sec)	N/A	0.296	0.383	1.009	0.265	0.269	0.237	0.290	14.410

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	52	40	91	51	68	44	52
N.S.	1	1.00	1.18	0.91	2.07	1.16	1.55	1.00	1.18
time (sec)	N/A	0.234	0.704	0.817	0.294	0.288	0.177	0.278	0.104

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	55	86	172	71	0	673	0
N.S.	1	1.00	0.85	1.32	2.65	1.09	0.00	10.35	0.00
time (sec)	N/A	0.345	0.216	0.928	0.333	0.271	0.000	0.301	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	78	124	196	105	0	535	0
N.S.	1	1.00	0.88	1.39	2.20	1.18	0.00	6.01	0.00
time (sec)	N/A	0.383	0.482	1.073	0.322	0.264	0.000	0.306	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	224	217	180	949	369	779	335	452
N.S.	1	0.95	0.92	0.76	4.00	1.56	3.29	1.41	1.91
time (sec)	N/A	0.500	1.659	1.568	0.403	0.289	0.444	0.310	15.277

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	193	122	494	212	456	203	255
N.S.	1	1.00	1.15	0.73	2.94	1.26	2.71	1.21	1.52
time (sec)	N/A	0.406	0.781	1.299	0.317	0.299	0.333	0.280	15.196

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	98	80	92	197	98	219	103	117
N.S.	1	0.83	0.68	0.78	1.67	0.83	1.86	0.87	0.99
time (sec)	N/A	0.312	0.932	0.977	0.272	0.287	0.229	0.297	14.941

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	139	114	197	339	145	0	6693	0
N.S.	1	0.96	0.79	1.36	2.34	1.00	0.00	46.16	0.00
time (sec)	N/A	0.563	0.777	1.057	0.446	0.304	0.000	0.428	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	160	206	276	372	219	0	1049	0
N.S.	1	1.01	1.30	1.74	2.34	1.38	0.00	6.60	0.00
time (sec)	N/A	0.538	0.854	1.159	0.465	0.307	0.000	0.448	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	146	151	364	935	422	0	0	0
N.S.	1	1.09	1.13	2.72	6.98	3.15	0.00	0.00	0.00
time (sec)	N/A	0.786	0.342	1.615	0.450	0.305	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	108	125	197	284	226	0	0	0
N.S.	1	1.07	1.24	1.95	2.81	2.24	0.00	0.00	0.00
time (sec)	N/A	0.582	0.359	1.259	0.368	0.279	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	51	70	40	160	58	70	194	65
N.S.	1	1.04	1.43	0.82	3.27	1.18	1.43	3.96	1.33
time (sec)	N/A	0.339	0.088	0.978	0.343	0.276	0.328	0.333	15.456

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	285	27	27	22	22
N.S.	1	1.00	1.10	1.00	14.25	1.35	1.35	1.10	1.10
time (sec)	N/A	0.230	3.932	0.416	0.659	0.280	1.046	0.291	14.949

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	442	51	58	22	22
N.S.	1	1.00	1.10	1.00	22.10	2.55	2.90	1.10	1.10
time (sec)	N/A	0.226	2.508	0.382	0.974	0.260	1.882	0.506	14.914

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	273	250	678	3275	771	0	0	0
N.S.	1	1.01	0.92	2.50	12.08	2.85	0.00	0.00	0.00
time (sec)	N/A	1.213	1.094	2.918	0.997	0.308	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	206	212	358	776	390	0	0	0
N.S.	1	0.97	1.00	1.69	3.66	1.84	0.00	0.00	0.00
time (sec)	N/A	0.919	1.175	2.546	0.607	0.274	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	119	113	74	763	118	146	661	175
N.S.	1	0.97	0.92	0.60	6.20	0.96	1.19	5.37	1.42
time (sec)	N/A	0.486	1.128	1.677	0.354	0.281	0.451	0.494	18.838

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	2913	57	54	22	22
N.S.	1	1.00	1.10	1.00	145.65	2.85	2.70	1.10	1.10
time (sec)	N/A	0.219	11.283	0.699	10.732	0.280	1.772	0.306	14.722

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	3521	99	105	22	22
N.S.	1	1.00	1.10	1.00	176.05	4.95	5.25	1.10	1.10
time (sec)	N/A	0.217	12.478	0.709	19.786	0.283	4.511	0.878	14.159

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	149	164	468	967	467	0	0	0
N.S.	1	1.12	1.23	3.52	7.27	3.51	0.00	0.00	0.00
time (sec)	N/A	0.776	1.590	1.335	0.448	0.295	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	110	292	247	310	283	0	0	0
N.S.	1	1.08	2.86	2.42	3.04	2.77	0.00	0.00	0.00
time (sec)	N/A	0.582	6.458	1.237	0.392	0.275	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	53	57	54	160	59	90	189	65
N.S.	1	1.06	1.14	1.08	3.20	1.18	1.80	3.78	1.30
time (sec)	N/A	0.341	0.866	1.194	0.264	0.302	0.353	0.330	13.633

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	285	28	29	25	23
N.S.	1	1.00	1.10	1.00	13.57	1.33	1.38	1.19	1.10
time (sec)	N/A	0.231	3.628	0.366	0.686	0.293	1.581	0.269	13.474

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	442	52	60	25	23
N.S.	1	1.00	1.10	1.00	21.05	2.48	2.86	1.19	1.10
time (sec)	N/A	0.230	2.516	0.375	0.954	0.266	3.184	0.480	13.597

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	120	53	132	206	0	0	98	83
N.S.	1	1.09	0.48	1.20	1.87	0.00	0.00	0.89	0.75
time (sec)	N/A	0.607	0.334	0.525	0.502	0.000	0.000	0.296	13.774

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	93	44	105	122	0	0	77	63
N.S.	1	1.06	0.50	1.19	1.39	0.00	0.00	0.88	0.72
time (sec)	N/A	0.470	0.247	0.385	0.465	0.000	0.000	0.286	13.779

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	66	34	80	61	0	0	57	46
N.S.	1	1.25	0.64	1.51	1.15	0.00	0.00	1.08	0.87
time (sec)	N/A	0.340	0.385	0.388	0.424	0.000	0.000	0.286	0.230

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	20	32	0	30	33
N.S.	1	1.00	1.12	1.65	0.77	1.23	0.00	1.15	1.27
time (sec)	N/A	0.187	0.079	0.679	0.390	0.310	0.000	0.273	13.148

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	58	55	0	61	0	0	166	0
N.S.	1	0.69	0.65	0.00	0.73	0.00	0.00	1.98	0.00
time (sec)	N/A	0.449	0.315	0.000	0.409	0.000	0.000	0.308	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	81	75	0	193	0	0	560	0
N.S.	1	0.74	0.68	0.00	1.75	0.00	0.00	5.09	0.00
time (sec)	N/A	0.531	0.246	0.000	0.446	0.000	0.000	0.327	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	108	98	0	227	0	0	662	0
N.S.	1	0.72	0.65	0.00	1.50	0.00	0.00	4.38	0.00
time (sec)	N/A	0.639	0.352	0.000	0.538	0.000	0.000	0.314	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	63	33	87	48	0	0	55	91
N.S.	1	0.93	0.49	1.28	0.71	0.00	0.00	0.81	1.34
time (sec)	N/A	0.485	0.134	0.542	0.354	0.000	0.000	0.285	14.227

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	49	29	70	36	0	0	43	70
N.S.	1	0.92	0.55	1.32	0.68	0.00	0.00	0.81	1.32
time (sec)	N/A	0.405	0.101	0.487	0.396	0.000	0.000	0.279	13.609

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	35	22	55	24	0	0	31	50
N.S.	1	1.09	0.69	1.72	0.75	0.00	0.00	0.97	1.56
time (sec)	N/A	0.307	0.074	0.469	0.360	0.000	0.000	0.299	14.284

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	18	25	12	18	0	17	34
N.S.	1	1.00	1.20	1.67	0.80	1.20	0.00	1.13	2.27
time (sec)	N/A	0.171	0.045	0.881	0.478	0.270	0.000	0.282	0.131

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	17	0	0	16	0
N.S.	1	1.00	1.00	0.00	0.74	0.00	0.00	0.70	0.00
time (sec)	N/A	0.281	0.009	0.000	0.489	0.000	0.000	0.296	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	39	33	0	23	0	0	34	0
N.S.	1	0.93	0.79	0.00	0.55	0.00	0.00	0.81	0.00
time (sec)	N/A	0.352	0.117	0.000	0.501	0.000	0.000	0.286	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	56	44	0	19	0	0	48	0
N.S.	1	0.84	0.66	0.00	0.28	0.00	0.00	0.72	0.00
time (sec)	N/A	0.427	0.131	0.000	0.475	0.000	0.000	0.280	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	64	34	86	129	0	0	55	92
N.S.	1	0.89	0.47	1.19	1.79	0.00	0.00	0.76	1.28
time (sec)	N/A	0.507	0.161	0.627	0.383	0.000	0.000	0.282	14.844

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	50	30	69	100	0	0	51	71
N.S.	1	0.89	0.54	1.23	1.79	0.00	0.00	0.91	1.27
time (sec)	N/A	0.410	0.126	0.583	0.353	0.000	0.000	0.289	14.619

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F(-2)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	36	23	54	72	0	0	31	48
N.S.	1	1.06	0.68	1.59	2.12	0.00	0.00	0.91	1.41
time (sec)	N/A	0.309	0.092	0.523	0.336	0.000	0.000	0.279	14.621

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	19	25	23	19	0	26	34
N.S.	1	1.00	1.19	1.56	1.44	1.19	0.00	1.62	2.12
time (sec)	N/A	0.173	0.059	0.941	0.423	0.274	0.000	0.289	14.520

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	0	0	16	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.67	0.00
time (sec)	N/A	0.292	0.086	0.000	0.000	0.000	0.000	0.290	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	40	34	0	0	0	0	34	0
N.S.	1	0.91	0.77	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.368	0.033	0.000	0.000	0.000	0.000	0.471	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	58	45	0	0	0	0	48	0
N.S.	1	0.83	0.64	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.437	0.149	0.000	0.000	0.000	0.000	0.362	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	164	67	0	98	0	0	113	0
N.S.	1	0.89	0.36	0.00	0.53	0.00	0.00	0.61	0.00
time (sec)	N/A	0.991	0.383	0.000	0.365	0.000	0.000	0.346	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	115	54	0	72	0	0	85	0
N.S.	1	0.79	0.37	0.00	0.50	0.00	0.00	0.59	0.00
time (sec)	N/A	0.654	0.296	0.000	0.344	0.000	0.000	0.363	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	73	45	0	48	0	0	59	0
N.S.	1	0.82	0.51	0.00	0.54	0.00	0.00	0.66	0.00
time (sec)	N/A	0.408	0.117	0.000	0.522	0.000	0.000	0.359	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	40	36	0	29	0	0	32	0
N.S.	1	0.73	0.65	0.00	0.53	0.00	0.00	0.58	0.00
time (sec)	N/A	0.348	0.024	0.000	0.393	0.000	0.000	0.285	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	58	53	0	37	0	0	62	0
N.S.	1	0.73	0.67	0.00	0.47	0.00	0.00	0.78	0.00
time (sec)	N/A	0.353	0.139	0.000	0.405	0.000	0.000	0.341	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	C	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	91	66	0	33	0	0	92	0
N.S.	1	0.83	0.61	0.00	0.30	0.00	0.00	0.84	0.00
time (sec)	N/A	0.510	0.065	0.000	0.386	0.000	0.000	0.366	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	374	241	199	0	0	0	0	0	0
N.S.	1	0.64	0.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.843	0.137	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	167	146	0	0	0	0	0	0
N.S.	1	0.64	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.605	0.083	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	103	89	0	0	0	0	0	0
N.S.	1	0.66	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.397	0.054	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	40	56	90	126	0	93	45
N.S.	1	1.00	0.87	1.22	1.96	2.74	0.00	2.02	0.98
time (sec)	N/A	0.205	0.018	0.491	0.386	0.283	0.000	0.349	14.153

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	30	17	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.67	0.94	1.00	1.00
time (sec)	N/A	0.239	1.091	0.172	0.578	0.255	1.864	0.563	14.357

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	235	150	170	0	0	0	0	0	0
N.S.	1	0.64	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.675	0.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	102	117	0	0	0	0	0	0
N.S.	1	0.63	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.514	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	97	64	83	0	0	0	0	0	0
N.S.	1	0.66	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	0.100	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	25	81	87	0	20	0
N.S.	1	1.00	0.97	0.68	2.19	2.35	0.00	0.54	0.00
time (sec)	N/A	0.193	0.073	0.911	0.488	0.264	0.000	0.416	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	13	15	25	15	15	15
N.S.	1	1.00	1.13	0.87	1.00	1.67	1.00	1.00	1.00
time (sec)	N/A	0.236	4.668	0.313	0.519	0.253	1.832	0.783	13.977

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	423	263	257	0	0	0	0	0	0
N.S.	1	0.62	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.061	0.458	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	257	160	185	0	0	0	0	0	0
N.S.	1	0.62	0.72	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.712	0.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	104	165	0	0	0	0	0	0
N.S.	1	0.69	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.453	0.232	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	36	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	2.57	1.00	1.00	1.00
time (sec)	N/A	0.238	11.208	0.400	0.531	0.252	9.327	1.071	14.491

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	0	15	18	18
N.S.	1	1.00	1.11	0.89	1.00	0.00	0.83	1.00	1.00
time (sec)	N/A	0.229	2.246	0.137	0.557	0.000	1.657	0.633	14.846

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	383	351	290	0	0	1030	0	0	0
N.S.	1	0.92	0.76	0.00	0.00	2.69	0.00	0.00	0.00
time (sec)	N/A	1.353	1.718	0.000	0.000	0.359	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	319	255	0	0	1263	0	0	0
N.S.	1	0.97	0.78	0.00	0.00	3.84	0.00	0.00	0.00
time (sec)	N/A	1.236	0.607	0.000	0.000	0.426	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	215	756	414	0	915	0	0	0
N.S.	1	1.00	3.53	1.93	0.00	4.28	0.00	0.00	0.00
time (sec)	N/A	0.797	0.942	0.845	0.000	0.419	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	13	10	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.08	0.83	1.17	1.17
time (sec)	N/A	0.206	1.453	0.201	0.358	0.313	2.084	0.277	14.017

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	296	306	933	674	0	1482	0	0	0
N.S.	1	1.03	3.15	2.28	0.00	5.01	0.00	0.00	0.00
time (sec)	N/A	1.179	10.621	3.345	0.000	0.458	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [16] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	12	12	1.11	14	0.857
2	A	10	10	1.09	14	0.714
3	A	7	7	1.06	14	0.500
4	A	5	5	1.00	12	0.417
5	A	5	5	1.00	14	0.357
6	A	8	8	1.04	14	0.571
7	A	10	10	1.01	14	0.714
8	A	13	13	1.06	14	0.929
9	A	9	9	1.03	16	0.562
10	A	6	6	0.96	16	0.375
11	A	6	6	1.02	16	0.375
12	A	3	3	1.00	14	0.214
13	A	3	3	1.00	16	0.188
14	A	8	8	1.04	16	0.500
15	A	6	6	1.30	16	0.375
16	A	25	24	1.35	16	1.500
17	A	18	18	1.23	16	1.125
18	A	12	11	1.11	16	0.688
19	A	7	7	0.99	14	0.500
20	A	3	3	1.00	16	0.188
21	A	3	3	1.02	16	0.188
22	A	9	9	1.32	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	12	12	1.38	12	1.000
24	A	12	12	1.35	12	1.000
25	A	5	5	1.06	10	0.500
26	A	3	3	1.00	12	0.250
27	A	3	3	1.06	12	0.250
28	A	5	5	1.53	12	0.417
29	A	7	6	1.09	14	0.429
30	A	6	5	1.06	14	0.357
31	A	5	4	1.00	12	0.333
32	N/A	2	0	1.00	14	0.000
33	A	10	9	1.20	16	0.562
34	A	9	8	1.18	16	0.500
35	A	5	5	1.00	14	0.357
36	N/A	2	0	1.00	16	0.000
37	A	11	10	1.06	16	0.625
38	A	9	8	1.07	16	0.500
39	A	7	6	1.01	14	0.429
40	N/A	2	0	1.00	16	0.000
41	A	16	15	1.04	16	0.938
42	A	13	12	1.04	16	0.750
43	A	11	10	1.02	16	0.625
44	A	8	7	1.00	16	0.438
45	A	11	10	1.04	16	0.625
46	A	13	12	1.05	16	0.750
47	A	16	15	1.07	16	0.938
48	A	6	6	1.03	18	0.333
49	A	6	6	1.03	18	0.333
50	A	3	3	1.00	18	0.167
51	A	3	3	1.00	18	0.167
52	A	11	10	1.04	18	0.556
53	A	6	6	1.25	18	0.333
54	A	14	13	1.03	18	0.722
55	A	9	9	1.20	18	0.500

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	20	19	1.40	18	1.056
57	A	17	16	1.42	18	0.889
58	A	3	3	1.00	18	0.167
59	A	3	3	1.00	18	0.167
60	A	3	3	1.06	18	0.167
61	A	12	11	1.52	18	0.611
62	A	15	14	1.42	18	0.778
63	A	9	8	1.00	8	1.000
64	A	7	6	1.00	8	0.750
65	A	4	3	1.00	8	0.375
66	A	7	6	1.00	8	0.750
67	A	9	9	1.05	16	0.562
68	A	6	6	1.05	16	0.375
69	A	6	6	1.05	16	0.375
70	A	4	4	1.00	16	0.250
71	A	4	4	1.00	16	0.250
72	A	6	6	1.05	16	0.375
73	A	6	6	1.06	16	0.375
74	A	9	9	1.06	16	0.562
75	N/A	2	0	1.00	12	0.000
76	A	2	2	1.00	10	0.200
77	N/A	2	0	1.00	14	0.000
78	N/A	4	0	1.00	12	0.000
79	A	4	4	1.00	10	0.400
80	N/A	2	0	1.00	14	0.000
81	A	1	1	1.00	28	0.036
82	N/A	4	0	1.00	10	0.000
83	N/A	2	0	1.00	12	0.000
84	A	2	2	1.00	10	0.200
85	N/A	2	0	1.00	14	0.000
86	N/A	4	0	1.00	12	0.000
87	A	4	4	1.00	10	0.400
88	N/A	2	0	1.00	14	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	1	1	1.00	25	0.040
90	A	1	1	1.00	17	0.059
91	A	1	1	1.00	20	0.050
92	A	1	1	1.00	20	0.050
93	A	1	1	1.00	21	0.048
94	A	1	1	1.00	20	0.050
95	A	1	1	1.00	20	0.050
96	A	1	1	1.00	20	0.050
97	A	1	1	1.00	24	0.042
98	N/A	2	0	1.00	18	0.000
99	A	3	3	1.00	16	0.188
100	A	3	3	1.00	16	0.188
101	A	4	4	1.00	14	0.286
102	N/A	2	0	1.00	14	0.000
103	N/A	2	0	1.00	16	0.000
104	A	4	4	1.00	12	0.333
105	A	4	4	1.00	12	0.333
106	A	4	4	1.00	12	0.333
107	A	4	4	1.00	10	0.400
108	A	4	4	1.00	12	0.333
109	A	4	4	1.00	12	0.333
110	A	4	4	1.00	12	0.333
111	A	3	3	1.00	14	0.214
112	A	3	3	1.00	14	0.214
113	A	3	3	1.00	14	0.214
114	A	3	3	1.00	12	0.250
115	A	3	3	1.00	14	0.214
116	A	3	3	1.00	14	0.214
117	A	3	3	1.00	14	0.214
118	A	3	3	1.00	18	0.167
119	A	3	3	1.00	18	0.167
120	A	3	3	1.00	16	0.188
121	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	3	3	1.00	18	0.167
123	A	3	3	0.95	20	0.150
124	A	3	3	1.00	20	0.150
125	A	3	3	0.83	18	0.167
126	A	5	5	0.96	20	0.250
127	A	5	5	1.01	20	0.250
128	A	12	11	1.09	20	0.550
129	A	11	10	1.07	20	0.500
130	A	7	7	1.04	18	0.389
131	N/A	2	0	1.00	20	0.000
132	N/A	2	0	1.00	20	0.000
133	A	15	14	1.01	20	0.700
134	A	15	14	0.97	20	0.700
135	A	9	9	0.97	18	0.500
136	N/A	2	0	1.00	20	0.000
137	N/A	2	0	1.00	20	0.000
138	A	12	11	1.12	21	0.524
139	A	11	10	1.08	21	0.476
140	A	7	7	1.06	19	0.368
141	N/A	2	0	1.00	21	0.000
142	N/A	2	0	1.00	21	0.000
143	A	12	12	1.09	18	0.667
144	A	9	9	1.06	18	0.500
145	A	7	7	1.25	16	0.438
146	A	2	2	1.00	14	0.143
147	A	7	7	0.69	18	0.389
148	A	10	10	0.74	18	0.556
149	A	12	12	0.72	18	0.667
150	A	12	12	0.93	14	0.857
151	A	9	9	0.92	14	0.643
152	A	7	7	1.09	12	0.583
153	A	2	2	1.00	10	0.200
154	A	4	4	1.00	14	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
155	A	7	7	0.93	14	0.500
156	A	9	9	0.84	14	0.643
157	A	11	11	0.89	15	0.733
158	A	9	9	0.89	15	0.600
159	A	6	6	1.06	13	0.462
160	A	2	2	1.00	11	0.182
161	A	4	4	1.00	15	0.267
162	A	6	6	0.91	15	0.400
163	A	9	9	0.83	15	0.600
164	A	20	20	0.89	14	1.429
165	A	14	13	0.79	14	0.929
166	A	9	9	0.82	12	0.750
167	A	5	5	0.73	14	0.357
168	A	5	5	0.73	14	0.357
169	A	8	8	0.83	14	0.571
170	A	9	8	0.64	18	0.444
171	A	8	7	0.64	18	0.389
172	A	7	6	0.66	16	0.375
173	A	4	3	1.00	14	0.214
174	N/A	2	0	1.00	18	0.000
175	A	9	8	0.64	15	0.533
176	A	8	7	0.63	15	0.467
177	A	7	6	0.66	13	0.462
178	A	4	3	1.00	11	0.273
179	N/A	2	0	1.00	15	0.000
180	A	13	12	0.62	14	0.857
181	A	11	10	0.62	14	0.714
182	A	9	8	0.69	12	0.667
183	N/A	2	0	1.00	14	0.000
184	N/A	2	0	1.00	18	0.000
185	A	10	9	0.92	12	0.750
186	A	9	8	0.97	16	0.500
187	A	8	7	1.00	14	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
188	N/A	2	0	1.00	12	0.000
189	A	13	12	1.03	18	0.667

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int (c+dx)^4 \cos(a+bx)\,dx$

3.1.1 Optimal result

Integrand size = 14, antiderivative size = 91

$$\int (c+dx)^4 \cos(a+bx)\,dx = -\frac{24d^3(c+dx)\cos(a+bx)}{b^4} + \frac{4d(c+dx)^3\cos(a+bx)}{b^2} + \frac{24d^4\sin(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} + \frac{(c+dx)^4\sin(a+bx)}{b}$$

output
```
-24*d^3*(d*x+c)*cos(b*x+a)/b^4+4*d*(d*x+c)^3*cos(b*x+a)/b^2+24*d^4*sin(b*x
+a)/b^5-12*d^2*(d*x+c)^2*sin(b*x+a)/b^3+(d*x+c)^4*sin(b*x+a)/b
```

3.1.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int (c+dx)^4 \cos(a+bx)\,dx = \frac{4bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx) + \left(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4\right)\sin(a+bx)}{b^5}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x],x]
```

output
```
(4*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (24*d^4 - 12*b^
2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Sin[a + b*x])/b^5
```

3.1.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^4\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{4d\int -(c+dx)^3\sin(a+bx)dx}{b}+\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\int (c+dx)^3\sin(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\int (c+dx)^3\sin(a+bx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\int (c+dx)^2\cos(a+bx)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x],x]
```

output
```
((c + d*x)^4*Sin[a + b*x])/b - (4*d*(-(((c + d*x)^3*Cos[a + b*x])/b) + (3*
d*(((c + d*x)^2*Sin[a + b*x])/b - (2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d
*Sin[a + b*x])/b^2))/b))/b))/b
```

3.1. $\int(c+dx)^4\cos(a+bx)\,dx$

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.1.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.57

method	result
parallelrisch	$\frac{-12b\,d^2\left(\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-2d^2\right)x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(2(dx+c)^4b^4-24d^2(dx+c)^2b^2+48d^4\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+8b\left(\left(x^2d^2+\right.\right.}{b^5\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$
risch	$\frac{4d\left(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c\right)\cos(bx+a)}{b^4}+\frac{\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12\right.}{b^5}$
norman	$\frac{\frac{\left(-8b^2c^3d+48c\,d^3\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^4}+\frac{4d^4x^3}{b^2}+\frac{2\left(b^4c^4-12b^2c^2d^2+24d^4\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^5}+\frac{12c\,d^3x^2}{b^2}+\frac{12d^2\left(b^2c^2-2d^2\right)x}{b^4}-\frac{4d^4x^3\left(\tan\right.}{}}{}$
parts	$\frac{\sin(bx+a)d^4x^4}{b}+\frac{4\sin(bx+a)c\,d^3x^3}{b}+\frac{6\sin(bx+a)c^2d^2x^2}{b}+\frac{4\sin(bx+a)c^3dx}{b}+\frac{\sin(bx+a)c^4}{b}-\frac{4d\left(\frac{a^3d^3\cos(bx\right.}{b^3}}{}$
meijerg	$\frac{16d^4\cos(a)\sqrt{\pi}\left(-\frac{x\left(b^2\right)^{\frac{5}{2}}\left(-\frac{5x^2b^2}{2}+15\right)\cos(bx)}{10\sqrt{\pi}\,b^4}+\frac{\left(b^2\right)^{\frac{5}{2}}\left(\frac{5}{8}x^4b^4-\frac{15}{2}x^2b^2+15\right)\sin(bx)}{10\sqrt{\pi}\,b^5}\right)}{b^4\sqrt{b^2}}-\frac{16d^4\sin(a)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}x^4\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^4d^4\sin(bx+a)}{b^4}-\frac{4a^3c\,d^3\sin(bx+a)}{b^3}-\frac{4a^3d^4(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^4}+\frac{6a^2c^2d^2\sin(bx+a)}{b^2}+\frac{12a^2c\,d^3(\cos(bx+a)+(bx+a)\sin}{b^3}}{}$
default	$\frac{\frac{a^4d^4\sin(bx+a)}{b^4}-\frac{4a^3c\,d^3\sin(bx+a)}{b^3}-\frac{4a^3d^4(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^4}+\frac{6a^2c^2d^2\sin(bx+a)}{b^2}+\frac{12a^2c\,d^3(\cos(bx+a)+(bx+a)\sin}{b^3}}{}$

input
```
int((d*x+c)^4*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(-12*b*d^2*((1/3*x^2*d^2+c*d*x+c^2)*b^2-2*d^2)*x*tan(1/2*b*x+1/2*a)^2+(2*(
d*x+c)^4*b^4-24*d^2*(d*x+c)^2*b^2+48*d^4)*tan(1/2*b*x+1/2*a)+8*b*((d^2*x^2
+c*d*x+c^2)*b^2-6*d^2)*d*(1/2*d*x+c))/b^5/(1+tan(1/2*b*x+1/2*a)^2)
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.86

$$\int (c+dx)^4\cos(a+bx)\,dx$$
$$=\frac{4\left(b^3d^4x^3+3\,b^3cd^3x^2+b^3c^3d-6\,bcd^3+3\left(b^3c^2d^2-2\,bd^4\right)x\right)\cos\left(bx+a\right)+\left(b^4d^4x^4+4\,b^4cd^3x^3+b^4c^4-1\right.}{b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a),x, algorithm="fricas")
```

output
```
(4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2
 - 2*b*d^4)*x)*cos(b*x + a) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 1
2*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d -
6*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(92) = 184.

Time = 0.35 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.42

$$\int (c+dx)^4\cos(a+bx)\,dx$$
$$=\begin{cases}\frac{c^4\sin\left(a+bx\right)}{b}+\frac{4c^3dx\sin\left(a+bx\right)}{b}+\frac{6c^2d^2x^2\sin\left(a+bx\right)}{b}+\frac{4cd^3x^3\sin\left(a+bx\right)}{b}+\frac{d^4x^4\sin\left(a+bx\right)}{b}+\frac{4c^3d\cos\left(a+bx\right)}{b^2}+\frac{12c^2d^2x\cos\left(a+\right.}{b^2}\\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\cos\left(a\right)\end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a),x)
```

output
```
Piecewise((c**4*sin(a + b*x)/b + 4*c**3*d*x*sin(a + b*x)/b + 6*c**2*d**2*x
**2*sin(a + b*x)/b + 4*c*d**3*x**3*sin(a + b*x)/b + d**4*x**4*sin(a + b*x)
/b + 4*c**3*d*cos(a + b*x)/b**2 + 12*c**2*d**2*x*cos(a + b*x)/b**2 + 12*c*
d**3*x**2*cos(a + b*x)/b**2 + 4*d**4*x**3*cos(a + b*x)/b**2 - 12*c**2*d**2
*sin(a + b*x)/b**3 - 24*c*d**3*x*sin(a + b*x)/b**3 - 12*d**4*x**2*sin(a +
b*x)/b**3 - 24*c*d**3*cos(a + b*x)/b**4 - 24*d**4*x*cos(a + b*x)/b**4 + 24
*d**4*sin(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2
*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a), True))
```

3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(91) = 182$.

Time = 0.29 (sec) , antiderivative size = 481, normalized size of antiderivative = 5.29

$$\int (c+dx)^4 \cos(a+bx)\,dx$$

$$= \frac{c^4 \sin(bx+a) - \frac{4\,ac^3d\sin(bx+a)}{b} + \frac{6\,a^2c^2d^2\sin(bx+a)}{b^2} - \frac{4\,a^3cd^3\sin(bx+a)}{b^3} + \frac{a^4d^4\sin(bx+a)}{b^4} + \frac{4\,((bx+a)\sin(bx+a)+\cos(bx+a))}{b}}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a),x, algorithm="maxima")
```

output
```
(c^4*sin(b*x + a) - 4*a*c^3*d*sin(b*x + a)/b + 6*a^2*c^2*d^2*sin(b*x + a)/
b^2 - 4*a^3*c*d^3*sin(b*x + a)/b^3 + a^4*d^4*sin(b*x + a)/b^4 + 4*((b*x +
a)*sin(b*x + a) + cos(b*x + a))*c^3*d/b - 12*((b*x + a)*sin(b*x + a) + cos
(b*x + a))*a*c^2*d^2/b^2 + 12*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a^2*
c*d^3/b^3 - 4*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a^3*d^4/b^4 + 6*(2*(
b*x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 12*(
2*(b*x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 + 6
*(2*(b*x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 +
 4*(3*((b*x + a)^2 - 2)*cos(b*x + a) + ((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x
 + a))*c*d^3/b^3 - 4*(3*((b*x + a)^2 - 2)*cos(b*x + a) + ((b*x + a)^3 - 6*
b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 + (4*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*
x + a) + ((b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.87

$$\int (c+dx)^4\cos(a+bx)\,dx$$
$$= \frac{4\,(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3)\cos{(bx+a)}}{b^5} + \frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4)\sin{(bx+}}{b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a),x, algorithm="giac")
```

output
```
4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x
 - 6*b*c*d^3)*cos(b*x + a)/b^5 + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^
2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12
*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

3.1.9 Mupad [B] (verification not implemented)

Time = 14.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.41

$$\int (c+dx)^4\cos(a+bx)\,dx = \frac{\sin{(a+b\,x)}\,\left(b^4\,c^4-12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5} - \frac{4\,\cos{(a+b\,x)}\,\left(6\,c\,d^3-b^2\,c^3\,d\right)}{b^4} + \frac{4\,d^4\,x^3\,\cos{(a+b\,x)}}{b^2} - \frac{12\,x\,\cos{(a+b\,x)}\,\left(2\,d^4-b^2\,c^2\,d^2\right)}{b^4} + \frac{d^4\,x^4\,\sin{(a+b\,x)}}{b} - \frac{4\,x\,\sin{(a+b\,x)}\,\left(6\,c\,d^3-b^2\,c^3\,d\right)}{b^3} - \frac{6\,x^2\,\sin{(a+b\,x)}\,\left(2\,d^4-b^2\,c^2\,d^2\right)}{b^3} + \frac{12\,c\,d^3\,x^2\,\cos{(a+b\,x)}}{b^2} + \frac{4\,c\,d^3\,x^3\,\sin{(a+b\,x)}}{b}$$

input
```
int(cos(a + b*x)*(c + d*x)^4,x)
```

output
```
(sin(a + b*x)*(24*d^4 + b^4*c^4 - 12*b^2*c^2*d^2))/b^5 - (4*cos(a + b*x)*(
6*c*d^3 - b^2*c^3*d))/b^4 + (4*d^4*x^3*cos(a + b*x))/b^2 - (12*x*cos(a + b
*x)*(2*d^4 - b^2*c^2*d^2))/b^4 + (d^4*x^4*sin(a + b*x))/b - (4*x*sin(a + b
*x)*(6*c*d^3 - b^2*c^3*d))/b^3 - (6*x^2*sin(a + b*x)*(2*d^4 - b^2*c^2*d^2)
)/b^3 + (12*c*d^3*x^2*cos(a + b*x))/b^2 + (4*c*d^3*x^3*sin(a + b*x))/b
```

3.2 $\int (c+dx)^3 \cos(a+bx)\,dx$

3.2.1 Optimal result

Integrand size = 14, antiderivative size = 70

$$\int (c+dx)^3\cos(a+bx)\,dx = -\frac{6d^3\cos(a+bx)}{b^4}+\frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{6d^2(c+dx)\sin(a+bx)}{b^3}+\frac{(c+dx)^3\sin(a+bx)}{b}$$

output
```
-6*d^3*cos(b*x+a)/b^4+3*d*(d*x+c)^2*cos(b*x+a)/b^2-6*d^2*(d*x+c)*sin(b*x+a
)/b^3+(d*x+c)^3*sin(b*x+a)/b
```

3.2.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (c+dx)^3\cos(a+bx)\,dx = \frac{3d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\sin(a+bx)}{b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x],x]
```

output
```
(3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + b*(c + d*x)*(-6*d^2 + b^2*(
c + d*x)^2)*Sin[a + b*x])/b^4
```

3.2.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{3d\int -(c+dx)^2\sin(a+bx)dx}{b}+\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int (c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x],x]
```

output
```
((c + d*x)^3*Sin[a + b*x])/b - (3*d*(-(((c + d*x)^2*Cos[a + b*x])/b) + (2*
d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b))/b
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.2.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.53

method	result
risch	$\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{b^4}+\frac{(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c)\sin(bx+a)}{b^3}$
parallelrisch	$\frac{-6b^2d^2\left(\frac{dx}{2}+c\right)x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2b(dx+c)\left((dx+c)^2b^2-6d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+6d\left(\left(\frac{1}{2}x^2d^2+cdx+c^2\right)b^2-2d^2\right)}{b^4\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$
norman	$\frac{\frac{(-6b^2c^2d+12d^3)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^4}+\frac{3d^3x^2}{b^2}+\frac{6c\,d^2x}{b^2}+\frac{2c(b^2c^2-6d^2)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{2d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{3d^3x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
parts	$\frac{\sin(bx+a)d^3x^3}{b}+\frac{3\sin(bx+a)c\,d^2x^2}{b}+\frac{3\sin(bx+a)c^2dx}{b}+\frac{\sin(bx+a)c^3}{b}-\frac{3d\left(-\frac{a^2d^2\cos(bx+a)}{b^2}+\frac{2acd\cos(bx+a)}{b}-\right.}{}$
derivativedivides	$\frac{-\frac{a^3d^3\sin(bx+a)}{b^3}+\frac{3a^2c\,d^2\sin(bx+a)}{b^2}+\frac{3a^2d^3(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^3}-\frac{3a\,c^2d\sin(bx+a)}{b}-\frac{6ac\,d^2(\cos(bx+a)+(bx+a)\sin(bx}{b^2}}{}$
default	$\frac{-\frac{a^3d^3\sin(bx+a)}{b^3}+\frac{3a^2c\,d^2\sin(bx+a)}{b^2}+\frac{3a^2d^3(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^3}-\frac{3a\,c^2d\sin(bx+a)}{b}-\frac{6ac\,d^2(\cos(bx+a)+(bx+a)\sin(bx}{b^2}}{}$
meijerg	$\frac{8d^3\cos(a)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3x^2b^2}{2}+3\right)\cos(bx)}{4\sqrt{\pi}}-\frac{xb\left(-\frac{x^2b^2}{2}+3\right)\sin(bx)}{4\sqrt{\pi}}\right)}{b^4}-\frac{8d^3\sin(a)\sqrt{\pi}\left(\frac{xb\left(-\frac{5x^2b^2}{2}+15\right)\cos(bx)}{20\sqrt{\pi}}-\frac{\left(-\frac{15}{}\right.}{}\right.}{b^4}$

input
```
int((d*x+c)^3*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
3*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*cos(b*x+a)+1/b^3*(b^2*d^3*
x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^2)*sin(b*x+a)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int (c+dx)^3\cos(a+bx)\,dx$$
$$=\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)+\left(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3-6\,bcd^2+3\left(b^3c^2d-2\,bd^3\right)\right.}{b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a),x, algorithm="fricas")
```

output
```
(3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x
)*sin(b*x + a))/b^4
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.89

$$\int (c + dx)^3 \cos(a + bx) \, dx$$
$$= \begin{cases} \frac{c^3 \sin(a+bx)}{b} + \frac{3c^2 dx \sin(a+bx)}{b} + \frac{3cd^2x^2 \sin(a+bx)}{b} + \frac{d^3x^3 \sin(a+bx)}{b} + \frac{3c^2 d \cos(a+bx)}{b^2} + \frac{6cd^2 x \cos(a+bx)}{b^2} + \frac{3d^3x^2 \cos(a+bx)}{b^2} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \cos(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a),x)
```

output
```
Piecewise((c**3*sin(a + b*x)/b + 3*c**2*d*x*sin(a + b*x)/b + 3*c*d**2*x**2
*sin(a + b*x)/b + d**3*x**3*sin(a + b*x)/b + 3*c**2*d*cos(a + b*x)/b**2 +
6*c*d**2*x*cos(a + b*x)/b**2 + 3*d**3*x**2*cos(a + b*x)/b**2 - 6*c*d**2*si
n(a + b*x)/b**3 - 6*d**3*x*sin(a + b*x)/b**3 - 6*d**3*cos(a + b*x)/b**4, N
e(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos(a),
True))
```

3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(70) = 140$.

Time = 0.23 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.97

$$\int (c + dx)^3 \cos(a + bx) \, dx$$
$$= \frac{c^3 \sin(bx+a) - \frac{3\,ac^2 d \sin(bx+a)}{b} + \frac{3\,a^2cd^2 \sin(bx+a)}{b^2} - \frac{a^3d^3 \sin(bx+a)}{b^3} + \frac{3\,((bx+a)\sin(bx+a)+\cos(bx+a))c^2d}{b} - \frac{6\,((bx+a)\sin(b}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a),x, algorithm="maxima")
```

output
```
(c^3*sin(b*x + a) - 3*a*c^2*d*sin(b*x + a)/b + 3*a^2*c*d^2*sin(b*x + a)/b^
2 - a^3*d^3*sin(b*x + a)/b^3 + 3*((b*x + a)*sin(b*x + a) + cos(b*x + a))*c
^2*d/b - 6*((b*x + a)*sin(b*x + a) + cos(b*x + a))*a*c*d^2/b^2 + 3*((b*x +
 a)*sin(b*x + a) + cos(b*x + a))*a^2*d^3/b^3 + 3*(2*(b*x + a)*cos(b*x + a)
 + ((b*x + a)^2 - 2)*sin(b*x + a))*c*d^2/b^2 - 3*(2*(b*x + a)*cos(b*x + a)
 + ((b*x + a)^2 - 2)*sin(b*x + a))*a*d^3/b^3 + (3*((b*x + a)^2 - 2)*cos(b*
x + a) + ((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.57

$$\int (c+dx)^3\cos(a+bx)\,dx = \frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)}{b^4} + \frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2\right)\sin\left(bx+a\right)}{b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a),x, algorithm="giac")
```

output
```
3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)/b^4 + (b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*
d^2)*sin(b*x + a)/b^4
```

3.2.9 Mupad [B] (verification not implemented)

Time = 14.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.10

$$\int (c+dx)^3\cos(a+bx)\,dx = \frac{3\,d^3\,x^2\,\cos\left(a+b\,x\right)}{b^2} - \frac{\sin\left(a+b\,x\right)\,\left(6\,c\,d^2-b^2\,c^3\right)}{b^3} - \frac{3\,\cos\left(a+b\,x\right)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^4} + \frac{d^3\,x^3\,\sin\left(a+b\,x\right)}{b} - \frac{3\,x\,\sin\left(a+b\,x\right)\,\left(2\,d^3-b^2\,c^2\,d\right)}{b^3} + \frac{6\,c\,d^2\,x\,\cos\left(a+b\,x\right)}{b^2} + \frac{3\,c\,d^2\,x^2\,\sin\left(a+b\,x\right)}{b}$$

input
```
int(cos(a + b*x)*(c + d*x)^3,x)
```

output
```
(3*d^3*x^2*cos(a + b*x))/b^2 - (sin(a + b*x)*(6*c*d^2 - b^2*c^3))/b^3 - (3
*cos(a + b*x)*(2*d^3 - b^2*c^2*d))/b^4 + (d^3*x^3*sin(a + b*x))/b - (3*x*s
in(a + b*x)*(2*d^3 - b^2*c^2*d))/b^3 + (6*c*d^2*x*cos(a + b*x))/b^2 + (3*c
*d^2*x^2*sin(a + b*x))/b
```

3.3 $\int (c+dx)^2 \cos(a+bx)\,dx$

3.3.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (c+dx)^2 \cos(a+bx)\,dx = \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{2d^2\sin(a+bx)}{b^3} + \frac{(c+dx)^2\sin(a+bx)}{b}$$

output
```
2*d*(d*x+c)*cos(b*x+a)/b^2-2*d^2*sin(b*x+a)/b^3+(d*x+c)^2*sin(b*x+a)/b
```

3.3.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \cos(a+bx)\,dx = \frac{2bd(c+dx)\cos(a+bx) + (-2d^2 + b^2(c+dx)^2)\sin(a+bx)}{b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x],x]
```

output
```
(2*b*d*(c + d*x)*Cos[a + b*x] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a + b*x])/b
^3
```

3.3.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{2d\int-((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin(a+bx+\frac{\pi}{2})dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x],x]
```

output
```
((c + d*x)^2*Sin[a + b*x])/b - (2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Si
n[a + b*x])/b^2))/b
```

3.3.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

method	result
risch	$\frac{2d(dx+c)\cos(bx+a)}{b^2}+\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{b^3}$
parallelrisch	$\frac{-2d^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)xb+\left(2(dx+c)^2b^2-4d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+4bd\left(\frac{dx}{2}+c\right)}{b^3\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$
parts	$\frac{\sin(bx+a)x^2d^2}{b}+\frac{2\sin(bx+a)cdx}{b}+\frac{\sin(bx+a)c^2}{b}-\frac{2d\left(\frac{da\cos(bx+a)}{b}-c\cos(bx+a)+\frac{d(\sin(bx+a)-(bx+a)\cos(bx+a))}{b}\right)}{b^2}$
norman	$\frac{\frac{4cd}{b^2}+\frac{2d^2x}{b^2}+\frac{2\left(b^2c^2-2d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{2d^2x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2d^2x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{4cdx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
derivativedivides	$\frac{\frac{a^2d^2\sin(bx+a)}{b^2}-\frac{2acd\sin(bx+a)}{b}-\frac{2a\,d^2(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^2}+c^2\sin(bx+a)+\frac{2cd(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}+\frac{d^2(}{}}{b}$
default	$\frac{\frac{a^2d^2\sin(bx+a)}{b^2}-\frac{2acd\sin(bx+a)}{b}-\frac{2a\,d^2(\cos(bx+a)+(bx+a)\sin(bx+a))}{b^2}+c^2\sin(bx+a)+\frac{2cd(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}+\frac{d^2(}{}}{b}$
meijerg	$\frac{4d^2\cos(a)\sqrt{\pi}\left(\frac{x\left(b^2\right)^{\frac{3}{2}}\cos(bx)}{2\sqrt{\pi}\,b^2}-\frac{\left(b^2\right)^{\frac{3}{2}}\left(-\frac{3x^2b^2}{2}+3\right)\sin(bx)}{6\sqrt{\pi}\,b^3}\right)}{b^2\sqrt{b^2}}-\frac{4d^2\sin(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-\frac{x^2b^2}{2}+1\right)\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(bx}{2\sqrt{\pi}}\right.}{b^3}$

input
```
int((d*x+c)^2*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*d*(d*x+c)*cos(b*x+a)/b^2+(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^3*sin
(b*x+a)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int (c+dx)^2\cos(a+bx)\,dx = \frac{2\,(bd^2x+bcd)\cos{(bx+a)}+(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin{(bx+a)}}{b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a),x, algorithm="fricas")
```

output
```
(2*(b*d^2*x + b*c*d)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 -
 2*d^2)*sin(b*x + a))/b^3
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (c+dx)^2\cos(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2\sin(a+bx)}{b} + \frac{2cdx\sin(a+bx)}{b} + \frac{d^2x^2\sin(a+bx)}{b} + \frac{2cd\cos(a+bx)}{b^2} + \frac{2d^2x\cos(a+bx)}{b^2} - \frac{2d^2\sin(a+bx)}{b^3} & \text{for } b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a),x)
```

output
```
Piecewise((c**2*sin(a + b*x)/b + 2*c*d*x*sin(a + b*x)/b + d**2*x**2*sin(a
+ b*x)/b + 2*c*d*cos(a + b*x)/b**2 + 2*d**2*x*cos(a + b*x)/b**2 - 2*d**2*s
in(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cos(a), Tr
ue))
```

3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(49) = 98$.

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.78

$$\int (c+dx)^2\cos(a+bx)\,dx$$
$$= \frac{c^2\sin(bx+a) - \frac{2\,acd\sin(bx+a)}{b} + \frac{a^2d^2\sin(bx+a)}{b^2} + \frac{2\,((bx+a)\sin(bx+a)+\cos(bx+a))cd}{b} - \frac{2\,((bx+a)\sin(bx+a)+\cos(bx+a))ad^2}{b^2} +}{b}$$

input
```
integrate((d*x+c)^2*cos(b*x+a),x, algorithm="maxima")
```

output
```
(c^2*sin(b*x + a) - 2*a*c*d*sin(b*x + a)/b + a^2*d^2*sin(b*x + a)/b^2 + 2*
((b*x + a)*sin(b*x + a) + cos(b*x + a))*c*d/b - 2*((b*x + a)*sin(b*x + a)
+ cos(b*x + a))*a*d^2/b^2 + (2*(b*x + a)*cos(b*x + a) + ((b*x + a)^2 - 2)*
sin(b*x + a))*d^2/b^2)/b
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int (c+dx)^2 \cos(a+bx)\,dx = \frac{2\,(bd^2x+bcd)\cos(bx+a)}{b^3} + \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a),x, algorithm="giac")
```

output
```
2*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 - 2*d^2)*sin(b*x + a)/b^3
```

3.3.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.71

$$\int (c+dx)^2 \cos(a+bx)\,dx = \frac{d^2\,x^2\,\sin(a+b\,x)}{b} - \frac{\sin(a+b\,x)\,(2\,d^2-b^2\,c^2)}{b^3} + \frac{2\,c\,d\,\cos(a+b\,x)}{b^2} + \frac{2\,d^2\,x\,\cos(a+b\,x)}{b^2} + \frac{2\,c\,d\,x\,\sin(a+b\,x)}{b}$$

input
```
int(cos(a + b*x)*(c + d*x)^2,x)
```

output
```
(d^2*x^2*sin(a + b*x))/b - (sin(a + b*x)*(2*d^2 - b^2*c^2))/b^3 + (2*c*d*c
os(a + b*x))/b^2 + (2*d^2*x*cos(a + b*x))/b^2 + (2*c*d*x*sin(a + b*x))/b
```

3.4 $\int (c+dx)\cos(a+bx)\,dx$

3.4.1 Optimal result

Integrand size = 12, antiderivative size = 27

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(a+bx)}{b^2} + \frac{(c+dx)\sin(a+bx)}{b}$$

output
```
d*cos(b*x+a)/b^2+(d*x+c)*sin(b*x+a)/b
```

3.4.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(a+bx)+b(c+dx)\sin(a+bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x],x]
```

output
```
(d*Cos[a + b*x] + b*(c + d*x)*Sin[a + b*x])/b^2
```

3.4.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 3777$$

$$\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}$$

$$\Big\downarrow \ 3118$$

$$\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}$$

input
```
Int[(c + d*x)*Cos[a + b*x],x]
```

output
```
(d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.4.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result
risch	$\frac{d\cos(bx+a)}{b^2}+\frac{(dx+c)\sin(bx+a)}{b}$
parallelrisch	$\frac{(dx+c)b\sin(bx+a)+d(\cos(bx+a)-1)}{b^2}$
parts	$\frac{\sin(bx+a)dx}{b}+\frac{\sin(bx+a)c}{b}+\frac{d\cos(bx+a)}{b^2}$
derivativedivides	$\frac{-\frac{da\sin(bx+a)}{b}+c\sin(bx+a)+\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}}{b}$
default	$\frac{-\frac{da\sin(bx+a)}{b}+c\sin(bx+a)+\frac{d(\cos(bx+a)+(bx+a)\sin(bx+a))}{b}}{b}$
norman	$\frac{\frac{2d}{b^2}+\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)}$
meijerg	$\frac{2d\cos(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(bx)}{2\sqrt{\pi}}+\frac{xb\sin(bx)}{2\sqrt{\pi}}\right)}{b^2}-\frac{2d\sin(a)\sqrt{\pi}\left(-\frac{xb\cos(bx)}{2\sqrt{\pi}}+\frac{\sin(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{c\cos(a)\sin(bx)}{b}-\frac{c\sin(a)\sqrt{\pi}\left(\right.}{}$

input
```
int((d*x+c)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
d*cos(b*x+a)/b^2+(d*x+c)*sin(b*x+a)/b
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(bx+a)+(bdx+bc)\sin(bx+a)}{b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a),x, algorithm="fricas")
```

output
```
(d*cos(b*x + a) + (b*d*x + b*c)*sin(b*x + a))/b^2
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int (c+dx)\cos(a+bx)\,dx = \begin{cases} \frac{c\sin(a+bx)}{b}+\frac{dx\sin(a+bx)}{b}+\frac{d\cos(a+bx)}{b^2} & \text{for } b\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\cos(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a),x)
```

output
```
Piecewise((c*sin(a + b*x)/b + d*x*sin(a + b*x)/b + d*cos(a + b*x)/b**2, Ne
(b, 0)), ((c*x + d*x**2/2)*cos(a), True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int (c+dx)\cos(a+bx)\,dx = \frac{c\sin(bx+a)-\frac{ad\sin(bx+a)}{b}+\frac{((bx+a)\sin(bx+a)+\cos(bx+a))d}{b}}{b}$$

input
```
integrate((d*x+c)*cos(b*x+a),x, algorithm="maxima")
```

output
```
(c*sin(b*x + a) - a*d*sin(b*x + a)/b + ((b*x + a)*sin(b*x + a) + cos(b*x +
 a))*d/b)/b
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int (c+dx)\cos(a+bx)\,dx = \frac{d\cos(bx+a)}{b^2} + \frac{(bdx+bc)\sin(bx+a)}{b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a),x, algorithm="giac")
```

output
```
d*cos(b*x + a)/b^2 + (b*d*x + b*c)*sin(b*x + a)/b^2
```

3.4.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int (c+dx)\cos(a+bx)\,dx = \frac{c\,\sin(a+b\,x)+d\,x\,\sin(a+b\,x)}{b} + \frac{d\,\cos(a+b\,x)}{b^2}$$

input
```
int(cos(a + b*x)*(c + d*x),x)
```

output
```
(c*sin(a + b*x) + d*x*sin(a + b*x))/b + (d*cos(a + b*x))/b^2
```

3.5 $\int \frac{\cos(a+bx)}{c+dx}\,dx$

3.5.1 Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{\cos(a+bx)}{c+dx}\,dx=\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

output
```
Ci(b*c/d+b*x)*cos(a-b*c/d)/d-Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\cos(a+bx)}{c+dx}\,dx=\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)-\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x),x]
```

output
```
(Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x] - Sin[a - (b*c)/d]*SinIntegra
l[(b*c)/d + b*x])/d
```

3.5.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3780}$$

$$\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[Cos[a + b*x]/(c + d*x),x]
```

output
```
(Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d - (Sin[a - (b*c)/d]*SinInt
egral[(b*c)/d + b*x])/d
```

3.5.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.5.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

method	result	size
derivativedivides	$-\frac{\operatorname{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}+\frac{\operatorname{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}$	77
default	$-\frac{\operatorname{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}+\frac{\operatorname{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}$	77
risch	$-\frac{\mathrm{e}^{-\frac{i(ad-bc)}{d}}\operatorname{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{2d}-\frac{\mathrm{e}^{\frac{i(ad-bc)}{d}}\operatorname{Ei}_1\left(-ixb-ia-\frac{-iad+ibc}{d}\right)}{2d}$	96

input
```
int(cos(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-
a*d+b*c)/d)/d
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \frac{\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)-\sin\left(-\frac{bc-ad}{d}\right)\operatorname{Si}\left(\frac{bdx+bc}{d}\right)}{d}$$

input
```
integrate(cos(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
(cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - sin(-(b*c - a*d)/d)*s
in_integral((b*d*x + b*c)/d))/d
```

3.5.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x), x)
```

3.5.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.73

$$\int \frac{\cos(a+bx)}{c+dx}\,dx =$$
$$-\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,b}{}\right.\right.}{2\,bd}$$

input
```
integrate(cos(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integr
al_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(I*ex
p_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(1, -
(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/(b*d)
```

3.5.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 577, normalized size of antiderivative = 11.10

$$\int \frac{\cos(a+bx)}{c+dx}\,dx$$
$$= \frac{\Re\left(\mathrm{Ci}\left(bx+\frac{bc}{d}\right)\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{bc}{2d}\right)^2+\Re\left(\mathrm{Ci}\left(-bx-\frac{bc}{d}\right)\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{bc}{2d}\right)^2-2\,\Im\left(\mathrm{Ci}\left(bx+\frac{bc}{d}\right)\right)\tan\left(\frac{1}{2}\right.}{}$$

input
```
integrate(cos(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 +
real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*im
ag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 4*sin_integra
l((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x
- b*c/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*ta
n(1/2*a)*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(1/2*a
)^2 - real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 + 4*real_part(cos
_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 4*real_part(cos_integr
al(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - real_part(cos_integral(b*x +
 b*c/d))*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b*x - b*c/d))*tan(1/2*
b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) + 2*imag_part
(cos_integral(-b*x - b*c/d))*tan(1/2*a) - 4*sin_integral((b*d*x + b*c)/d)*
tan(1/2*a) + 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) - 2*ima
g_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) + 4*sin_integral((b*d*x
+ b*c)/d)*tan(1/2*b*c/d) + real_part(cos_integral(b*x + b*c/d)) + real_par
t(cos_integral(-b*x - b*c/d)))/(d*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/
2*a)^2 + d*tan(1/2*b*c/d)^2 + d)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x),x)
```

output
```
int(cos(a + b*x)/(c + d*x), x)
```

3.6 $\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx$

3.6.1 Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx=-\frac{\cos(a+bx)}{d(c+dx)}-\frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

output
```
-cos(b*x+a)/d/(d*x+c)-b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2-b*Ci(b*c/d+b*x)*sin
(a-b*c/d)/d^2
```

3.6.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = -\frac{\frac{d\cos(a+bx)}{c+dx}+b\,\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^2,x]
```

output
```
-(((d*Cos[a + b*x])/(c + d*x) + b*CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d
] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])/d^2)
```

3.6.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{b\int -\frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 3784}$$

$$-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 3780}$$

$$-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^2,x]
```

output
```
-(Cos[a + b*x]/(d*(c + d*x))) - (b*((CosIntegral[(b*c)/d + b*x]*Sin[a - (b
*c)/d])/d + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d))/d
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.6.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

method	result	size
derivativedivides	$b\left(-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}}{d}\right)$	114
default	$b\left(-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d}}{d}\right)$	114
risch	$\frac{ib\,\mathrm{e}^{-\frac{i(ad-bc)}{d}}\mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{2d^2}-\frac{ib\,\mathrm{e}^{\frac{i(ad-bc)}{d}}\mathrm{Ei}_1\left(-ixb-ia-\frac{-iad+ibc}{d}\right)}{2d^2}-\frac{(-2bxd-2bc)\cos(bx+a)}{2d(dx+c)(-bxd-bc)}$	140

input
```
int(cos(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-cos(b*x+a)/(-a*d+b*c+d*(b*x+a))/d-(-Si(-b*x-a-(-a*d+b*c)/d)*cos((-a*d+
b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = -\frac{(bdx+bc)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)+d\cos(bx+a)}{d^3x+cd^2}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + (b*d*x
 + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + d*cos(b*x + a)
)/(d^3*x + c*d^2)
```

3.6.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**2, x)
```

3.6.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.25

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = -\frac{b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(-i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_2\left(\right.\right.}{2\,(bcd+(bx+a)d^2-ad^2)b}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2*(
-I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e
(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b*c*d + (b
*x + a)*d^2 - a*d^2)*b)
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(73) = 146.

Time = 0.31 (sec) , antiderivative size = 523, normalized size of antiderivative = 7.16

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = -\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\operatorname{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+b^3c\operatorname{Ci}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right.}{d}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(((d*x + c
)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d)
+ b^3*c*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d)*sin(-(b*c - a*d)/d) - a*b^2*d*cos_integral(((d*x + c)*(b - b*c/(
d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) - (d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)/d)*sin_integral(-
((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) - b^3*c*cos
(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
c)) + b*c - a*d)/d) + a*b^2*d*cos(-(b*c - a*d)/d)*sin_integral(-((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + b^2*d*cos(-(d*x + c
)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/((((d*x + c)*(b - b*c/(d*x +
c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)/(c + d*x)^2, x)
```

3.7 $\int \frac{\cos(a+bx)}{(c+dx)^3} dx$

3.7.1 Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{2d^3} + \frac{b\sin(a+bx)}{2d^2(c+dx)} + \frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

output
```
-1/2*b^2*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^3-1/2*cos(b*x+a)/d/(d*x+c)^2+1/2*b^2
*Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+1/2*b*sin(b*x+a)/d^2/(d*x+c)
```

3.7.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = \frac{-b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) + \frac{d(-d\cos(a+bx)+b(c+dx)\sin(a+bx))}{(c+dx)^2} + b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^3,x]
```

output
```
(-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + (d*(-(d*Cos[a + b*x])
+ b*(c + d*x)*Sin[a + b*x]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegra
l[b*(c/d + x)])/(2*d^3)
```

3.7.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3778, 25, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{b\int -\frac{\sin(a+bx)}{(c+dx)^2}dx}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}dx}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}dx}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{b\left(\frac{b\int \frac{\cos(a+bx)}{c+dx}dx}{d}-\frac{\sin(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\frac{b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}-\frac{\sin(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{b\left(\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{b\left(\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\sin(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{b\left(\frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\sin(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\cos(a+bx)}{2d(c+dx)^2}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*Cos[a + b*x]/(d*(c + d*x)^2) - (b*(-(Sin[a + b*x]/(d*(c + d*x))) + (b
*((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d - (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d))/d))/(2*d)
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.42

method	result
derivativedivides	$b^2 \Bigg(-\frac{\cos(bx+a)}{2(-ad+bc+d(bx+a))^2 d} - \frac{-\frac{\sin(bx+a)}{(-ad+bc+d(bx+a))d} + \frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d} + \frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos}{d}}{d}}{2d}$
default	$b^2 \Bigg(-\frac{\cos(bx+a)}{2(-ad+bc+d(bx+a))^2 d} - \frac{-\frac{\sin(bx+a)}{(-ad+bc+d(bx+a))d} + \frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\sin\left(\frac{-ad+bc}{d}\right)}{d} + \frac{\mathrm{Ci}\left(bx+a+\frac{-ad+bc}{d}\right)\cos}{d}}{d}}{2d}$
risch	$\frac{b^2 \mathrm{e}^{-\frac{i(ad-bc)}{d}} \mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{4d^3} + \frac{b^2 \mathrm{e}^{\frac{i(ad-bc)}{d}} \mathrm{Ei}_1\left(-ixb-ia-\frac{-iad+ibc}{d}\right)}{4d^3} + \frac{(-2b^2d^3x^2-4b^2c\,d^2x-2b^2c^2d)\,\mathrm{co}}{4d^2(dx+c)^2(x^2d^2b^2+2b^2cdx+}$

input
```
int(cos(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*(-1/2*cos(b*x+a)/(-a*d+b*c+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-a*d+b*c+d
*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b
*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.59

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{d^2\cos(bx+a) + (b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right) - (b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\sin\left(-\frac{b}{}\right.}{2\,(d^5x^2 + 2\,cd^4x + c^2d^3)}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(d^2*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c -
 a*d)/d)*cos_integral((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - (b*d^2*x + b*c*d)
*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.7.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**3, x)
```

3.7.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.91

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = -\frac{b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) + E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) + b^3\left(-i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) + i\,E_3\left(\right.\right.}{2\left(b^2c^2d - 2\,abcd^2 + (bx+a)^2d^3 + a^2d^3 + 2\,(bcd^2 - ad^3)(bx+a)\right)b}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3*(
-I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e
(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*b)
```

3.7.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 5518, normalized size of antiderivative = 53.06

$$\int \frac{\cos(a + bx)}{(c + dx)^3} \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/4*(b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/
d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part
(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2
*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*
a)^2*tan(1/2*b*c/d) - 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*
b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*imag_part(cos_integral(
b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c
/d)^2 + 4*b^2*d^2*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2
*a)*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(b*x + b*c/d))*ta
n(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_int
egral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - b^2*d^
2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2 - b
^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)
^2 + 4*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan
(1/2*a)*tan(1/2*b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d
))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^2*c*d*x*imag_part(cos_in
tegral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*b^2*c*
d*x*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*t...
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)/(c + d*x)^3, x)
```

3.8 $\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$

3.8.1 Optimal result

Integrand size = 14, antiderivative size = 127

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = -\frac{\cos(a+bx)}{3d(c+dx)^3}+\frac{b^2\cos(a+bx)}{6d^3(c+dx)}+\frac{b^3\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{6d^4}+\frac{b\sin(a+bx)}{6d^2(c+dx)^2}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{6d^4}$$

output
```
-1/3*cos(b*x+a)/d/(d*x+c)^3+1/6*b^2*cos(b*x+a)/d^3/(d*x+c)+1/6*b^3*cos(a-b
*c/d)*Si(b*c/d+b*x)/d^4+1/6*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^4+1/6*b*sin(b
*x+a)/d^2/(d*x+c)^2
```

3.8.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = \frac{d\cos(bx)\left(\left(-2d^2+b^2(c+dx)^2\right)\cos(a)+bd(c+dx)\sin(a)\right)+d\left(bd(c+dx)\cos(a)-\left(-2d^2+b^2(c+dx)^2\right)\right.}{6d^4(c+dx}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^4,x]
```

output
```
(d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d
*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3
*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]
*SinIntegral[b*(c/d + x)]))/(6*d^4*(c + d*x)^3)
```

3.8.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^4} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{b\int -\frac{\sin(a+bx)}{(c+dx)^3}dx}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}dx}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}dx}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{b\left(\frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}dx}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\frac{b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$\Big\downarrow$ 3778

$$-\frac{b\left(\frac{b\left(\frac{b\int -\frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$\Big\downarrow$ 25

$$-\frac{b\left(\frac{b\left(-\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$\Big\downarrow$ 3042

$$-\frac{b\left(\frac{b\left(-\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$\Big\downarrow$ 3784

$$-\frac{b\left(\frac{b\left(-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$\Big\downarrow$ 3042

$$-\frac{b\left(\frac{b\left(-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$\Big\downarrow$ 3780

3.8. $\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$

$$-\frac{b\left(\frac{b\left(-\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \; 3783$$

$$-\frac{b\left(\frac{b\left(-\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{\cos(a+bx)}{d(c+dx)}\right)}{2d}-\frac{\sin(a+bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\cos(a+bx)}{3d(c+dx)^3}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^4,x]
```

output
```
-1/3*Cos[a + b*x]/(d*(c + d*x)^3) - (b*(-1/2*Sin[a + b*x]/(d*(c + d*x)^2)
+ (b*(-(Cos[a + b*x]/(d*(c + d*x))) - (b*((CosIntegral[(b*c)/d + b*x]*Sin[
a - (b*c)/d])/d + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d))/d))/(2
*d)))/(3*d)
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

3.8.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.45

method	result
derivativedivides	$b^3\left(-\frac{\cos(bx+a)}{3(-ad+bc+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-ad+bc+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right.}{d}}{d}}{2d}}{3d}\right.$
default	$b^3\left(-\frac{\cos(bx+a)}{3(-ad+bc+d(bx+a))^3d}-\frac{-\frac{\sin(bx+a)}{2(-ad+bc+d(bx+a))^2d}+\frac{-\frac{\cos(bx+a)}{(-ad+bc+d(bx+a))d}-\frac{-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\cos\left(\frac{-ad+bc}{d}\right.}{d}}{d}}{2d}}{3d}\right.$
risch	$-\frac{ib^3\mathrm{e}^{-\frac{i(ad-bc)}{d}}\mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{12d^4}+\frac{ib^3\mathrm{e}^{\frac{i(ad-bc)}{d}}\mathrm{Ei}_1\left(-ixb-ia-\frac{-iad+ibc}{d}\right)}{12d^4}-\frac{(-2b^5d^5x^5-10b^5c\,d^4x^4-20b}{}$

input
```
int(cos(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
b^3*(-1/3*cos(b*x+a)/(-a*d+b*c+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a)/(-a*d+b
*c+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+b*c+d*(b*x+a))/d-(-Si(-b*x-a-(-a*
d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/
d)/d)/d)
```

3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(117) = 234$.

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.85

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{co}}{6\left(d^7x^3+3\,cd^6x^2+3\right.}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integra
l((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d)
 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (b*d^3
*x + b*c*d^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4
)
```

3.8.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**4, x)
```

3.8.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.96

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(-i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+i\,E_4\left(\right.\right.}{2\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\left(bcd^3-ad^4\right)(bx+a)^2+3\left(b^2c^2d^2-2\right.\right.}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/2*(b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^4*(
-I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e
(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d))/((b^3*c^3*d
- 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))
*b)
```

3.8.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 8378, normalized size of antiderivative = 65.97

$$\int \frac{\cos(a+bx)}{(c+dx)^4} \, dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/12*(b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/
d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*sin_integ
ral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*b^3*
d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(1/2*b*c/d) + 2*b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(1/
2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b
*c/d)^2 + 3*b^3*c*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 3*b^3*c*d^2*x^2*imag_part(cos_integral
(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 6*b^3*c*d^2
*x^2*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*b*c
/d)^2 - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*x
)^2*tan(1/2*a)^2 - 2*b^3*d^3*x^3*sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x
)^2*tan(1/2*a)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(
1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*b^3*d^3*x^3*
sin_integral((b*d*x + b*c)/d)*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*b*c/d) ...
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^4}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)/(c + d*x)^4, x)
```

3.9 $\int (c+dx)^4 \cos^2(a+bx)\,dx$

3.9.1 Optimal result

Integrand size = 16, antiderivative size = 161

$$\int (c+dx)^4 \cos^2(a+bx)\,dx = \frac{3d^4x}{4b^4}-\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{3d^4\cos(a+bx)\sin(a+bx)}{4b^5}-\frac{3d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b^3}+\frac{(c+dx)^4\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
3/4*d^4*x/b^4-1/2*d*(d*x+c)^3/b^2+1/10*(d*x+c)^5/d-3/2*d^3*(d*x+c)*cos(b*x
+a)^2/b^4+d*(d*x+c)^3*cos(b*x+a)^2/b^2+3/4*d^4*cos(b*x+a)*sin(b*x+a)/b^5-3
/2*d^2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3+1/2*(d*x+c)^4*cos(b*x+a)*sin(b*
x+a)/b
```

3.9.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int (c+dx)^4 \cos^2(a+bx)\,dx$$
$$= \frac{8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)+20bd(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+1}{80b^5}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2,x]
```

output
```
(8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) + 2
0*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 10*(3*d^4
- 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(80*b^5)
```

3.9.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3792, 17, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^4 \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2\int (c+dx)^2\cos^2(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^4dx+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\int (c+dx)^2\cos^2(a+bx)dx}{b^2}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d^2\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{b^2}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2\left(-\frac{d^2\int\cos^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)}{b^2}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow 17$$

$$-\frac{3d^2\left(-\frac{d^2\int\cos^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow 3042$$

$$-\frac{3d^2\left(-\frac{d^2\int\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow 3115$$

$$-\frac{3d^2\left(-\frac{d^2\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow 24$$

$$-\frac{3d^2\left(\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}+\frac{d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{(c+dx)^4\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^2,x]
```

output
```
(c + d*x)^5/(10*d) + (d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + ((c + d*x)^4*Cos
[a + b*x]*Sin[a + b*x])/(2*b) - (3*d^2*((c + d*x)^3/(6*d) + (d*(c + d*x)*C
os[a + b*x]^2)/(2*b^2) + ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (
d^2*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/b^2
```

3.9. $\int (c+dx)^4\cos^2(a+bx)\,dx$

3.9.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.9.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{\left(2(dx+c)^4 b^4-6d^2(dx+c)^2 b^2+3d^4\right)\sin(2bx+2a)+4b\left((dx+c)\left((dx+c)^2 b^2-\frac{3d^2}{2}\right)d\cos(2bx+2a)+x\left(\frac{1}{5}d^4x^4+c\,d^3x^3+2c^2\right.\right.}{8b^5}$
risch	$\frac{d^4x^5}{10}+\frac{c\,d^3x^4}{2}+d^2c^2x^3+d\,c^3x^2+\frac{c^4x}{2}+\frac{c^5}{10d}+\frac{d\left(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3d^2c\right)\cos}{4b^4}$
norman	$\frac{-\frac{\left(2b^4c^4-6b^2c^2d^2+3d^4\right)\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b^5}+\frac{d^4x^5\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{10}+\frac{d^4x^4\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4c\,d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{d^2\left(2b^2c^2+d^2\right)x^3}{2b^2}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(d*x+c)^4*b^4-6*d^2*(d*x+c)^2*b^2+3*d^4)*sin(2*b*x+2*a)+4*b*((d*x+
c)*((d*x+c)^2*b^2-3/2*d^2)*d*cos(2*b*x+2*a)+x*(1/5*d^4*x^4+c*d^3*x^3+2*c^2
*d^2*x^2+2*c^3*d*x+c^4)*b^4-b^2*c^3*d+3/2*c*d^3))/b^5
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.78

$$\int (c+dx)^4\cos^2(a+bx)\,dx$$
$$= \frac{2\,b^5d^4x^5 + 10\,b^5cd^3x^4 + 10\,(2\,b^5c^2d^2 - b^3d^4)x^3 + 10\,(2\,b^5c^3d - 3\,b^3cd^3)x^2 + 10\,(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 2\,b^3}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 10*(2*b^5*c^2*d^2 - b^3*d^4)*x^3
+ 10*(2*b^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2
 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 +
 5*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 +
6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x
 + a)*sin(b*x + a) + 5*(2*b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.

Time = 0.52 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.10

$$\int (c+dx)^4\cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4x\sin^2(a+bx)}{2} + \frac{c^4x\cos^2(a+bx)}{2} + c^3dx^2\sin^2(a+bx) + c^3dx^2\cos^2(a+bx) + c^2d^2x^3\sin^2(a+bx) + c^2d^2x^3\,\mathrm{c} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**2,x)
```

output
```
Piecewise((c**4*x*sin(a + b*x)**2/2 + c**4*x*cos(a + b*x)**2/2 + c**3*d*x*
*2*sin(a + b*x)**2 + c**3*d*x**2*cos(a + b*x)**2 + c**2*d**2*x**3*sin(a +
b*x)**2 + c**2*d**2*x**3*cos(a + b*x)**2 + c*d**3*x**4*sin(a + b*x)**2/2 +
 c*d**3*x**4*cos(a + b*x)**2/2 + d**4*x**5*sin(a + b*x)**2/10 + d**4*x**5*
cos(a + b*x)**2/10 + c**4*sin(a + b*x)*cos(a + b*x)/(2*b) + 2*c**3*d*x*sin
(a + b*x)*cos(a + b*x)/b + 3*c**2*d**2*x**2*sin(a + b*x)*cos(a + b*x)/b +
2*c*d**3*x**3*sin(a + b*x)*cos(a + b*x)/b + d**4*x**4*sin(a + b*x)*cos(a +
 b*x)/(2*b) + c**3*d*cos(a + b*x)**2/b**2 - 3*c**2*d**2*x*sin(a + b*x)**2/
(2*b**2) + 3*c**2*d**2*x*cos(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*sin(a +
b*x)**2/(2*b**2) + 3*c*d**3*x**2*cos(a + b*x)**2/(2*b**2) - d**4*x**3*sin(
a + b*x)**2/(2*b**2) + d**4*x**3*cos(a + b*x)**2/(2*b**2) - 3*c**2*d**2*si
n(a + b*x)*cos(a + b*x)/(2*b**3) - 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)/b*
*3 - 3*d**4*x**2*sin(a + b*x)*cos(a + b*x)/(2*b**3) - 3*c*d**3*cos(a + b*x
)**2/(2*b**4) + 3*d**4*x*sin(a + b*x)**2/(4*b**4) - 3*d**4*x*cos(a + b*x)*
*2/(4*b**4) + 3*d**4*sin(a + b*x)*cos(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4
*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a)*
*2, True))
```

3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(147) = 294$.

Time = 0.28 (sec) , antiderivative size = 717, normalized size of antiderivative = 4.45

$$\int (c+dx)^4 \cos^2(a+bx)\,dx$$

$$= \frac{10\,(2\,bx+2\,a+\sin\left(2\,bx+2\,a\right))c^4 - \frac{40\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ac^3d}{b} + \frac{60\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2c^2d^2}{b^2} - \frac{40\,(2\,bx+2\,a+\sin}{}}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/40*(10*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^4 - 40*(2*b*x + 2*a + sin(2*b*
x + 2*a))*a*c^3*d/b + 60*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*c^2*d^2/b^2
- 40*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^3*c*d^3/b^3 + 10*(2*b*x + 2*a + si
n(2*b*x + 2*a))*a^4*d^4/b^4 + 20*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x +
2*a) + cos(2*b*x + 2*a))*c^3*d/b - 60*(2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b
*x + 2*a) + cos(2*b*x + 2*a))*a*c^2*d^2/b^2 + 60*(2*(b*x + a)^2 + 2*(b*x +
 a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^2*c*d^3/b^3 - 20*(2*(b*x + a)^2
 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^3*d^4/b^4 + 10*(4*(b
*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x
 + 2*a))*c^2*d^2/b^2 - 20*(4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) +
3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 10*(4*(b*x + a)^3 +
6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a^2
*d^4/b^4 + 10*(2*(b*x + a)^4 + 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*
(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*c*d^3/b^3 - 10*(2*(b*x + a
)^4 + 3*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^3 - 3*b*x -
3*a)*sin(2*b*x + 2*a))*a*d^4/b^4 + (4*(b*x + a)^5 + 10*(2*(b*x + a)^3 - 3*
b*x - 3*a)*cos(2*b*x + 2*a) + 5*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*sin(2*
b*x + 2*a))*d^4/b^4)/b
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.38

$$\int (c+dx)^4\cos^2(a+bx)\,dx = \frac{1}{10}\,d^4x^5+\frac{1}{2}\,cd^3x^4+c^2d^2x^3+c^3dx^2+\frac{1}{2}\,c^4x + \frac{(2\,b^3d^4x^3+6\,b^3cd^3x^2+6\,b^3c^2d^2x+2\,b^3c^3d-3\,bd^4x-3\,bcd^3)\cos(2\,bx+2\,a)}{4\,b^5} + \frac{(2\,b^4d^4x^4+8\,b^4cd^3x^3+12\,b^4c^2d^2x^2+8\,b^4c^3dx+2\,b^4c^4-6\,b^2d^4x^2-12\,b^2cd^3x-6\,b^2c^2d^2+3\,d^4)\sin(2\,b}{8\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x + 1/4*(
2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*b^3*c^3*d - 3*b*d^4*
x - 3*b*c*d^3)*cos(2*b*x + 2*a)/b^5 + 1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3
 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2
*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*sin(2*b*x + 2*a)/b^5
```

3.9.9 Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.17

$$\int (c+dx)^4\cos^2(a+bx)\,dx$$

$$= \frac{\frac{15\,d^4\,\sin(2\,a+2\,b\,x)}{2}+10\,b^5\,c^4\,x+5\,b^4\,c^4\,\sin\left(2\,a+2\,b\,x\right)+2\,b^5\,d^4\,x^5+10\,b^3\,c^3\,d\,\cos\left(2\,a+2\,b\,x\right)+20\,b^5\,c^3\,d\,x^2}{}$$

input
```
int(cos(a + b*x)^2*(c + d*x)^4,x)
```

output
```
((15*d^4*sin(2*a + 2*b*x))/2 + 10*b^5*c^4*x + 5*b^4*c^4*sin(2*a + 2*b*x) +
 2*b^5*d^4*x^5 + 10*b^3*c^3*d*cos(2*a + 2*b*x) + 20*b^5*c^3*d*x^2 + 10*b^5
*c*d^3*x^4 - 15*b^2*c^2*d^2*sin(2*a + 2*b*x) + 10*b^3*d^4*x^3*cos(2*a + 2*
b*x) + 20*b^5*c^2*d^2*x^3 - 15*b^2*d^4*x^2*sin(2*a + 2*b*x) + 5*b^4*d^4*x^
4*sin(2*a + 2*b*x) - 15*b*c*d^3*cos(2*a + 2*b*x) - 15*b*d^4*x*cos(2*a + 2*
b*x) + 30*b^4*c^2*d^2*x^2*sin(2*a + 2*b*x) - 30*b^2*c*d^3*x*sin(2*a + 2*b*
x) + 20*b^4*c^3*d*x*sin(2*a + 2*b*x) + 30*b^3*c^2*d^2*x*cos(2*a + 2*b*x) +
 30*b^3*c*d^3*x^2*cos(2*a + 2*b*x) + 20*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/(2
0*b^5)
```

3.10 $\int (c+dx)^3 \cos^2(a+bx)\,dx$

3.10.1 Optimal result

Integrand size = 16, antiderivative size = 134

$$\begin{aligned}\int (c+dx)^3\cos^2(a+bx)\,dx = &-\frac{3cd^2x}{4b^2}-\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}\\&-\frac{3d^3\cos^2(a+bx)}{8b^4}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}\\&-\frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{4b^3}\\&+\frac{(c+dx)^3\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

output
```
-3/4*c*d^2*x/b^2-3/8*d^3*x^2/b^2+1/8*(d*x+c)^4/d-3/8*d^3*cos(b*x+a)^2/b^4+
3/4*d*(d*x+c)^2*cos(b*x+a)^2/b^2-3/4*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^3
+1/2*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b
```

3.10.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

$$\begin{aligned}&\int (c+dx)^3\cos^2(a+bx)\,dx\\&=\frac{2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+3d(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))+2b(c+dx)\left(-3d^2+2b^2(c\right.}{16b^4}\end{aligned}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2,x]
```

output
```
(2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(
c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*
Sin[2*(a + b*x)])/(16*b^4)
```

3.10.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2\int (c+dx)\cos^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^3dx+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\int (c+dx)\cos^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2,x]
```

output
```
(c + d*x)^4/(8*d) + (3*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)^
3*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (3*d^2*((c + d*x)^2/(4*d) + (d*Cos[a
+ b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)
```

3.10.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.10.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{4b\sin(2bx+2a)(dx+c)\left((dx+c)^2b^2-\frac{3d^2}{2}\right)+6d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\cos(2bx+2a)+2\left(d^3x^4+4d^2c\,x^3+6d\,c^2x^2+4c^3x\right)b^4-6b^2}{16b^4}$
risch	$\frac{d^3x^4}{8}+\frac{d^2c\,x^3}{2}+\frac{3d\,c^2x^2}{4}+\frac{c^3x}{2}+\frac{c^4}{8d}+\frac{3d\left(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2\right)\cos(2bx+2a)}{16b^4}+\frac{\left(2b^2d^3x^3+6b^2c\,d^2x^2\right.}{}$
norman	$d^2c\,x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{d^3x^3\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{d^3x^4}{8}+\frac{d^2c\,x^3}{2}+\frac{d^3x^4\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{d^3x^4\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8}+\frac{\left(-6b^2c^2d+3d^3\right)(}{2b^4}$
derivativedivides	$-\frac{a^3d^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{3a^2c\,d^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{3a^2d^3\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right.\right.}{b^3}$
default	$-\frac{a^3d^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^3}+\frac{3a^2c\,d^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}+\frac{3a^2d^3\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right.\right.}{b^3}$

input
```
int((d*x+c)^3*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*b*sin(2*b*x+2*a)*(d*x+c)*((d*x+c)^2*b^2-3/2*d^2)+6*d*((d*x+c)^2*b^
2-1/2*d^2)*cos(2*b*x+2*a)+2*(d^3*x^4+4*c*d^2*x^3+6*c^2*d*x^2+4*c^3*x)*b^4-
6*b^2*c^2*d+3*d^3)/b^4
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.42

$$\int (c+dx)^3\cos^2(a+bx)\,dx$$
$$=\frac{b^4d^3x^4+4\,b^4cd^2x^3+3\,(2\,b^4c^2d-b^2d^3)x^2+3\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\cos{(bx+a)}^2+2\,(2\,b^3d^3}{8\,b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 3*(2*b^4*c^2*d - b^2*d^3)*x^2 + 3*(2*
b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(2*b^3
*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^
3)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.

Time = 0.40 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.40

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3x\sin^2(a+bx)}{2} + \frac{c^3x\cos^2(a+bx)}{2} + \frac{3c^2dx^2\sin^2(a+bx)}{4} + \frac{3c^2dx^2\cos^2(a+bx)}{4} + \frac{cd^2x^3\sin^2(a+bx)}{2} + \frac{cd^2x^3\cos^2(a+bx)}{2} + \frac{d^3x^4\sin \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**2,x)
```

output
```
Piecewise((c**3*x*sin(a + b*x)**2/2 + c**3*x*cos(a + b*x)**2/2 + 3*c**2*d*
x**2*sin(a + b*x)**2/4 + 3*c**2*d*x**2*cos(a + b*x)**2/4 + c*d**2*x**3*sin
(a + b*x)**2/2 + c*d**2*x**3*cos(a + b*x)**2/2 + d**3*x**4*sin(a + b*x)**2
/8 + d**3*x**4*cos(a + b*x)**2/8 + c**3*sin(a + b*x)*cos(a + b*x)/(2*b) +
3*c**2*d*x*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c*d**2*x**2*sin(a + b*x)*co
s(a + b*x)/(2*b) + d**3*x**3*sin(a + b*x)*cos(a + b*x)/(2*b) + 3*c**2*d*co
s(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sin(a + b*x)**2/(4*b**2) + 3*c*d**2*x*
cos(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sin(a + b*x)**2/(8*b**2) + 3*d**3*x
**2*cos(a + b*x)**2/(8*b**2) - 3*c*d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3)
 - 3*d**3*x*sin(a + b*x)*cos(a + b*x)/(4*b**3) - 3*d**3*cos(a + b*x)**2/(8
*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)
*cos(a)**2, True))
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(120) = 240$.

Time = 0.32 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.19

$$\int (c+dx)^3 \cos^2(a+bx)\,dx$$
$$= \frac{4\,(2\,bx + 2\,a + \sin(2\,bx + 2\,a))c^3 - \frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ac^2d}{b} + \frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2cd^2}{b^2} - \frac{4\,(2\,bx+2\,a+\sin(2\,b}{b^3}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/16*(4*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^3 - 12*(2*b*x + 2*a + sin(2*b*x
 + 2*a))*a*c^2*d/b + 12*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*c*d^2/b^2 - 4
*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^3*d^3/b^3 + 6*(2*(b*x + a)^2 + 2*(b*x
+ a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)^2 + 2*
(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a
)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a^2*d^3/b^3 + 2*(4*
(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b
*x + 2*a))*c*d^2/b^2 - 2*(4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3
*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(b*x + a)^4 + 3*(2*(
b*x + a)^2 - 1)*cos(2*b*x + 2*a) + 2*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b
*x + 2*a))*d^3/b^3)/b
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.14

$$\int (c+dx)^3\cos^2(a+bx)\,dx$$
$$=\frac{1}{8}d^3x^4+\frac{1}{2}cd^2x^3+\frac{3}{4}c^2dx^2+\frac{1}{2}c^3x+\frac{3\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\cos(2\,bx+2\,a)}{16\,b^4}$$
$$+\frac{(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2)\sin(2\,bx+2\,a)}{8\,b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x + 3/16*(2*b^2*d^3*
x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(2*b*x + 2*a)/b^4 + 1/8*(2*b^3
*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c
*d^2)*sin(2*b*x + 2*a)/b^4
```

3.10.9 Mupad [B] (verification not implemented)

Time = 14.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int (c+dx)^3\cos^2(a+bx)\,dx$$
$$=\frac{4\,b^4\,c^3\,x-\frac{3\,d^3\,\cos(2\,a+2\,b\,x)}{2}+2\,b^3\,c^3\,\sin(2\,a+2\,b\,x)+b^4\,d^3\,x^4+3\,b^2\,c^2\,d\,\cos(2\,a+2\,b\,x)+6\,b^4\,c^2\,d\,x^2+4\,b}{}$$

input
```
int(cos(a + b*x)^2*(c + d*x)^3,x)
```

output
```
(4*b^4*c^3*x - (3*d^3*cos(2*a + 2*b*x))/2 + 2*b^3*c^3*sin(2*a + 2*b*x) + b
^4*d^3*x^4 + 3*b^2*c^2*d*cos(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d^2*
x^3 + 3*b^2*d^3*x^2*cos(2*a + 2*b*x) + 2*b^3*d^3*x^3*sin(2*a + 2*b*x) - 3*
b*c*d^2*sin(2*a + 2*b*x) - 3*b*d^3*x*sin(2*a + 2*b*x) + 6*b^2*c*d^2*x*cos(
2*a + 2*b*x) + 6*b^3*c^2*d*x*sin(2*a + 2*b*x) + 6*b^3*c*d^2*x^2*sin(2*a +
2*b*x))/(8*b^4)
```

3.10. $\int (c+dx)^3 \cos^2(a+bx)\,dx$

3.11 $\int (c+dx)^2 \cos^2(a+bx)\,dx$

3.11.1 Optimal result

Integrand size = 16, antiderivative size = 95

$$\int (c+dx)^2\cos^2(a+bx)\,dx = -\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(a+bx)}{4b^3} + \frac{(c+dx)^2\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
-1/4*d^2*x/b^2+1/6*(d*x+c)^3/d+1/2*d*(d*x+c)*cos(b*x+a)^2/b^2-1/4*d^2*cos(
b*x+a)*sin(b*x+a)/b^3+1/2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b
```

3.11.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int (c+dx)^2\cos^2(a+bx)\,dx = \frac{4b^3x(3c^2+3cdx+d^2x^2)+6bd(c+dx)\cos(2(a+bx))+3(-d^2+2b^2(c+dx)^2)\sin(2(a+bx))}{24b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^2,x]
```

output
```
(4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) + 6*b*d*(c + d*x)*Cos[2*(a + b*x)] +
3*(-d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(24*b^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{d^2\int \cos^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^2dx+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{d^2\int \cos^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{d^2\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{24}$$

$$\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2,x]
```

output
```
(c + d*x)^3/(6*d) + (d*(c + d*x)*Cos[a + b*x]^2)/(2*b^2) + ((c + d*x)^2*Co
s[a + b*x]*Sin[a + b*x])/(2*b) - (d^2*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(
2*b)))/(2*b^2)
```

3.11.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.11.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{\left(2(dx+c)^2b^2-d^2\right)\sin(2bx+2a)+4b\left(\frac{d(dx+c)\cos(2bx+2a)}{2}+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-\frac{cd}{2}\right)}{8b^3}$
risch	$\frac{d^2x^3}{6}+\frac{cd\,x^2}{2}+\frac{c^2x}{2}+\frac{c^3}{6d}+\frac{d(dx+c)\cos(2bx+2a)}{4b^2}+\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{\frac{a^2d^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-(b\ldots\right.}{b^2}}{}$
default	$\frac{\frac{a^2d^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b^2}-\frac{2acd\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2a\,d^2\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-(b\ldots\right.}{b^2}}{}$
norman	$\frac{cd\,x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{d^2x^2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{d^2x^3}{6}+\frac{cd\,x^2}{2}+\frac{d^2x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3}+\frac{d^2x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{6}+\frac{(2b^2c^2-d^2)\tan\left(\frac{bx}{2}\ldots\right.}{2b^3}}{}$

input
```
int((d*x+c)^2*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(d*x+c)^2*b^2-d^2)*sin(2*b*x+2*a)+4*b*(1/2*d*(d*x+c)*cos(2*b*x+2*a
)+x*(1/3*x^2*d^2+c*d*x+c^2)*b^2-1/2*c*d))/b^3
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int (c+dx)^2\cos^2(a+bx)\,dx$$
$$=\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,(bd^2x+bcd)\cos{(bx+a)}^2+3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos{(bx+a)}\sin{(b\ldots}}{12\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 +
 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x +
a) + 3*(2*b^3*c^2 - b*d^2)*x)/b^3
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.78

$$\int (c+dx)^2 \cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2x\sin^2(a+bx)}{2} + \frac{c^2x\cos^2(a+bx)}{2} + \frac{cdx^2\sin^2(a+bx)}{2} + \frac{cdx^2\cos^2(a+bx)}{2} + \frac{d^2x^3\sin^2(a+bx)}{6} + \frac{d^2x^3\cos^2(a+bx)}{6} + \frac{c^2\sin(a+bx)c}{2b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2,x)
```

output
```
Piecewise((c**2*x*sin(a + b*x)**2/2 + c**2*x*cos(a + b*x)**2/2 + c*d*x**2*
sin(a + b*x)**2/2 + c*d*x**2*cos(a + b*x)**2/2 + d**2*x**3*sin(a + b*x)**2
/6 + d**2*x**3*cos(a + b*x)**2/6 + c**2*sin(a + b*x)*cos(a + b*x)/(2*b) +
c*d*x*sin(a + b*x)*cos(a + b*x)/b + d**2*x**2*sin(a + b*x)*cos(a + b*x)/(2
*b) + c*d*cos(a + b*x)**2/(2*b**2) - d**2*x*sin(a + b*x)**2/(4*b**2) + d**
2*x*cos(a + b*x)**2/(4*b**2) - d**2*sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne
(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cos(a)**2, True))
```

3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(85) = 170$.

Time = 0.24 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.34

$$\int (c+dx)^2 \cos^2(a+bx)\,dx$$
$$= \frac{6\,(2\,bx + 2\,a + \sin(2\,bx + 2\,a))c^2 - \frac{12\,(2\,bx+2\,a+\sin(2\,bx+2\,a))acd}{b} + \frac{6\,(2\,bx+2\,a+\sin(2\,bx+2\,a))a^2d^2}{b^2} + \frac{6\left(2\,(bx+a)^2+2\,(bx+a\right.}{}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/24*(6*(2*b*x + 2*a + sin(2*b*x + 2*a))*c^2 - 12*(2*b*x + 2*a + sin(2*b*x
 + 2*a))*a*c*d/b + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2*d^2/b^2 + 6*(2*(
b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*c*d/b - 6*(2
*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a*d^2/b^2
+ (4*(b*x + a)^3 + 6*(b*x + a)*cos(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*si
n(2*b*x + 2*a))*d^2/b^2)/b
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int (c+dx)^2\cos^2(a+bx)\,dx = \frac{1}{6}\,d^2x^3 + \frac{1}{2}\,cdx^2 + \frac{1}{2}\,c^2x + \frac{(bd^2x+bcd)\cos(2\,bx+2\,a)}{4\,b^3} + \frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\sin(2\,bx+2\,a)}{8\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x + 1/4*(b*d^2*x + b*c*d)*cos(2*b*x +
2*a)/b^3 + 1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*sin(2*b*x +
 2*a)/b^3
```

3.11.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.88

$$\int (c+dx)^2\cos^2(a+bx)\,dx = x\left(\frac{c^2}{4}-\frac{d^2}{8\,b^2}\right)+x\left(\frac{c^2}{4}+\frac{d^2}{8\,b^2}\right) + \frac{d^2\,x^3}{6} - \frac{\sin(2\,a+2\,b\,x)\,\left(d^2-2\,b^2\,c^2\right)}{8\,b^3} - \frac{x\,\cos(2\,a+2\,b\,x)\,\left(\frac{c^2}{2}-\frac{d^2}{4\,b^2}\right)}{2} + \frac{x\,\cos(2\,a+2\,b\,x)\,\left(\frac{c^2}{2}+\frac{d^2}{4\,b^2}\right)}{2} + \frac{c\,d\,x^2}{2} + \frac{d^2\,x^2\,\sin(2\,a+2\,b\,x)}{4\,b} + \frac{c\,d\,\cos(2\,a+2\,b\,x)}{4\,b^2} + \frac{c\,d\,x\,\sin(2\,a+2\,b\,x)}{2\,b}$$

input
```
int(cos(a + b*x)^2*(c + d*x)^2,x)
```

output
```
x*(c^2/4 - d^2/(8*b^2)) + x*(c^2/4 + d^2/(8*b^2)) + (d^2*x^3)/6 - (sin(2*a
 + 2*b*x)*(d^2 - 2*b^2*c^2))/(8*b^3) - (x*cos(2*a + 2*b*x)*(c^2/2 - d^2/(4
*b^2)))/2 + (x*cos(2*a + 2*b*x)*(c^2/2 + d^2/(4*b^2)))/2 + (c*d*x^2)/2 + (
d^2*x^2*sin(2*a + 2*b*x))/(4*b) + (c*d*cos(2*a + 2*b*x))/(4*b^2) + (c*d*x*
sin(2*a + 2*b*x))/(2*b)
```

3.12 $\int (c+dx)\cos^2(a+bx)\,dx$

3.12.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int (c+dx)\cos^2(a+bx)\,dx=\frac{cx}{2}+\frac{dx^2}{4}+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
1/2*c*x+1/4*d*x^2+1/4*d*cos(b*x+a)^2/b^2+1/2*(d*x+c)*cos(b*x+a)*sin(b*x+a)
/b
```

3.12.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int (c+dx)\cos^2(a+bx)\,dx =\frac{d\cos(2(a+bx))+2b(2ac+bx(2c+dx)+(c+dx)\sin(2(a+bx)))}{8b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2,x]
```

output
```
(d*Cos[2*(a + b*x)] + 2*b*(2*a*c + b*x*(2*c + d*x) + (c + d*x)*Sin[2*(a +
b*x)]))/(8*b^2)
```

3.12.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{1}{2}\int (c+dx)dx+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2,x]
```

output
```
(c + d*x)^2/(4*d) + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*x]*S
in[a + b*x])/(2*b)
```

3.12.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.12.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result
risch	$\frac{d\,x^{2}}{4}+\frac{c x}{2}+\frac{d \cos\left(2 b x +2 a \right)}{8 b^{2}}+\frac{\left(d x +c \right) \sin\left(2 b x +2 a \right)}{4 b}$
parallelrisch	$\frac{2 b \sin\left(2 b x +2 a \right) \left(d x +c \right)+d \cos\left(2 b x +2 a \right)+\left(2 d\,x^{2}+4 c x \right) b^{2}-d}{8 b^{2}}$
derivativedivides	$\frac{-\frac{d a \left(\frac{\cos\left(b x +a \right) \sin\left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{b}+c \left(\frac{\cos\left(b x +a \right) \sin\left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}\right)+\frac{d ((b x +a) \left(\frac{\cos\left(b x +a \right) \sin\left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{\left(b x +a \right)^{2}}{4}-}{b}}{b}$
default	$\frac{-\frac{d a \left(\frac{\cos\left(b x +a \right) \sin\left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}\right)}{b}+c \left(\frac{\cos\left(b x +a \right) \sin\left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}\right)+\frac{d ((b x +a) \left(\frac{\cos\left(b x +a \right) \sin\left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}\right)-\frac{\left(b x +a \right)^{2}}{4}-}{b}}{b}$
norman	$\frac{\frac{c \tan\left(\frac{b x}{2}+\frac{a}{2}\right)}{b}+c x \left(\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)-\frac{d \left(\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)}{b^{2}}+\frac{d x \tan\left(\frac{b x}{2}+\frac{a}{2}\right)}{b}+\frac{c x}{2}+\frac{d\,x^{2}}{4}-\frac{c \left(\tan^{3}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)}{b}+\frac{c x \left(\tan^{4}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)}{2}+}{\left(1+\tan^{2}\left(\frac{b x}{2}+\frac{a}{2}\right)\right)^{2}}$

input
```
int((d*x+c)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d*x^2+1/2*c*x+1/8*d/b^2*cos(2*b*x+2*a)+1/4/b*(d*x+c)*sin(2*b*x+2*a)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (c+dx)\cos^2(a+bx)\,dx = \frac{b^2dx^2 + 2\,b^2cx + d\cos(bx+a)^2 + 2\,(bdx+bc)\cos(bx+a)\sin(bx+a)}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b^2*d*x^2 + 2*b^2*c*x + d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(b*x +
a)*sin(b*x + a))/b^2
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(49) = 98$.

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int (c+dx)\cos^2(a+bx)\,dx$$
$$= \begin{cases} \frac{cx\sin^2(a+bx)}{2} + \frac{cx\cos^2(a+bx)}{2} + \frac{dx^2\sin^2(a+bx)}{4} + \frac{dx^2\cos^2(a+bx)}{4} + \frac{c\sin(a+bx)\cos(a+bx)}{2b} + \frac{dx\sin(a+bx)\cos(a+bx)}{2b} + \frac{d\cos^{2}\ldots \\ \left(cx + \frac{dx^2}{2}\right)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**2,x)
```

output
```
Piecewise((c*x*sin(a + b*x)**2/2 + c*x*cos(a + b*x)**2/2 + d*x**2*sin(a +
b*x)**2/4 + d*x**2*cos(a + b*x)**2/4 + c*sin(a + b*x)*cos(a + b*x)/(2*b) +
 d*x*sin(a + b*x)*cos(a + b*x)/(2*b) + d*cos(a + b*x)**2/(4*b**2), Ne(b, 0
)), ((c*x + d*x**2/2)*cos(a)**2, True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.64

$$\int (c+dx)\cos^2(a+bx)\,dx$$
$$= \frac{2\,(2\,bx+2\,a+\sin(2\,bx+2\,a))c - \frac{2\,(2\,bx+2\,a+\sin(2\,bx+2\,a))ad}{b} + \frac{\left(2\,(bx+a)^2+2\,(bx+a)\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)d}{b}}{8\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/8*(2*(2*b*x + 2*a + sin(2*b*x + 2*a))*c - 2*(2*b*x + 2*a + sin(2*b*x + 2
*a))*a*d/b + (2*(b*x + a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2
*a))*d/b)/b
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int (c+dx)\cos^2(a+bx)\,dx = \frac{1}{4}\,dx^2 + \frac{1}{2}\,cx + \frac{d\cos(2\,bx+2\,a)}{8\,b^2} + \frac{(bdx+bc)\sin(2\,bx+2\,a)}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*d*x^2 + 1/2*c*x + 1/8*d*cos(2*b*x + 2*a)/b^2 + 1/4*(b*d*x + b*c)*sin(2
*b*x + 2*a)/b^2
```

3.12.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos^2(a+bx)\,dx = \frac{c\,x}{2} + \frac{d\,x^2}{4} + \frac{d\,\cos(2\,a+2\,b\,x)}{8\,b^2} + \frac{c\,\sin(2\,a+2\,b\,x)}{4\,b} + \frac{d\,x\,\sin(2\,a+2\,b\,x)}{4\,b}$$

input
```
int(cos(a + b*x)^2*(c + d*x),x)
```

output
```
(c*x)/2 + (d*x^2)/4 + (d*cos(2*a + 2*b*x))/(8*b^2) + (c*sin(2*a + 2*b*x))/
(4*b) + (d*x*sin(2*a + 2*b*x))/(4*b)
```

3.13 $\int \frac{\cos^2(a+bx)}{c+dx} \, dx$

3.13.1 Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\cos^2(a+bx)}{c+dx} \, dx = \frac{\cos\left(2a-\frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{\log(c+dx)}{2d} - \frac{\sin\left(2a-\frac{2bc}{d}\right) \text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

output
```
1/2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d+1/2*ln(d*x+c)/d-1/2*Si(2*b*c/d+2*
b*x)*sin(2*a-2*b*c/d)/d
```

3.13.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)}{c+dx} \, dx = \frac{\cos\left(2a-\frac{2bc}{d}\right) \text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \log(c+dx) - \sin\left(2a-\frac{2bc}{d}\right) \text{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

input
```
Integrate[Cos[a + b*x]^2/(c + d*x),x]
```

output
```
(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - Sin[
2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{c+dx}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{\cos(2a+2bx)}{2(c+dx)}+\frac{1}{2(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

input
```
Int[Cos[a + b*x]^2/(c + d*x),x]
```

output
```
(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]
/(2*d) - (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.13.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

method	result	s
risch	$\frac{\ln(dx+c)}{2d}-\frac{\mathrm{e}^{-\frac{2i(ad-bc)}{d}}\operatorname{Ei}_1\left(2ixb+2ia-\frac{2i(ad-bc)}{d}\right)}{4d}-\frac{\mathrm{e}^{\frac{2i(ad-bc)}{d}}\operatorname{Ei}_1\left(-2ixb-2ia-\frac{2(-iad+ibc)}{d}\right)}{4d}$	1
derivativedivides	$\frac{\frac{b\left(-\frac{2\operatorname{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}+\frac{2\operatorname{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{4}+\frac{b\ln(-ad+bc+d(bx+a))}{2d}}{b}$	1
default	$\frac{\frac{b\left(-\frac{2\operatorname{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}+\frac{2\operatorname{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{4}+\frac{b\ln(-ad+bc+d(bx+a))}{2d}}{b}$	1

input
```
int(cos(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d*x+c)/d-1/4/d*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*x*b+2*I*a-2*I*(a*d-b*
c)/d)-1/4/d*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*x*b-2*I*a-2*(-I*a*d+I*b*c)/d)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \frac{\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-\sin\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\log\left(dx+c\right)}{2\,d}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - sin(-2*(b*c -
 a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + log(d*x + c))/d
```

3.13.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \int \frac{\cos^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)**2/(c + d*x), x)
```

3.13.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.09

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = -\frac{b\left(E_1\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_1\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{2(-ibc-i(bx+a)d+i}{d}\right.\right.}{4\,bd}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*(b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d) -
 b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_int
egral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d) -
 2*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

3.13.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 610, normalized size of antiderivative = 7.82

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(2*log(abs(d*x + c))*tan(a)^2*tan(b*c/d)^2 + real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(a)^2*tan(b*c/d) + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^
2*tan(b*c/d) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) + 2*i
mag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*sin_integral(2*(b*
d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*log(abs(d*x + c))*tan(a)^2 - real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(a)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)
 + 2*log(abs(d*x + c))*tan(b*c/d)^2 - real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*c/d)^2 - real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2
 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(a) - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)
 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*c/d) + 2*log(abs(d*x + c)) + real_part(cos_integral(2*b*x + 2*b*c/
d)) + real_part(cos_integral(-2*b*x - 2*b*c/d)))/(d*tan(a)^2*tan(b*c/d)^2
+ d*tan(a)^2 + d*tan(b*c/d)^2 + d)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2/(c + d*x),x)
```

output
```
int(cos(a + b*x)^2/(c + d*x), x)
```

3.14 $\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx$

3.14.1 Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\cos^2(a+bx)}{d(c+dx)} - \frac{b\,\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} - \frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output
```
-cos(b*x+a)^2/d/(d*x+c)-b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-b*Ci(2*b*
c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2
```

3.14.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\frac{d\cos^2(a+bx)}{c+dx} + b\,\mathrm{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) + b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

input
```
Integrate[Cos[a + b*x]^2/(c + d*x)^2,x]
```

output
```
-(((d*Cos[a + b*x]^2)/(c + d*x) + b*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a
 - (2*b*c)/d] + b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^2
)
```

3.14.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3794, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3794}$$

$$\frac{2b\int -\frac{\sin(2a+2bx)}{2(c+dx)}dx}{d}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{b\left(\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\cos^2(a+bx)}{d(c+dx)}$$

input
```
Int[Cos[a + b*x]^2/(c + d*x)^2,x]
```

output
```
-(Cos[a + b*x]^2/(d*(c + d*x))) - (b*((CosIntegral[(2*b*c)/d + 2*b*x]*Sin[
2*a - (2*b*c)/d])/d + (Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x]
)/d))/d
```

3.14.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.14.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{1}{2d(dx+c)}+\frac{ib\,\mathrm{e}^{-\frac{2i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(2ixb+2ia-\frac{2i(ad-bc)}{d}\right)}{2d^2}-\frac{ib\,\mathrm{e}^{\frac{2i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-2ixb-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^2}-\frac{(-2bx\cdots}{4d(\cdots}$
derivativedivides	$\frac{\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-ad+bc+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{d}\right)\cdots}{4}}{b}$
default	$\frac{\frac{b^2\left(-\frac{2\cos(2bx+2a)}{(-ad+bc+d(bx+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\cos\left(\frac{-2ad+2bc}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2bc}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}\right)}{d}\right)\cdots}{4}}{b}$

input
```
int(cos(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(d*x+c)+1/2*I*b/d^2*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*x*b+2*I*a-2*I*(a
*d-b*c)/d)-1/2*I*b/d^2*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*x*b-2*I*a-2*(-I*a*d+
I*b*c)/d)-1/4/d*(-2*b*d*x-2*b*c)/(-b*d*x-b*c)/(d*x+c)*cos(2*b*x+2*a)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{d\cos{(bx+a)}^2 + (bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) + (bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{d^3x+cd^2}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(d*cos(b*x + a)^2 + (b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*
(b*c - a*d)/d) + (b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x
 + b*c)/d))/(d^3*x + c*d^2)
```

3.14.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos^2{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)**2/(c + d*x)**2, x)
```

3.14.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.06

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{b^2\left(E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right) + E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right) + b^2\left(i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i}{d}\right.\right.}{4\,(bcd+(bx+a)d^2-ad^2)b}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_i
ntegral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d)
 + b^2*(I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_
integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d
) + 2*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(83) = 166$.

Time = 0.36 (sec) , antiderivative size = 534, normalized size of antiderivative = 6.43

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\,\mathrm{Ci}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\,b^3c\,\mathrm{Ci}\left(\frac{2\left((dx+c)\cdots\right.}{}\right.\cdots}{}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(2*(
(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c
- a*d)/d) + 2*b^3*c*cos_integral(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*
x + c)) + b*c - a*d)/d)*sin(-2*(b*c - a*d)/d) - 2*a*b^2*d*cos_integral(2*(
(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-2*(b*c
- a*d)/d) - 2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-2*(b*
c - a*d)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))
 + b*c - a*d)/d) - 2*b^3*c*cos(-2*(b*c - a*d)/d)*sin_integral(-2*((d*x + c
)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 2*a*b^2*d*cos(-2*(
b*c - a*d)/d)*sin_integral(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c
)) + b*c - a*d)/d) + b^2*d*cos(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x
+ c))/d) + b^2*d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4
+ b*c*d^4 - a*d^5)*b)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(cos(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)^2/(c + d*x)^2, x)
```

3.15 $\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx$

3.15.1 Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{\cos^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

output
```
-b^2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^3-1/2*cos(b*x+a)^2/d/(d*x+c)^2+b
^2*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3+b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x
+c)
```

3.15.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = \frac{-2b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(-d\cos^2(a+bx)+b(c+dx)\sin(2(a+bx)))}{(c+dx)^2} + 2b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+d}{d}\right.}{2d^3}$$

input
```
Integrate[Cos[a + b*x]^2/(c + d*x)^3,x]
```

output
```
(-2*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(-(d*Cos[
a + b*x]^2) + b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sin[2*a -
 (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

3.15.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3795, 16, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{2b^2\int \frac{\cos^2(a+bx)}{c+dx}dx}{d^2}+\frac{b^2\int \frac{1}{c+dx}dx}{d^2}+\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\cos^2(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{2b^2\int \frac{\cos^2(a+bx)}{c+dx}dx}{d^2}+\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\cos^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{c+dx}dx}{d^2}+\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\cos^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{2b^2\int \left(\frac{\cos(2a+2bx)}{2(c+dx)}+\frac{1}{2(c+dx)}\right)dx}{d^2}+\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\cos^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^2\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}\right)}{d^2}+\frac{b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}-\frac{\cos^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\log(c+dx)}{d^3}$$

input
```
Int[Cos[a + b*x]^2/(c + d*x)^3,x]
```

output
```
-1/2*Cos[a + b*x]^2/(d*(c + d*x)^2) + (b^2*Log[c + d*x])/d^3 + (b*Cos[a +
b*x]*Sin[a + b*x])/(d^2*(c + d*x)) - (2*b^2*((Cos[2*a - (2*b*c)/d]*CosInte
gral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]/(2*d) - (Sin[2*a - (2*b*c)/d
]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)))/d^2
```

3.15.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.15.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-ad+bc+d(bx+a))^2 d}-\frac{-\frac{2\sin(2bx+2a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2b}{d}\right.}{d}}{d}}{d}\right.}{4}}{b}$
default	$\frac{\frac{b^3\left(-\frac{\cos(2bx+2a)}{(-ad+bc+d(bx+a))^2 d}-\frac{-\frac{2\sin(2bx+2a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2bx-2a-\frac{2(-ad+bc)}{d}\right)\sin\left(\frac{-2ad+2bc}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2ad+2b}{d}\right.}{d}}{d}}{d}\right.}{4}}{b}$
risch	$-\frac{1}{4d(dx+c)^2}+\frac{b^2 e^{-\frac{2i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(2ixb+2ia-\frac{2i(ad-bc)}{d}\right)}{2d^3}+\frac{b^2 e^{\frac{2i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-2ixb-2ia-\frac{2(-iad+ibc)}{d}\right)}{2d^3}+\frac{(-2b}{8}$

input
```
int(cos(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b^3*(-cos(2*b*x+2*a)/(-a*d+b*c+d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/
(-a*d+b*c+d*(b*x+a))/d+2*(-2*Si(-2*b*x-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c
)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)-1/4*b^3
/(-a*d+b*c+d*(b*x+a))^2/d)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.57

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{d^2\cos(bx+a)^2+2\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-2\left(bd^2x+bcd\right)\cos(bx+}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(d^2*cos(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*
(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 2*(b*d^2*x + b*c*d)*cos(b
*x + a)*sin(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c
 - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3
)
```

3.15.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(cos(a + b*x)**2/(c + d*x)**3, x)
```

3.15.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.82

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = -\frac{b^3\left(E_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+E_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(iE_3\left(\frac{2(-ibc-i(bx+a)d+i}{d}\right.\right.}{4\left(b^2c^2d-2abcd^2+(bx+a)^2d^3+a^2d^3+2(bcd^2-ad^3)\right.}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_i
ntegral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)/d)
 + b^3*(I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_
integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d
) + b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*b)
```

3.15.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 5136, normalized size of antiderivative = 45.86

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b^2*d^2*
x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2
*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*
c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2*tan(a)*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*real_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*real
_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b^2*d^2*x^2*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4
*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) - 4*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^2*c*d*x*sin_integral(2*(b*d*x + b
*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b^2*d^2*x^2*real_part(cos_integ...
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int(cos(a + b*x)^2/(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)^2/(c + d*x)^3, x)
```

3.16 $\int (c+dx)^4 \cos^3(a+bx)\,dx$

3.16.1 Optimal result

Integrand size = 16, antiderivative size = 225

$$\int (c+dx)^4 \cos^3(a+bx)\,dx = -\frac{160d^3(c+dx)\cos(a+bx)}{9b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{3b^2} - \frac{8d^3(c+dx)\cos^3(a+bx)}{27b^4} + \frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2} + \frac{488d^4\sin(a+bx)}{27b^5} - \frac{80d^2(c+dx)^2\sin(a+bx)}{9b^3} + \frac{2(c+dx)^4\sin(a+bx)}{3b} - \frac{4d^2(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{9b^3} + \frac{(c+dx)^4\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{8d^4\sin^3(a+bx)}{81b^5}$$

output
```
-160/9*d^3*(d*x+c)*cos(b*x+a)/b^4+8/3*d*(d*x+c)^3*cos(b*x+a)/b^2-8/27*d^3*
(d*x+c)*cos(b*x+a)^3/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)^3/b^2+488/27*d^4*sin(b
*x+a)/b^5-80/9*d^2*(d*x+c)^2*sin(b*x+a)/b^3+2/3*(d*x+c)^4*sin(b*x+a)/b-4/9
*d^2*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b^3+1/3*(d*x+c)^4*cos(b*x+a)^2*sin(
b*x+a)/b-8/81*d^4*sin(b*x+a)^3/b^5
```

3.16.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.71

$$\int (c+dx)^4\cos^3(a+bx)\,dx$$
$$=\frac{972bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)+12bd(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))+2}{}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^3,x]
```

output
```
(972*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 12*b*d*(c + d
*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 243*b^4*c^4*Sin[a + b*
x] - 2916*b^2*c^2*d^2*Sin[a + b*x] + 5832*d^4*Sin[a + b*x] + 972*b^4*c^3*d
*x*Sin[a + b*x] - 5832*b^2*c*d^3*x*Sin[a + b*x] + 1458*b^4*c^2*d^2*x^2*Sin
[a + b*x] - 2916*b^2*d^4*x^2*Sin[a + b*x] + 972*b^4*c*d^3*x^3*Sin[a + b*x]
 + 243*b^4*d^4*x^4*Sin[a + b*x] + 27*b^4*c^4*Sin[3*(a + b*x)] - 36*b^2*c^2
*d^2*Sin[3*(a + b*x)] + 8*d^4*Sin[3*(a + b*x)] + 108*b^4*c^3*d*x*Sin[3*(a
+ b*x)] - 72*b^2*c*d^3*x*Sin[3*(a + b*x)] + 162*b^4*c^2*d^2*x^2*Sin[3*(a +
 b*x)] - 36*b^2*d^4*x^2*Sin[3*(a + b*x)] + 108*b^4*c*d^3*x^3*Sin[3*(a + b*
x)] + 27*b^4*d^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

3.16.3 Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.35, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 3792, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\cos^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^4\sin\left(a+bx+\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{3792}$$

$$-\frac{4d^2\int(c+dx)^2\cos^3(a+bx)dx}{3b^2}+\frac{2}{3}\int(c+dx)^4\cos(a+bx)dx+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{4d^2\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^4\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$-\frac{4d^2\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{4d\int-(c+dx)^3\sin(a+bx)dx}{b}+\frac{(c+dx)^4\sin(a+bx)}{b}\right)+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 25

$$-\frac{4d^2\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\int(c+dx)^3\sin(a+bx)dx}{b}\right)+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{4d^2\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\int(c+dx)^3\sin(a+bx)dx}{b}\right)+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$-\frac{4d^2\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\int(c+dx)^2\cos(a+bx)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

3.16. $\int(c+dx)^4\cos^3(a+bx)\,dx$

$$-\frac{4d^2\int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)^3 dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{4d^2\int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)^3 dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4d^2\int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)^3 dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4d^2\int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)^3 dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

3.16. $\int (c+dx)^4\cos^3(a+bx)\,dx$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+ -\frac{4d^2\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+ -\frac{4d^2\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{4d^2\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

3.16. $\int(c+dx)^4\cos^3(a+bx)\,dx$

$$\Big\downarrow \text{3792}$$

$$-\frac{4d^2\left(-\frac{2d^2\int \cos^3(a+bx)dx}{9b^2}+\frac{2}{3}\int (c+dx)^2\cos(a+bx)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4d^2\left(-\frac{2d^2\int \sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{9b^2}+\frac{2}{3}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3113}$$

3.16. $\int (c+dx)^4\cos^3(a+bx)\,dx$

$$-\frac{4d^2\left(\frac{2d^2\int\left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(}{3b}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+$$

$$\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow\ 2009$$

$$-\frac{4d^2\left(\frac{2}{3}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+$$

$$\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow\ 3777$$

3.16. $\int(c+dx)^4\cos^3(a+bx)\,dx$

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a\cdots}{}\right.\cdots}{3b^2}$$

$$\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 25$$

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx\cdots}{3b}\right.\cdots}{3b^2}$$

$$\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3042$$

3.16. $\int(c+dx)^4\cos^3(a+bx)\,dx$

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)}{3b}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+$$

$$\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c\text{-}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+$$

$$\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

3.16. $\int(c+dx)^4\cos^3(a+bx)\,dx$

$$-\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)+$$

$$\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{4d(c+dx)^3\cos^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}\right)-$$

$$\frac{4d^2\left(\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^2 s\right.}{3b^2}$$

$$\frac{(c+dx)^4\sin(a+bx)\cos^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^3,x]
```

3.16. $\int(c+dx)^4\cos^3(a+bx)\,dx$

output
```
(4*d*(c + d*x)^3*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)^4*Cos[a + b*x]^2*Sin
[a + b*x])/(3*b) - (4*d^2*((2*d*(c + d*x)*Cos[a + b*x]^3)/(9*b^2) + ((c +
d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*d^2*(-Sin[a + b*x] + Sin[a
+ b*x]^3/3))/(9*b^3) + (2*(((c + d*x)^2*Sin[a + b*x])/b - (2*d*(-(((c + d*
x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/3))/(3*b^2) + (2*(((c + d
*x)^4*Sin[a + b*x])/b - (4*d*(-(((c + d*x)^3*Cos[a + b*x])/b) + (3*d*(((c
+ d*x)^2*Sin[a + b*x])/b - (2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a
+ b*x])/b^2))/b))/b))/b))/3
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.16.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{\left(27(dx+c)^4b^4-36d^2(dx+c)^2b^2+8d^4\right)\sin(3bx+3a)+36b(dx+c)d\left((dx+c)^2b^2-\frac{2d^2}{3}\right)\cos(3bx+3a)+243\left((dx+c)^4b^4-12d\right.}{324b^5}$
risch	$\frac{3d(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c)\cos(bx+a)}{b^4}+\frac{3(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-1}{4b^5}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/324*((27*(d*x+c)^4*b^4-36*d^2*(d*x+c)^2*b^2+8*d^4)*sin(3*b*x+3*a)+36*b*(
d*x+c)*d*((d*x+c)^2*b^2-2/3*d^2)*cos(3*b*x+3*a)+243*((d*x+c)^4*b^4-12*d^2*
(d*x+c)^2*b^2+24*d^4)*sin(b*x+a)+972*b*((d*x+c)*((d*x+c)^2*b^2-6*d^2)*cos(
b*x+a)+28/27*b^2*c^3-488/81*d^2*c)*d)/b^5
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.56

$$\int (c+dx)^4\cos^3(a+bx)\,dx$$
$$=\frac{12\,(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-2\,bcd^3+(9\,b^3c^2d^2-2\,bd^4)x)\cos{(bx+a)}^3+72\,(3\,b^3d^4x^3+9\,b^3cd^3x^2}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b
^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 + 72*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*
x^2 + 3*b^3*c^3*d - 20*b*c*d^3 + (9*b^3*c^2*d^2 - 20*b*d^4)*x)*cos(b*x + a
) + (54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 720*b^2*c^2*d^2 + 1
456*d^4 + 36*(9*b^4*c^2*d^2 - 20*b^2*d^4)*x^2 + (27*b^4*d^4*x^4 + 108*b^4*
c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^
2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 72*(3*b^4*
c^3*d - 20*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.

Time = 0.73 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.43

$$\int (c+dx)^4 \cos^3(a+bx)\,dx$$
$$= \begin{cases} \frac{2c^4\sin^3(a+bx)}{3b} + \frac{c^4\sin(a+bx)\cos^2(a+bx)}{b} + \frac{8c^3dx\sin^3(a+bx)}{3b} + \frac{4c^3dx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{4c^2d^2x^2\sin^3(a+bx)}{b} + \frac{6c^2d^2x^2\text{si}}{} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**3,x)
```

output
```
Piecewise((2*c**4*sin(a + b*x)**3/(3*b) + c**4*sin(a + b*x)*cos(a + b*x)**
2/b + 8*c**3*d*x*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)*cos(a + b
*x)**2/b + 4*c**2*d**2*x**2*sin(a + b*x)**3/b + 6*c**2*d**2*x**2*sin(a + b
*x)*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**3/(3*b) + 4*c*d**3*x**
3*sin(a + b*x)*cos(a + b*x)**2/b + 2*d**4*x**4*sin(a + b*x)**3/(3*b) + d**
4*x**4*sin(a + b*x)*cos(a + b*x)**2/b + 8*c**3*d*sin(a + b*x)**2*cos(a + b
*x)/(3*b**2) + 28*c**3*d*cos(a + b*x)**3/(9*b**2) + 8*c**2*d**2*x*sin(a +
b*x)**2*cos(a + b*x)/b**2 + 28*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 8*c*
d**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 28*c*d**3*x**2*cos(a + b*x)*
*3/(3*b**2) + 8*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 28*d**4*
x**3*cos(a + b*x)**3/(9*b**2) - 80*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 28
*c**2*d**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 160*c*d**3*x*sin(a + b*
x)**3/(9*b**3) - 56*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 80*d*
*4*x**2*sin(a + b*x)**3/(9*b**3) - 28*d**4*x**2*sin(a + b*x)*cos(a + b*x)*
*2/(3*b**3) - 160*c*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 488*c*d**
3*cos(a + b*x)**3/(27*b**4) - 160*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b
**4) - 488*d**4*x*cos(a + b*x)**3/(27*b**4) + 1456*d**4*sin(a + b*x)**3/(8
1*b**5) + 488*d**4*sin(a + b*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c*
*4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cos(a
)**3, True))
```

3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(205) = 410$.

Time = 0.33 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.11

$$\int (c+dx)^4 \cos^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/324*(108*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^4 - 432*(sin(b*x + a)^3 -
3*sin(b*x + a))*a*c^3*d/b + 648*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*c^2*
d^2/b^2 - 432*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^3*c*d^3/b^3 + 108*(sin(b
*x + a)^3 - 3*sin(b*x + a))*a^4*d^4/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a)
 + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*c^3*d/b
 + 108*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b
*x + 3*a) + 27*cos(b*x + a))*a*c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x +
3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a^2
*c*d^3/b^3 + 36*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) + 27*cos(b*x + a))*a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*
b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x +
3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 + 36*(6*(b*x + a)*co
s(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*
x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 - 18*(6*(b*x + a
)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 - 12*((9*(b*
x + a)^2 - 2)*cos(3*b*x + 3*a) + 243*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3
*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 81*((b*x + a)^3 - 6*b*x - 6
*a)*sin(b*x + a))*c*d^3/b^3 + 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 2
43*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin...
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.56

$$\int (c+dx)^4 \cos^3(a+bx)\,dx$$
$$= \frac{(3\,b^3d^4x^3 + 9\,b^3cd^3x^2 + 9\,b^3c^2d^2x + 3\,b^3c^3d - 2\,bd^4x - 2\,bcd^3)\cos(3\,bx+3\,a)}{27\,b^5}$$
$$+ \frac{3\,(b^3d^4x^3 + 3\,b^3cd^3x^2 + 3\,b^3c^2d^2x + b^3c^3d - 6\,bd^4x - 6\,bcd^3)\cos(bx+a)}{b^5}$$
$$+ \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 + 108\,b^4c^3dx + 27\,b^4c^4 - 36\,b^2d^4x^2 - 72\,b^2cd^3x - 36\,b^2c^2d^2 + 8}{324\,b^5}$$
$$+ \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^4c^3dx + b^4c^4 - 12\,b^2d^4x^2 - 24\,b^2cd^3x - 12\,b^2c^2d^2 + 24\,d^4)\sin(bx}{4\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*
b*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + 3*(b^3*d^4*x^3 + 3*b^3*c*d^3*x
^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5
 + 1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b
^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2
 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 3/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^
4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x
- 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

3.16.9 Mupad [B] (verification not implemented)

Time = 15.57 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.36

$$\int (c+dx)^4\cos^3(a+bx)\,dx = \frac{2\sin(a+b\,x)^3\,(27\,b^4\,c^4-360\,b^2\,c^2\,d^2+728\,d^4)}{81\,b^5} - \frac{4\cos(a+b\,x)^3\,(122\,c\,d^3-21\,b^2\,c^3\,d)}{27\,b^4} + \frac{\cos(a+b\,x)^2\,\sin(a+b\,x)\,\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{27\,b^5} - \frac{8\,\cos(a+b\,x)\,\sin(a+b\,x)^2\,(20\,c\,d^3-3\,b^2\,c^3\,d)}{9\,b^4} + \frac{28\,d^4\,x^3\cos(a+b\,x)^3}{9\,b^2} - \frac{4\,x\cos(a+b\,x)^3\,(122\,d^4-63\,b^2\,c^2\,d^2)}{27\,b^4} + \frac{2\,d^4\,x^4\sin(a+b\,x)^3}{3\,b} - \frac{8\,x\sin(a+b\,x)^3\,(20\,c\,d^3-3\,b^2\,c^3\,d)}{9\,b^3} - \frac{4\,x^2\sin(a+b\,x)^3\,(20\,d^4-9\,b^2\,c^2\,d^2)}{9\,b^3} - \frac{2\,x^2\cos(a+b\,x)^2\,\sin(a+b\,x)\,\left(14\,d^4-9\,b^2\,c^2\,d^2\right)}{3\,b^3} + \frac{28\,c\,d^3\,x^2\cos(a+b\,x)^3}{3\,b^2} + \frac{d^4\,x^4\cos(a+b\,x)^2\,\sin(a+b\,x)}{b} + \frac{8\,d^4\,x^3\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{3\,b^2} + \frac{8\,c\,d^3\,x^3\sin(a+b\,x)^3}{3\,b} - \frac{8\,x\,\cos(a+b\,x)\,\sin(a+b\,x)^2\,(20\,d^4-9\,b^2\,c^2\,d^2)}{9\,b^4} - \frac{4\,x\cos(a+b\,x)^2\,\sin(a+b\,x)\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{3\,b^3} + \frac{4\,c\,d^3\,x^3\cos(a+b\,x)^2\,\sin(a+b\,x)}{b} + \frac{8\,c\,d^3\,x^2\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{b^2}$$

input
```
int(cos(a + b*x)^3*(c + d*x)^4,x)
```

output
```
(2*sin(a + b*x)^3*(728*d^4 + 27*b^4*c^4 - 360*b^2*c^2*d^2))/(81*b^5) - (4*
cos(a + b*x)^3*(122*c*d^3 - 21*b^2*c^3*d))/(27*b^4) + (cos(a + b*x)^2*sin(
a + b*x)*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(27*b^5) - (8*cos(a + b
*x)*sin(a + b*x)^2*(20*c*d^3 - 3*b^2*c^3*d))/(9*b^4) + (28*d^4*x^3*cos(a +
 b*x)^3)/(9*b^2) - (4*x*cos(a + b*x)^3*(122*d^4 - 63*b^2*c^2*d^2))/(27*b^4
) + (2*d^4*x^4*sin(a + b*x)^3)/(3*b) - (8*x*sin(a + b*x)^3*(20*c*d^3 - 3*b
^2*c^3*d))/(9*b^3) - (4*x^2*sin(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(9*b^
3) - (2*x^2*cos(a + b*x)^2*sin(a + b*x)*(14*d^4 - 9*b^2*c^2*d^2))/(3*b^3)
+ (28*c*d^3*x^2*cos(a + b*x)^3)/(3*b^2) + (d^4*x^4*cos(a + b*x)^2*sin(a +
b*x))/b + (8*d^4*x^3*cos(a + b*x)*sin(a + b*x)^2)/(3*b^2) + (8*c*d^3*x^3*s
in(a + b*x)^3)/(3*b) - (8*x*cos(a + b*x)*sin(a + b*x)^2*(20*d^4 - 9*b^2*c^
2*d^2))/(9*b^4) - (4*x*cos(a + b*x)^2*sin(a + b*x)*(14*c*d^3 - 3*b^2*c^3*d
))/(3*b^3) + (4*c*d^3*x^3*cos(a + b*x)^2*sin(a + b*x))/b + (8*c*d^3*x^2*co
s(a + b*x)*sin(a + b*x)^2)/b^2
```

3.16. $\int (c+dx)^4 \cos^3(a+bx)\,dx$

3.17 $\int (c+dx)^3 \cos^3(a+bx)\,dx$

3.17.1 Optimal result

Integrand size = 16, antiderivative size = 175

$$\int (c+dx)^3 \cos^3(a+bx)\,dx = -\frac{40d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{b^2} - \frac{2d^3\cos^3(a+bx)}{27b^4} + \frac{d(c+dx)^2\cos^3(a+bx)}{3b^2} - \frac{40d^2(c+dx)\sin(a+bx)}{9b^3} + \frac{2(c+dx)^3\sin(a+bx)}{3b} - \frac{2d^2(c+dx)\cos^2(a+bx)\sin(a+bx)}{9b^3} + \frac{(c+dx)^3\cos^2(a+bx)\sin(a+bx)}{3b}$$

output
```
-40/9*d^3*cos(b*x+a)/b^4+2*d*(d*x+c)^2*cos(b*x+a)/b^2-2/27*d^3*cos(b*x+a)^
3/b^4+1/3*d*(d*x+c)^2*cos(b*x+a)^3/b^2-40/9*d^2*(d*x+c)*sin(b*x+a)/b^3+2/3
*(d*x+c)^3*sin(b*x+a)/b-2/9*d^2*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^3+1/3*(d
*x+c)^3*cos(b*x+a)^2*sin(b*x+a)/b
```

3.17.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.69

$$\int (c+dx)^3\cos^3(a+bx)\,dx$$
$$=\frac{243d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+6b(c+dx)\left(-82d^2+15\right.}{108b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^3,x]
```

output
```
(243*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(c + d*
x)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2
*d^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^4)
```

3.17.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.23, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3042, 3792, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{2d^2\int (c+dx)\cos^3(a+bx)dx}{3b^2}+\frac{2}{3}\int (c+dx)^3\cos(a+bx)dx+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int-(c+dx)^2\sin(a+bx)dx}{b}+\frac{(c+dx)^3\sin(a+bx)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 25

$$-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int(c+dx)^2\sin(a+bx)dx}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int(c+dx)^2\sin(a+bx)dx}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3777

3.17. $\int(c+dx)^3\cos^3(a+bx)\,dx$

$$-\frac{2d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{2d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3791}$$

3.17. $\int (c+dx)^3\cos^3(a+bx)\,dx$

$$-\frac{2d^2\left(\frac{2}{3}\int (c+dx)\cos(a+bx)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2d^2\left(\frac{2}{3}\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 25$$

3.17. $\int (c+dx)^3\cos^3(a+bx)\,dx$

$$-\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3118

$$-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^3,x]
```

3.17. $\int (c+dx)^3\cos^3(a+bx)\,dx$

output
```
(d*(c + d*x)^2*Cos[a + b*x]^3)/(3*b^2) + ((c + d*x)^3*Cos[a + b*x]^2*Sin[a
 + b*x])/(3*b) - (2*d^2*((d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Cos[a + b
*x]^2*Sin[a + b*x])/(3*b) + (2*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a +
b*x])/b))/3))/(3*b^2) + (2*(((c + d*x)^3*Sin[a + b*x])/b - (3*d*(-(((c + d
*x)^2*Cos[a + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b
*x])/b))/b))/b))/3
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.17.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.28

method	result
risch	$\frac{9d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(bx+a)}{4b^4}+\frac{3(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6d^2c)\sin(bx+a)}{4b^3}+\frac{d(9x^2}{}$
parallelrisch	$\frac{-126b^2d^2\left(\frac{dx}{2}+c\right)x\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+54b(dx+c)\left((dx+c)^2b^2-\frac{14d^2}{3}\right)\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(\left(-27d^3x^2-54c\,d^2x+162c^2d\right)b^2\right.}{}$
derivativedivides	$\frac{-\frac{a^3d^3\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b^3}+\frac{a^2c\,d^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{b^2}+\frac{3a^2d^3\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}\right.}{b^3}}{}$
default	$\frac{-\frac{a^3d^3\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b^3}+\frac{a^2c\,d^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{b^2}+\frac{3a^2d^3\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}\right.}{b^3}}{}$
norman	$\frac{\frac{d^3x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{126b^2c^2d-244d^3}{27b^4}+\frac{7d^3x^2}{3b^2}+\frac{\left(18b^2c^2d-28d^3\right)\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^4}+\frac{\left(72b^2c^2d-160d^3\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{9b^4}+\frac{14c\,d^{}}{3b^2}}{}$

input
```
int((d*x+c)^3*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
9/4*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*cos(b*x+a)+3/4/b^3*(b^2*
d^3*x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^2)*sin(b*x+a)+
1/108*d*(9*b^2*d^2*x^2+18*b^2*c*d*x+9*b^2*c^2-2*d^2)/b^4*cos(3*b*x+3*a)+1/
36/b^3*(3*b^2*d^3*x^3+9*b^2*c*d^2*x^2+9*b^2*c^2*d*x+3*b^2*c^3-2*d^3*x-2*c*
d^2)*sin(3*b*x+3*a)
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.30

$$\int (c+dx)^3\cos^3(a+bx)\,dx$$
$$=\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3+6\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-20\,d^3\right)\cos\left(bx+\right.}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^
3 + 6*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 20*d^3)*cos(b*x + a)
 + 3*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - 40*b*c*d^2 + (3*b^3*d
^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)
*x)*cos(b*x + a)^2 + 2*(9*b^3*c^2*d - 20*b*d^3)*x)*sin(b*x + a))/b^4
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.

Time = 0.49 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.83

$$\int (c+dx)^3 \cos^3(a+bx)\,dx$$
$$= \begin{cases} \frac{2c^3\sin^3(a+bx)}{3b} + \frac{c^3\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2c^2dx\sin^3(a+bx)}{b} + \frac{3c^2dx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2cd^2x^2\sin^3(a+bx)}{b} + \frac{3cd^2x^2\sin(\\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**3,x)
```

output
```
Piecewise((2*c**3*sin(a + b*x)**3/(3*b) + c**3*sin(a + b*x)*cos(a + b*x)**
2/b + 2*c**2*d*x*sin(a + b*x)**3/b + 3*c**2*d*x*sin(a + b*x)*cos(a + b*x)*
*2/b + 2*c*d**2*x**2*sin(a + b*x)**3/b + 3*c*d**2*x**2*sin(a + b*x)*cos(a
+ b*x)**2/b + 2*d**3*x**3*sin(a + b*x)**3/(3*b) + d**3*x**3*sin(a + b*x)*c
os(a + b*x)**2/b + 2*c**2*d*sin(a + b*x)**2*cos(a + b*x)/b**2 + 7*c**2*d*c
os(a + b*x)**3/(3*b**2) + 4*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)/b**2 + 1
4*c*d**2*x*cos(a + b*x)**3/(3*b**2) + 2*d**3*x**2*sin(a + b*x)**2*cos(a +
b*x)/b**2 + 7*d**3*x**2*cos(a + b*x)**3/(3*b**2) - 40*c*d**2*sin(a + b*x)*
*3/(9*b**3) - 14*c*d**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 40*d**3*x*
sin(a + b*x)**3/(9*b**3) - 14*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3)
 - 40*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 122*d**3*cos(a + b*x)**
3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x*
*4/4)*cos(a)**3, True))
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(161) = 322$.

Time = 0.28 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.06

$$\int (c+dx)^3 \cos^3(a+bx)\, dx =$$

$$-\frac{36\left(\sin\left(bx+a\right)^3 - 3\sin\left(bx+a\right)\right)c^3 - \frac{108\left(\sin(bx+a)^3 - 3\sin(bx+a)\right)ac^2d}{b} + \frac{108\left(\sin(bx+a)^3 - 3\sin(bx+a)\right)a^2cd^2}{b^2} - \frac{36\,(}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/108*(36*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^3 - 108*(sin(b*x + a)^3 - 3
*sin(b*x + a))*a*c^2*d/b + 108*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*c*d^2
/b^2 - 36*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^3*d^3/b^3 - 9*(3*(b*x + a)*s
in(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*
x + a))*c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x
+ a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*si
n(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x
 + a))*a^2*d^3/b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b
*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(
b*x + a))*c*d^2/b^2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(
b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin
(b*x + a))*a*d^3/b^3 - ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 243*((b*x +
 a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a)
 + 81*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.32

$$\begin{aligned}
&\int (c+dx)^3 \cos^3(a+bx)\, dx \\
&= \frac{\left(9\,b^2d^3x^2 + 18\,b^2cd^2x + 9\,b^2c^2d - 2\,d^3\right)\cos\left(3\,bx + 3\,a\right)}{108\,b^4} \\
&+ \frac{9\left(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d - 2\,d^3\right)\cos\left(bx + a\right)}{4\,b^4} \\
&+ \frac{\left(3\,b^3d^3x^3 + 9\,b^3cd^2x^2 + 9\,b^3c^2dx + 3\,b^3c^3 - 2\,bd^3x - 2\,bcd^2\right)\sin\left(3\,bx + 3\,a\right)}{36\,b^4} \\
&+ \frac{3\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 6\,bd^3x - 6\,bcd^2\right)\sin\left(bx + a\right)}{4\,b^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3
*a)/b^4 + 9/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x +
a)/b^4 + 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3
 - 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 3/4*(b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/
b^4
```

3.17.9 Mupad [B] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.08

$$\int (c+dx)^3\cos^3(a+bx)\,dx = \frac{7\,d^3\,x^2\cos\left(a+b\,x\right)^3}{3\,b^2} - \frac{2\sin\left(a+b\,x\right)^3\left(20\,c\,d^2-3\,b^2\,c^3\right)}{9\,b^3} - \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,c\,d^2-3\,b^2\,c^3\right)}{3\,b^3} - \frac{2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\left(20\,d^3-9\,b^2\,c^2\,d\right)}{9\,b^4} - \frac{2\,x\sin\left(a+b\,x\right)^3\left(20\,d^3-9\,b^2\,c^2\,d\right)}{9\,b^3} - \frac{\cos\left(a+b\,x\right)^3\left(122\,d^3-63\,b^2\,c^2\,d\right)}{27\,b^4} + \frac{2\,d^3\,x^3\sin\left(a+b\,x\right)^3}{3\,b} + \frac{14\,c\,d^2\,x\cos\left(a+b\,x\right)^3}{3\,b^2} - \frac{x\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,d^3-9\,b^2\,c^2\,d\right)}{3\,b^3} + \frac{d^3\,x^3\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{b} + \frac{2\,d^3\,x^2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b^2} + \frac{2\,c\,d^2\,x^2\sin\left(a+b\,x\right)^3}{b} + \frac{3\,c\,d^2\,x^2\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{b} + \frac{4\,c\,d^2\,x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b^2}$$

input
```
int(cos(a + b*x)^3*(c + d*x)^3,x)
```

output
```
(7*d^3*x^2*cos(a + b*x)^3)/(3*b^2) - (2*sin(a + b*x)^3*(20*c*d^2 - 3*b^2*c
^3))/(9*b^3) - (cos(a + b*x)^2*sin(a + b*x)*(14*c*d^2 - 3*b^2*c^3))/(3*b^3
) - (2*cos(a + b*x)*sin(a + b*x)^2*(20*d^3 - 9*b^2*c^2*d))/(9*b^4) - (2*x*
sin(a + b*x)^3*(20*d^3 - 9*b^2*c^2*d))/(9*b^3) - (cos(a + b*x)^3*(122*d^3
- 63*b^2*c^2*d))/(27*b^4) + (2*d^3*x^3*sin(a + b*x)^3)/(3*b) + (14*c*d^2*x
*cos(a + b*x)^3)/(3*b^2) - (x*cos(a + b*x)^2*sin(a + b*x)*(14*d^3 - 9*b^2*
c^2*d))/(3*b^3) + (d^3*x^3*cos(a + b*x)^2*sin(a + b*x))/b + (2*d^3*x^2*cos
(a + b*x)*sin(a + b*x)^2)/b^2 + (2*c*d^2*x^2*sin(a + b*x)^3)/b + (3*c*d^2*
x^2*cos(a + b*x)^2*sin(a + b*x))/b + (4*c*d^2*x*cos(a + b*x)*sin(a + b*x)^
2)/b^2
```

3.17. $\int (c+dx)^3 \cos^3(a+bx)\,dx$

3.18 $\int (c+dx)^2 \cos^3(a+bx)\,dx$

3.18.1 Optimal result

Integrand size = 16, antiderivative size = 123

$$\int (c+dx)^2\cos^3(a+bx)\,dx = \frac{4d(c+dx)\cos(a+bx)}{3b^2}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2} - \frac{14d^2\sin(a+bx)}{9b^3}+\frac{2(c+dx)^2\sin(a+bx)}{3b} + \frac{(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b}+\frac{2d^2\sin^3(a+bx)}{27b^3}$$

output
```
4/3*d*(d*x+c)*cos(b*x+a)/b^2+2/9*d*(d*x+c)*cos(b*x+a)^3/b^2-14/9*d^2*sin(b
*x+a)/b^3+2/3*(d*x+c)^2*sin(b*x+a)/b+1/3*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)
/b+2/27*d^2*sin(b*x+a)^3/b^3
```

3.18.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int (c+dx)^2\cos^3(a+bx)\,dx = \frac{162bd(c+dx)\cos(a+bx)+6bd(c+dx)\cos(3(a+bx))+2(-82d^2+45b^2(c+dx)^2+(-2d^2+9b^2(c+d}{108b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^3,x]
```

output
```
(162*b*d*(c + d*x)*Cos[a + b*x] + 6*b*d*(c + d*x)*Cos[3*(a + b*x)] + 2*(-8
2*d^2 + 45*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)]
)*Sin[a + b*x])/(108*b^3)
```

3.18.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{2d^2\int\cos^3(a+bx)dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\cos(a+bx)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d^2\int\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3113}$$

$$\frac{2d^2\int\left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\int(c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3777$$

$$\frac{2}{3}\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 25$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3777$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3117$$

$$\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^3,x]
```

3.18. $\int (c+dx)^2\cos^3(a+bx)\,dx$

output
```
(2*d*(c + d*x)*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)^2*Cos[a + b*x]^2*Sin[a
 + b*x])/(3*b) + (2*d^2*(-Sin[a + b*x] + Sin[a + b*x]^3/3))/(9*b^3) + (2*(
((c + d*x)^2*Sin[a + b*x])/b - (2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Si
n[a + b*x])/b^2))/b))/3
```

3.18.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.18.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

method	result
risch	$\frac{3d(dx+c)\cos(bx+a)}{2b^2}+\frac{3(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(bx+a)}{4b^3}+\frac{d(dx+c)\cos(3bx+3a)}{18b^2}+\frac{(9x^2d^2b^2+18b^2cdx+9}{10}$
parallelrisch	$\frac{-42d^2\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)xb+\left(54(dx+c)^2b^2-84d^2\right)\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+108b\left(-\frac{dx}{6}+c\right)d\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\left(36(dx+c)^2b^2-\right.}{27b^3\left(1+\tan^2\left(\frac{bx}{2}\right.\right.}$
derivativedivides	$\frac{\frac{a^2d^2(2+\cos^2(bx+a))\sin(bx+a)}{3b^2}-\frac{2acd(2+\cos^2(bx+a))\sin(bx+a)}{3b}-\frac{2a\,d^2\left(\frac{(bx+a)(2+\cos^2(bx+a))\sin(bx+a)}{3}+\frac{(\cos^3(bx+a))}{9}+\frac{2c}{}\right.}{b^2}}{}$
default	$\frac{\frac{a^2d^2(2+\cos^2(bx+a))\sin(bx+a)}{3b^2}-\frac{2acd(2+\cos^2(bx+a))\sin(bx+a)}{3b}-\frac{2a\,d^2\left(\frac{(bx+a)(2+\cos^2(bx+a))\sin(bx+a)}{3}+\frac{(\cos^3(bx+a))}{9}+\frac{2c}{}\right.}{b^2}}{}$
norman	$\frac{4cd\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{28cd}{9b^2}+\frac{14d^2x}{9b^2}+\frac{4(9b^2c^2-38d^2)\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{27b^3}+\frac{2(9b^2c^2-14d^2)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{9b^3}+\frac{2(9b^2c^2-14d^2)\left(\tan^5\left(\frac{bx}{2}\right.\right.}{9b^3}$

input
```
int((d*x+c)^2*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3/2*d*(d*x+c)*cos(b*x+a)/b^2+3/4*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b
^3*sin(b*x+a)+1/18/b^2*d*(d*x+c)*cos(3*b*x+3*a)+1/108*(9*b^2*d^2*x^2+18*b^
2*c*d*x+9*b^2*c^2-2*d^2)/b^3*sin(3*b*x+3*a)
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04

$$\int (c+dx)^2\cos^3(a+bx)\,dx$$
$$=\frac{6\,(bd^2x+bcd)\cos{(bx+a)}^3+36\,(bd^2x+bcd)\cos{(bx+a)}+\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2+\left(9\,b^2d^2x^2\right.\right.}{27\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 + 36*(b*d^2*x + b*c*d)*cos(b*x +
a) + (18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 + (9*b^2*d^2*x^2 + 18*b^2
*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 40*d^2)*sin(b*x + a))/b^3
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.

Time = 0.38 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.31

$$\int (c+dx)^2\cos^3(a+bx)\,dx$$
$$= \begin{cases} \frac{2c^2\sin^3(a+bx)}{3b} + \frac{c^2\sin(a+bx)\cos^2(a+bx)}{b} + \frac{4cdx\sin^3(a+bx)}{3b} + \frac{2cdx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2d^2x^2\sin^3(a+bx)}{3b} + \frac{d^2x^2\sin(a+bx)}{b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**3,x)
```

output
```
Piecewise((2*c**2*sin(a + b*x)**3/(3*b) + c**2*sin(a + b*x)*cos(a + b*x)**
2/b + 4*c*d*x*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)*cos(a + b*x)**2
/b + 2*d**2*x**2*sin(a + b*x)**3/(3*b) + d**2*x**2*sin(a + b*x)*cos(a + b*
x)**2/b + 4*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 14*c*d*cos(a + b*x
)**3/(9*b**2) + 4*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 14*d**2*x
*cos(a + b*x)**3/(9*b**2) - 40*d**2*sin(a + b*x)**3/(27*b**3) - 14*d**2*si
n(a + b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2
*x**3/3)*cos(a)**3, True))
```

3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(111) = 222$.

Time = 0.25 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.17

$$\int (c+dx)^2\cos^3(a+bx)\,dx =$$
$$-\frac{36\left(\sin(bx+a)^3 - 3\sin(bx+a)\right)c^2 - \frac{72\left(\sin(bx+a)^3-3\sin(bx+a)\right)acd}{b} + \frac{36\left(\sin(bx+a)^3-3\sin(bx+a)\right)a^2d^2}{b^2} - \frac{6(3(bx\cdots}{}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/108*(36*(sin(b*x + a)^3 - 3*sin(b*x + a))*c^2 - 72*(sin(b*x + a)^3 - 3*
sin(b*x + a))*a*c*d/b + 36*(sin(b*x + a)^3 - 3*sin(b*x + a))*a^2*d^2/b^2 -
 6*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x + a) + cos(3*b*x +
 3*a) + 27*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x
 + a)*sin(b*x + a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*a*d^2/b^2 - (6*(b
*x + a)*cos(3*b*x + 3*a) + 162*(b*x + a)*cos(b*x + a) + (9*(b*x + a)^2 - 2
)*sin(3*b*x + 3*a) + 81*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.11

$$\int (c+dx)^2\cos^3(a+bx)\,dx = \frac{(bd^2x+bcd)\cos(3\,bx+3\,a)}{18\,b^3}+\frac{3\,(bd^2x+bcd)\cos(bx+a)}{2\,b^3}+\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\sin(3\,bx+3\,a)}{108\,b^3}+\frac{3\,(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{4\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 3/2*(b*d^2*x + b*c*d)*cos(b*
x + a)/b^3 + 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(
3*b*x + 3*a)/b^3 + 3/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b
*x + a)/b^3
```

3.18.9 Mupad [B] (verification not implemented)

Time = 14.47 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.41

$$\int (c+dx)^2\cos^3(a+bx)\,dx = \frac{\frac{d^2\,x\,\cos(3\,a+3\,b\,x)}{18}+\frac{3\,c\,d\,\cos(a+b\,x)}{2}+\frac{c\,d\,\cos(3\,a+3\,b\,x)}{18}+\frac{3\,d^2\,x\,\cos(a+b\,x)}{2}}{b^2}+\frac{\frac{3\,c^2\,\sin(a+b\,x)}{4}+\frac{c^2\,\sin(3\,a+3\,b\,x)}{12}+\frac{3\,d^2\,x^2\,\sin(a+b\,x)}{4}+\frac{d^2\,x^2\,\sin(3\,a+3\,b\,x)}{12}+\frac{3\,c\,d\,x\,\sin(a+b\,x)}{2}+\frac{c\,d\,x\,\sin(3\,a+3\,b\,x)}{6}}{b}-\frac{3\,d^2\,\sin(a+b\,x)}{2\,b^3}-\frac{d^2\,\sin(3\,a+3\,b\,x)}{54\,b^3}$$

input
```
int(cos(a + b*x)^3*(c + d*x)^2,x)
```

output
```
((d^2*x*cos(3*a + 3*b*x))/18 + (3*c*d*cos(a + b*x))/2 + (c*d*cos(3*a + 3*b
*x))/18 + (3*d^2*x*cos(a + b*x))/2)/b^2 + ((3*c^2*sin(a + b*x))/4 + (c^2*s
in(3*a + 3*b*x))/12 + (3*d^2*x^2*sin(a + b*x))/4 + (d^2*x^2*sin(3*a + 3*b*
x))/12 + (3*c*d*x*sin(a + b*x))/2 + (c*d*x*sin(3*a + 3*b*x))/6)/b - (3*d^2
*sin(a + b*x))/(2*b^3) - (d^2*sin(3*a + 3*b*x))/(54*b^3)
```

3.18. $\int (c+dx)^2 \cos^3(a+bx)\,dx$

3.19 $\int (c+dx)\cos^3(a+bx)\,dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 75

$$\int (c+dx)\cos^3(a+bx)\,dx=\frac{2d\cos(a+bx)}{3b^2}+\frac{d\cos^3(a+bx)}{9b^2}+\frac{2(c+dx)\sin(a+bx)}{3b}+\frac{(c+dx)\cos^2(a+bx)\sin(a+bx)}{3b}$$

output
```
2/3*d*cos(b*x+a)/b^2+1/9*d*cos(b*x+a)^3/b^2+2/3*(d*x+c)*sin(b*x+a)/b+1/3*(
d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b
```

3.19.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int (c+dx)\cos^3(a+bx)\,dx = \frac{27d\cos(a+bx)+d\cos(3(a+bx))+3b(c+dx)(9\sin(a+bx)+\sin(3(a+bx)))}{36b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^3,x]
```

output
```
(27*d*Cos[a + b*x] + d*Cos[3*(a + b*x)] + 3*b*(c + d*x)*(9*Sin[a + b*x] +
Sin[3*(a + b*x)]))/(36*b^2)
```

3.19.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \ 3791$$

$$\frac{2}{3}\int (c+dx)\cos(a+bx)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \ 3777$$

$$\frac{2}{3}\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \ 25$$

$$\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \ 3118$$

$$\frac{2}{3}\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^3,x]
```

output
```
(d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b)
 + (2*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/3
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.19.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{3b(dx+c)\sin(3bx+3a)+d\cos(3bx+3a)+27(dx+c)b\sin(bx+a)+27d\cos(bx+a)+28d}{36b^2}$
risch	$\frac{3d\cos(bx+a)}{4b^2}+\frac{3(dx+c)\sin(bx+a)}{4b}+\frac{d\cos(3bx+3a)}{36b^2}+\frac{(dx+c)\sin(3bx+3a)}{12b}$
derivativedivides	$-\frac{da\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b}+\frac{c\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+d\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}+\frac{2\cos(bx+a}{3}\right.$
default	$-\frac{da\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3b}+\frac{c\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+d\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{9}+\frac{2\cos(bx+a}{3}\right.$
norman	$\frac{\frac{2d\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}+\frac{14d}{9b^2}+\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4c\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b}+\frac{2c\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b}+\frac{8d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{3b^2}+\frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+4d}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^3}$

input
```
int((d*x+c)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*b*(d*x+c)*sin(3*b*x+3*a)+d*cos(3*b*x+3*a)+27*(d*x+c)*b*sin(b*x+a)+
27*d*cos(b*x+a)+28*d)/b^2
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int (c+dx)\cos^3(a+bx)\,dx$$
$$=\frac{d\cos(bx+a)^3+6\,d\cos(bx+a)+3\left(2\,bdx+(bdx+bc)\cos(bx+a)^2+2\,bc\right)\sin(bx+a)}{9\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/9*(d*cos(b*x + a)^3 + 6*d*cos(b*x + a) + 3*(2*b*d*x + (b*d*x + b*c)*cos(
b*x + a)^2 + 2*b*c)*sin(b*x + a))/b^2
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.68

$$\int (c+dx)\cos^3(a+bx)\,dx$$
$$=\begin{cases} \frac{2c\sin^3(a+bx)}{3b} + \frac{c\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2dx\sin^3(a+bx)}{3b} + \frac{dx\sin(a+bx)\cos^2(a+bx)}{b} + \frac{2d\sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{7d\cos^3(a+b\ldots}{9b^2} \\ \left(cx+\frac{dx^2}{2}\right)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**3,x)
```

output
```
Piecewise((2*c*sin(a + b*x)**3/(3*b) + c*sin(a + b*x)*cos(a + b*x)**2/b +
2*d*x*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)*cos(a + b*x)**2/b + 2*d*sin
(a + b*x)**2*cos(a + b*x)/(3*b**2) + 7*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0
)), ((c*x + d*x**2/2)*cos(a)**3, True))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.37

$$\int (c+dx)\cos^3(a+bx)\,dx =$$
$$-\frac{12\left(\sin(bx+a)^3-3\sin(bx+a)\right)c-\frac{12\left(\sin(bx+a)^3-3\sin(bx+a)\right)ad}{b}-\frac{(3(bx+a)\sin(3bx+3a)+27(bx+a)\sin(bx+a)+\cos\ldots}{b}}{36b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/36*(12*(sin(b*x + a)^3 - 3*sin(b*x + a))*c - 12*(sin(b*x + a)^3 - 3*sin
(b*x + a))*a*d/b - (3*(b*x + a)*sin(3*b*x + 3*a) + 27*(b*x + a)*sin(b*x +
a) + cos(3*b*x + 3*a) + 27*cos(b*x + a))*d/b)/b
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int (c+dx)\cos^3(a+bx)\,dx = \frac{d\cos(3\,bx+3\,a)}{36\,b^2} + \frac{3\,d\cos(bx+a)}{4\,b^2} + \frac{(bdx+bc)\sin(3\,bx+3\,a)}{12\,b^2} + \frac{3\,(bdx+bc)\sin(bx+a)}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
1/36*d*cos(3*b*x + 3*a)/b^2 + 3/4*d*cos(b*x + a)/b^2 + 1/12*(b*d*x + b*c)*
sin(3*b*x + 3*a)/b^2 + 3/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

3.19.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int (c+dx)\cos^3(a+bx)\,dx = \frac{\frac{3\,c\,\sin(a+b\,x)}{4} + \frac{c\,\sin(3\,a+3\,b\,x)}{12} + \frac{d\,x\,\sin(3\,a+3\,b\,x)}{12} + \frac{3\,d\,x\,\sin(a+b\,x)}{4}}{b} + \frac{d\,\cos(3\,a+3\,b\,x)}{36\,b^2} + \frac{3\,d\,\cos(a+b\,x)}{4\,b^2}$$

input
```
int(cos(a + b*x)^3*(c + d*x),x)
```

output
```
((3*c*sin(a + b*x))/4 + (c*sin(3*a + 3*b*x))/12 + (d*x*sin(3*a + 3*b*x))/1
2 + (3*d*x*sin(a + b*x))/4)/b + (d*cos(3*a + 3*b*x))/(36*b^2) + (3*d*cos(a
 + b*x))/(4*b^2)
```

3.20 $\int \frac{\cos^3(a+bx)}{c+dx} \, dx$

3.20.1 Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{\cos^3(a+bx)}{c+dx} \, dx = \frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

output
```
1/4*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d+3/4*Ci(b*c/d+b*x)*cos(a-b*c/d)/d-
1/4*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-3/4*Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

3.20.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)}{c+dx} \, dx = \frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right.}{4d}$$

input
```
Integrate[Cos[a + b*x]^3/(c + d*x),x]
```

output
```
(3*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*CosInt
egral[(3*b*(c + d*x))/d] - 3*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - S
in[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

3.20.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx$$
$$\Big\downarrow \; 3793$$
$$\int \left(\frac{3\cos(a+bx)}{4(c+dx)}+\frac{\cos(3a+3bx)}{4(c+dx)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

input
```
Int[Cos[a + b*x]^3/(c + d*x),x]
```

output
```
(3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)
/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (3*Sin[a - (b*c)/d]*SinIntegra
l[(b*c)/d + b*x])/(4*d) - (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*
b*x])/(4*d)
```

3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.20.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.41

method	result
derivativedivides	$\frac{\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{12}+3b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\ldots}{d}\right.\ldots}{b}$
default	$\frac{\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{12}+3b\left(-\frac{\mathrm{Si}\left(-bx-a-\frac{-ad+bc}{d}\right)\ldots}{d}\right.\ldots}{b}$
risch	$-\frac{e^{-\frac{3i(ad-bc)}{d}}\mathrm{Ei}_1\left(3ixb+3ia-\frac{3i(ad-bc)}{d}\right)}{8d}-\frac{3\,e^{-\frac{i(ad-bc)}{d}}\mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{8d}-\frac{3\,e^{\frac{i(ad-bc)}{d}}\mathrm{Ei}_1\left(-ixb-ia-\ldots\right)}{8d}\ldots$

input
```
int(cos(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3
*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)+3/4*b*(-Si(-b*x-a-(-a*d+b*
c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx$$
$$= \frac{\cos\left(-\frac{3\,(bc-ad)}{d}\right)\,\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+3\,\cos\left(-\frac{bc-ad}{d}\right)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)-\sin\left(-\frac{3\,(bc-ad)}{d}\right)\,\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-3\,\sin\left(-\frac{bc-}{}\right.}{4\,d}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d) + 3*cos(-(b*c -
 a*d)/d)*cos_integral((b*d*x + b*c)/d) - sin(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) - 3*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d)
)/d
```

3.20.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx = \int \frac{\cos^3\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)**3/(c + d*x), x)
```

3.20.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.30

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx =$$
$$-\frac{3\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*(3*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(ex
p_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1,
-3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b*(-I*ex
p_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -
(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(-I*exp_integr
al_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I
*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

3.20.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 6075, normalized size of antiderivative = 50.21

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(cos_integral(b*x + b*c/d))*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*real_part(c
os_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*imag_part(cos_integral(b*x
+ b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*im
ag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d) - 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*im
ag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*
b*c/d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*imag_part(cos_integral(b*x
 + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*i
mag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d
)^2*tan(1/2*b*c/d)^2 + 12*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1
/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/...
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{c+dx}\,dx=\int \frac{{\cos\left(a+b\,x\right)}^3}{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^3/(c + d*x),x)
```

output
```
int(cos(a + b*x)^3/(c + d*x), x)
```

3.21 $\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx$

3.21.1 Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{\cos^3(a+bx)}{d(c+dx)} - \frac{3b\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2} - \frac{3b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d^2} - \frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2} - \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

output
```
-cos(b*x+a)^3/d/(d*x+c)-3/4*b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2-3/4*b*cos(3*a
-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2-3/4*b*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d
^2-3/4*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

3.21.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{3d\cos(a+bx) + d\cos(3(a+bx)) + 3b(c+dx)\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right) + 3b(c+dx)\,\text{CosI}}{}$$

input
```
Integrate[Cos[a + b*x]^3/(c + d*x)^2,x]
```

output
```
-1/4*(3*d*Cos[a + b*x] + d*Cos[3*(a + b*x)] + 3*b*(c + d*x)*CosIntegral[(3
*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 3*b*(c + d*x)*CosIntegral[b*(c/d +
 x)]*Sin[a - (b*c)/d] + 3*b*c*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] +
3*b*d*x*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*c*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*(c + d*x))/d] + 3*b*d*x*Cos[3*a - (3*b*c)/d]*SinInte
gral[(3*b*(c + d*x))/d])/(d^2*(c + d*x))
```

3.21.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3794}$$

$$\frac{3b\int \left(-\frac{\sin(a+bx)}{4(c+dx)}-\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx}{d}-\frac{\cos^3(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b\left(-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}\right)}{d}$$

$$\frac{\cos^3(a+bx)}{d(c+dx)}$$

input
```
Int[Cos[a + b*x]^3/(c + d*x)^2,x]
```

output
```
-(Cos[a + b*x]^3/(d*(c + d*x))) + (3*b*(-1/4*(CosIntegral[(3*b*c)/d + 3*b*
x]*Sin[3*a - (3*b*c)/d])/d - (CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])
/(4*d) - (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4*d)))/d
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.21.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.70

method	result
derivativedivides	$\dfrac{\dfrac{b^2\left(-\frac{3\cos(3bx+3a)}{(-ad+bc+d(bx+a))d}-\frac{3\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{d}\cdots\right)}{12}\cdots}{b}$
default	$\dfrac{\dfrac{b^2\left(-\frac{3\cos(3bx+3a)}{(-ad+bc+d(bx+a))d}-\frac{3\left(-\frac{3\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\cos\left(\frac{-3ad+3bc}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3ad+3bc}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}\right)}{d}\cdots\right)}{12}\cdots}{b}$
risch	$\frac{3ib\,e^{-\frac{3i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(3ixb+3ia-\frac{3i(ad-bc)}{d}\right)}{8d^2}+\frac{3ib\,e^{-\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{8d^2}-\frac{3ib\,e^{\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-ixb-ia\cdots\right.}{8d^2}\cdots$

input
```
int(cos(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b^2*(-3*cos(3*b*x+3*a)/(-a*d+b*c+d*(b*x+a))/d-3*(-3*Si(-3*b*x-3*
a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin
(3*(-a*d+b*c)/d)/d)/d)+3/4*b^2*(-cos(b*x+a)/(-a*d+b*c+d*(b*x+a))/d-(-Si(-b
*x-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*
c)/d)/d)/d))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.22

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{4\,d\cos\left(bx+a\right)^3+3\left(bdx+bc\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+3\left(bdx+bc\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+}{4\left(d^3x+cd^2\right)}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*d*cos(b*x + a)^3 + 3*(b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)*s
in(-(b*c - a*d)/d) + 3*(b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d)*sin(-
3*(b*c - a*d)/d) + 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b
*d*x + b*c)/d) + 3*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x +
 b*c)/d))/(d^3*x + c*d^2)
```

3.21.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)**3/(c + d*x)**2, x)
```

3.21.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.10

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = -\frac{3\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\right.}{}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*(3*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_in
tegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2
*(exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e
(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b^2*
(-I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_
e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(I*exp
_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d +
(b*x + a)*d^2 - a*d^2)*b)
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1000 vs. 2(137) = 274.

Time = 0.44 (sec) , antiderivative size = 1000, normalized size of antiderivative = 6.90

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(((d
*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*
d)/d) + 3*b^3*c*cos_integral(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)
) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) - 3*a*b^2*d*cos_integral(((d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-(b*c - a*d)/d) +
3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos_integral(3*((d*x +
 c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-3*(b*c - a*d)
/d) + 3*b^3*c*cos_integral(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)
) + b*c - a*d)/d)*sin(-3*(b*c - a*d)/d) - 3*a*b^2*d*cos_integral(3*((d*x +
 c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*sin(-3*(b*c - a*d)
/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-(b*c - a*d)
/d)*sin_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a
*d)/d) - 3*b^3*c*cos(-(b*c - a*d)/d)*sin_integral(-((d*x + c)*(b - b*c/(d*
x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*cos(-(b*c - a*d)/d)*si
n_integral(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)
 - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*cos(-3*(b*c - a*d)/
d)*sin_integral(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
a*d)/d) - 3*b^3*c*cos(-3*(b*c - a*d)/d)*sin_integral(-3*((d*x + c)*(b - b*
c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d) + 3*a*b^2*d*cos(-3*(b*c - a*d
)/d)*sin_integral(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b...
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^2} \, dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2} \,dx$$

input
```
int(cos(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)^3/(c + d*x)^2, x)
```

3.22 $\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx$

3.22.1 Optimal result

Integrand size = 16, antiderivative size = 184

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = & -\frac{\cos^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3} \\ & - \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3} \\ & + \frac{3b\cos^2(a+bx)\sin(a+bx)}{2d^2(c+dx)} + \frac{3b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3} \\ & + \frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\end{aligned}$$

output
```
-9/8*b^2*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^3-3/8*b^2*Ci(b*c/d+b*x)*cos(
a-b*c/d)/d^3-1/2*cos(b*x+a)^3/d/(d*x+c)^2+9/8*b^2*Si(3*b*c/d+3*b*x)*sin(3*
a-3*b*c/d)/d^3+3/8*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+3/2*b*cos(b*x+a)^2*s
in(b*x+a)/d^2/(d*x+c)
```

3.22.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{6d\cos(bx)(-d\cos(a)+b(c+dx)\sin(a))+2d\cos(3bx)(-d\cos(3a)+3b(c+dx)\sin(3a))+6d(b(c+dx)\cos\ldots}{}$$

input
```
Integrate[Cos[a + b*x]^3/(c + d*x)^3,x]
```

output
```
(6*d*Cos[b*x]*(-(d*Cos[a]) + b*(c + d*x)*Sin[a]) + 2*d*Cos[3*b*x]*(-(d*Cos
[3*a]) + 3*b*(c + d*x)*Sin[3*a]) + 6*d*(b*(c + d*x)*Cos[a] + d*Sin[a])*Sin
[b*x] + 2*d*(3*b*(c + d*x)*Cos[3*a] + d*Sin[3*a])*Sin[3*b*x] - 6*b^2*(c +
d*x)^2*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)
] - 3*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]))/(16*d^3*(c + d
*x)^2)
```

3.22.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.32, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3795, 3042, 3784, 3042, 3780, 3783, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{9b^2\int \frac{\cos^3(a+bx)}{c+dx}dx}{2d^2}+\frac{3b^2\int \frac{\cos(a+bx)}{c+dx}dx}{d^2}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx}{d^2}-\frac{9b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{9b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}+\frac{3b^2\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{9b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}+\frac{3b^2\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{3b^2\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}-\frac{9b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{9b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}+\frac{3b^2\left(\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{9b^2\int \left(\frac{3\cos(a+bx)}{4(c+dx)}+\frac{\cos(3a+3bx)}{4(c+dx)}\right)dx}{2d^2}+\frac{3b^2\left(\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2009}$$

3.22. $\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx$

$$\frac{3b^2\left(\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}-\frac{9b^2\left(\frac{3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}\right)}{2d^2}+\frac{3b\sin(a+bx)\cos^2(a+bx)}{2d^2(c+dx)}-\frac{\cos^3(a+bx)}{2d(c+dx)^2}$$

input
```
Int[Cos[a + b*x]^3/(c + d*x)^3,x]
```

output
```
-1/2*Cos[a + b*x]^3/(d*(c + d*x)^2) + (3*b*Cos[a + b*x]^2*Sin[a + b*x])/(2
*d^2*(c + d*x)) + (3*b^2*((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d
- (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d))/d^2 - (9*b^2*((3*Cos[a
 - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*CosI
ntegral[(3*b*c)/d + 3*b*x])/(4*d) - (3*Sin[a - (b*c)/d]*SinIntegral[(b*c)/
d + b*x])/(4*d) - (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(4
*d)))/(2*d^2)
```

3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.22.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{b^3\left(-\frac{3\cos(3bx+3a)}{2(-ad+bc+d(bx+a))^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3a\cdots}{}\right)\cdots}{}}{d}\cdots\right)}{2d}\cdots\right)}{12}\cdots$
default	$\frac{b^3\left(-\frac{3\cos(3bx+3a)}{2(-ad+bc+d(bx+a))^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{(-ad+bc+d(bx+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3bx-3a-\frac{3(-ad+bc)}{d}\right)\sin\left(\frac{-3ad+3bc}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3a\cdots}{}\right)\cdots}{}}{d}\cdots\right)}{2d}\cdots\right)}{12}\cdots$
risch	$\frac{9b^2 e^{-\frac{3i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(3ixb+3ia-\frac{3i(ad-bc)}{d}\right)}{16d^3}+\frac{3b^2 e^{-\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(ixb+ia-\frac{i(ad-bc)}{d}\right)}{16d^3}+\frac{3b^2 e^{\frac{i(ad-bc)}{d}}\,\mathrm{Ei}_1\left(-ixb-ia\cdots\right)}{16d^3}\cdots$

input
```
int(cos(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b^3*(-3/2*cos(3*b*x+3*a)/(-a*d+b*c+d*(b*x+a))^2/d-3/2*(-3*sin(3*
b*x+3*a)/(-a*d+b*c+d*(b*x+a))/d+3*(-3*Si(-3*b*x-3*a-3*(-a*d+b*c)/d)*sin(3*
(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d
)+3/4*b^3*(-1/2*cos(b*x+a)/(-a*d+b*c+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-a*d
+b*c+d*(b*x+a))/d+(-Si(-b*x-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(
-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.57

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{4\,d^2\cos\left(bx+a\right)^3 - 12\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2\sin\left(bx+a\right) + 9\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{3\,(bc-\cdots}{d}\right.\cdots}{\cdots}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(4*d^2*cos(b*x + a)^3 - 12*(b*d^2*x + b*c*d)*cos(b*x + a)^2*sin(b*x +
 a) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*cos_in
tegral(3*(b*d*x + b*c)/d) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(
b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 9*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 3*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x
+ b*c)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

3.22.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos^3{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(cos(a + b*x)**3/(c + d*x)**3, x)
```

3.22.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.84

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{3\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\right.}{}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_in
tegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3
*(exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e
(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + 3*b^3*
(-I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_
e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(I*exp
_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*
d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*b)
```

3.22.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.42 (sec) , antiderivative size = 115446, normalized size of antiderivative = 627.42

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/16*(9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + 3*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 3*b^2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b
^2*d^2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/
2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 6*b
^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b^2*d^2*
x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 12*b^2*d^2*x^2*s
in_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_
integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_in
tegral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b^2*d^2*x^2*imag_part(cos_integral(b*x...
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^3}\,dx=\int \frac{{\cos\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(cos(a + b*x)^3/(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)^3/(c + d*x)^3, x)
```

3.23 $\int x^3 \cos^4(a+bx)\,dx$

3.23.1 Optimal result

Integrand size = 12, antiderivative size = 172

$$\int x^3 \cos^4(a+bx)\,dx = -\frac{45x^2}{128b^2}+\frac{3x^4}{32}-\frac{45\cos^2(a+bx)}{128b^4}+\frac{9x^2\cos^2(a+bx)}{16b^2} - \frac{3\cos^4(a+bx)}{128b^4}+\frac{3x^2\cos^4(a+bx)}{16b^2} - \frac{45x\cos(a+bx)\sin(a+bx)}{64b^3}+\frac{3x^3\cos(a+bx)\sin(a+bx)}{8b} - \frac{3x\cos^3(a+bx)\sin(a+bx)}{32b^3}+\frac{x^3\cos^3(a+bx)\sin(a+bx)}{4b}$$

output
```
-45/128*x^2/b^2+3/32*x^4-45/128*cos(b*x+a)^2/b^4+9/16*x^2*cos(b*x+a)^2/b^2
-3/128*cos(b*x+a)^4/b^4+3/16*x^2*cos(b*x+a)^4/b^2-45/64*x*cos(b*x+a)*sin(b
*x+a)/b^3+3/8*x^3*cos(b*x+a)*sin(b*x+a)/b-3/32*x*cos(b*x+a)^3*sin(b*x+a)/b
^3+1/4*x^3*cos(b*x+a)^3*sin(b*x+a)/b
```

3.23.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

$$\int x^3 \cos^4(a+bx)\,dx = \frac{192(-1+2b^2x^2)\cos(2(a+bx))+3(-1+8b^2x^2)\cos(4(a+bx))+4bx(24b^3x^3+32(-3+2b^2x^2)\sin(2(a+\cdots}{1024b^4}$$

input
```
Integrate[x^3*Cos[a + b*x]^4,x]
```

output
```
(192*(-1 + 2*b^2*x^2)*Cos[2*(a + b*x)] + 3*(-1 + 8*b^2*x^2)*Cos[4*(a + b*x
)] + 4*b*x*(24*b^3*x^3 + 32*(-3 + 2*b^2*x^2)*Sin[2*(a + b*x)] + (-3 + 8*b^
2*x^2)*Sin[4*(a + b*x)]))/(1024*b^4)
```

3.23.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3792, 3042, 3791, 3042, 3791, 15, 3792, 15, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \sin\left(a+bx+\frac{\pi}{2}\right)^4 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3\int x\cos^4(a+bx)dx}{8b^2}+\frac{3}{4}\int x^3\cos^2(a+bx)dx+\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int x\sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{8b^2}+\frac{3}{4}\int x^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{3\left(\frac{3}{4}\int x\cos^2(a+bx)dx+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int x^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{3}{4}\int x\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int x^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3791}$$

$$-\frac{3\left(\frac{3}{4}\left(\frac{\int xdx}{2}+\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int x^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3x^2\cos^4(a+bx)}{16b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\int x^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3x^2\cos^4(a+bx)}{16b^2}-\frac{3\left(\frac{3}{4}\left(\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{3}{4}\left(-\frac{3\int x\cos^2(a+bx)dx}{2b^2}+\frac{\int x^3dx}{2}+\frac{3x^2\cos^2(a+bx)}{4b^2}+\frac{x^3\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{3x^2\cos^4(a+bx)}{16b^2}-\frac{3\left(\frac{3}{4}\left(\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\left(-\frac{3\int x\cos^2(a+bx)dx}{2b^2}+\frac{3x^2\cos^2(a+bx)}{4b^2}+\frac{x^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^4}{8}\right)+\frac{3x^2\cos^4(a+bx)}{16b^2}-\frac{3\left(\frac{3}{4}\left(\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(-\frac{3\int x\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{3x^2\cos^2(a+bx)}{4b^2}+\frac{x^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^4}{8}\right)+\frac{3x^2\cos^4(a+bx)}{16b^2}-\frac{3\left(\frac{3}{4}\left(\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

3.23. $\int x^3\cos^4(a+bx)\,dx$

$$\Big\downarrow \text{3791}$$

$$\frac{3}{4}\left(-\frac{3\left(\frac{\int xdx}{2}+\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3x^2\cos^2(a+bx)}{4b^2}+\frac{x^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^4}{8}\right)+\frac{3x^2\cos^4(a+bx)}{16b^2}-\frac{3\left(\frac{3}{4}\left(\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{3x^2\cos^4(a+bx)}{16b^2}-\frac{3\left(\frac{3}{4}\left(\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\left(\frac{3x^2\cos^2(a+bx)}{4b^2}-\frac{3\left(\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)}{2b^2}+\frac{x^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^4}{8}\right)+\frac{x^3\sin(a+bx)\cos^3(a+bx)}{4b}$$

input
```
Int[x^3*Cos[a + b*x]^4,x]
```

output
```
(3*x^2*Cos[a + b*x]^4)/(16*b^2) + (x^3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b)
- (3*(Cos[a + b*x]^4/(16*b^2) + (x*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) + (3
*(x^2/4 + Cos[a + b*x]^2/(4*b^2) + (x*Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4
))/(8*b^2) + (3*(x^4/8 + (3*x^2*Cos[a + b*x]^2)/(4*b^2) + (x^3*Cos[a + b*x
]*Sin[a + b*x])/(2*b) - (3*(x^2/4 + Cos[a + b*x]^2/(4*b^2) + (x*Cos[a + b*
x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/4
```

3.23.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.23.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{(384x^2b^2-192)\cos(2bx+2a)+(24x^2b^2-3)\cos(4bx+4a)+(256x^3b^3-384bx)\sin(2bx+2a)+(32x^3b^3-12bx)\sin(4bx+4a)}{1024b^4}$
risch	$\frac{3x^4}{32}+\frac{3(8x^2b^2-1)\cos(4bx+4a)}{1024b^4}+\frac{x(8x^2b^2-3)\sin(4bx+4a)}{256b^3}+\frac{3(2x^2b^2-1)\cos(2bx+2a)}{16b^4}+\frac{x(2x^2b^2-3)\sin(2bx-}{8b^3}$
derivativedivides	$-a^3\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)+3a^2\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)\right.$
default	$-a^3\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)+3a^2\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)\right.$

input
```
int(x^3*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/1024*((384*b^2*x^2-192)*cos(2*b*x+2*a)+(24*b^2*x^2-3)*cos(4*b*x+4*a)+(25
6*b^3*x^3-384*b*x)*sin(2*b*x+2*a)+(32*b^3*x^3-12*b*x)*sin(4*b*x+4*a)+96*x^
4*b^4+195)/b^4
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

$$\int x^3\cos^4(a+bx)\,dx$$
$$=\frac{12\,b^4x^4+3\,(8\,b^2x^2-1)\cos{(bx+a)}^4-45\,b^2x^2+9\,(8\,b^2x^2-5)\cos{(bx+a)}^2+2\,\left(2\,(8\,b^3x^3-3\,bx)\cos{(bx}}{128\,b^4}$$

input
```
integrate(x^3*cos(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/128*(12*b^4*x^4 + 3*(8*b^2*x^2 - 1)*cos(b*x + a)^4 - 45*b^2*x^2 + 9*(8*b
^2*x^2 - 5)*cos(b*x + a)^2 + 2*(2*(8*b^3*x^3 - 3*b*x)*cos(b*x + a)^3 + 3*(
8*b^3*x^3 - 15*b*x)*cos(b*x + a))*sin(b*x + a))/b^4
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.47

$$\int x^3\cos^4(a+bx)\,dx$$
$$=\begin{cases}\frac{3x^4\sin^4{(a+bx)}}{32}+\frac{3x^4\sin^2{(a+bx)}\cos^2{(a+bx)}}{16}+\frac{3x^4\cos^4{(a+bx)}}{32}+\frac{3x^3\sin^3{(a+bx)}\cos{(a+bx)}}{8b}+\frac{5x^3\sin{(a+bx)}\cos^3{(a+bx)}}{8b}-\frac{45x^2}{} \\ \frac{x^4\cos^4{(a)}}{4}\end{cases}$$

input
```
integrate(x**3*cos(b*x+a)**4,x)
```

output
```
Piecewise((3*x**4*sin(a + b*x)**4/32 + 3*x**4*sin(a + b*x)**2*cos(a + b*x)
**2/16 + 3*x**4*cos(a + b*x)**4/32 + 3*x**3*sin(a + b*x)**3*cos(a + b*x)/(
8*b) + 5*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 45*x**2*sin(a + b*x)**4
/(128*b**2) - 9*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2) + 51*x**2*c
os(a + b*x)**4/(128*b**2) - 45*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) -
51*x*sin(a + b*x)*cos(a + b*x)**3/(64*b**3) + 45*sin(a + b*x)**4/(256*b**4
) - 51*cos(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*cos(a)**4/4, True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.76

$$\int x^3 \cos^4(a+bx)\,dx$$
$$= \frac{96\,(bx+a)^4 - 32\,(12\,bx + 12\,a + \sin\left(4\,bx + 4\,a\right) + 8\,\sin\left(2\,bx + 2\,a\right))a^3 + 24\,\left(24\,(bx+a)^2 + 4\,(bx+a)\,\mathrm{si}\right.}{}$$

input
```
integrate(x^3*cos(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/1024*(96*(b*x + a)^4 - 32*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*
x + 2*a))*a^3 + 24*(24*(b*x + a)^2 + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*
x + a)*sin(2*b*x + 2*a) + cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))*a^2 - 12
*(32*(b*x + a)^3 + 4*(b*x + a)*cos(4*b*x + 4*a) + 64*(b*x + a)*cos(2*b*x +
 2*a) + (8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 32*(2*(b*x + a)^2 - 1)*sin(
2*b*x + 2*a))*a + 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 192*(2*(b*x + a
)^2 - 1)*cos(2*b*x + 2*a) + 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*
a) + 128*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))/b^4
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.63

$$\int x^3 \cos^4(a+bx)\,dx = \frac{3}{32}\,x^4 + \frac{3\,(8\,b^2x^2-1)\cos\left(4\,bx+4\,a\right)}{1024\,b^4} + \frac{3\,(2\,b^2x^2-1)\cos\left(2\,bx+2\,a\right)}{16\,b^4} + \frac{(8\,b^3x^3-3\,bx)\sin\left(4\,bx+4\,a\right)}{256\,b^4} + \frac{(2\,b^3x^3-3\,bx)\sin\left(2\,bx+2\,a\right)}{8\,b^4}$$

input
```
integrate(x^3*cos(b*x+a)^4,x, algorithm="giac")
```

output
```
3/32*x^4 + 3/1024*(8*b^2*x^2 - 1)*cos(4*b*x + 4*a)/b^4 + 3/16*(2*b^2*x^2 -
 1)*cos(2*b*x + 2*a)/b^4 + 1/256*(8*b^3*x^3 - 3*b*x)*sin(4*b*x + 4*a)/b^4
+ 1/8*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a)/b^4
```

3.23.9 Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.80

$$\int x^3 \cos^4(a+bx)\,dx$$

$$= \frac{\frac{3\sin(2a+2bx)^2}{512} - b^2\left(\frac{3x^2\left(2\sin(2a+2bx)^2-1\right)}{128} + \frac{3x^2\left(2\sin(a+bx)^2-1\right)}{8}\right) - b\left(\frac{3x\,\sin(2a+2bx)}{8} + \frac{3x\,\sin(4a+4bx)}{256}\right) + b^3\left(\frac{3}{}\right.}{b^4}$$

$$+ \frac{3x^4}{32}$$

input
```
int(x^3*cos(a + b*x)^4,x)
```

output
```
((3*sin(2*a + 2*b*x)^2)/512 - b^2*((3*x^2*(2*sin(2*a + 2*b*x)^2 - 1))/128
+ (3*x^2*(2*sin(a + b*x)^2 - 1))/8) - b*((3*x*sin(2*a + 2*b*x))/8 + (3*x*s
in(4*a + 4*b*x))/256) + b^3*((x^3*sin(2*a + 2*b*x))/4 + (x^3*sin(4*a + 4*b
*x))/32) + (3*sin(a + b*x)^2)/8)/b^4 + (3*x^4)/32
```

3.24 $\int x^2 \cos^4(a+bx)\,dx$

3.24.1 Optimal result

Integrand size = 12, antiderivative size = 134

$$\begin{aligned}\int x^2 \cos^4(a+bx)\,dx = &-\frac{15x}{64b^2}+\frac{x^3}{8}+\frac{3x\cos^2(a+bx)}{8b^2}+\frac{x\cos^4(a+bx)}{8b^2}\\ &-\frac{15\cos(a+bx)\sin(a+bx)}{64b^3}+\frac{3x^2\cos(a+bx)\sin(a+bx)}{8b}\\ &-\frac{\cos^3(a+bx)\sin(a+bx)}{32b^3}+\frac{x^2\cos^3(a+bx)\sin(a+bx)}{4b}\end{aligned}$$

output
```
-15/64*x/b^2+1/8*x^3+3/8*x*cos(b*x+a)^2/b^2+1/8*x*cos(b*x+a)^4/b^2-15/64*c
os(b*x+a)*sin(b*x+a)/b^3+3/8*x^2*cos(b*x+a)*sin(b*x+a)/b-1/32*cos(b*x+a)^3
*sin(b*x+a)/b^3+1/4*x^2*cos(b*x+a)^3*sin(b*x+a)/b
```

3.24.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int x^2 \cos^4(a+bx)\,dx$$
$$=\frac{32b^3x^3+64bx\cos(2(a+bx))+4bx\cos(4(a+bx))-32\sin(2(a+bx))+64b^2x^2\sin(2(a+bx))-\sin(4(a}{256b^3}$$

input
```
Integrate[x^2*Cos[a + b*x]^4,x]
```

output
```
(32*b^3*x^3 + 64*b*x*Cos[2*(a + b*x)] + 4*b*x*Cos[4*(a + b*x)] - 32*Sin[2*
(a + b*x)] + 64*b^2*x^2*Sin[2*(a + b*x)] - Sin[4*(a + b*x)] + 8*b^2*x^2*Si
n[4*(a + b*x)])/(256*b^3)
```

3.24.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sin\left(a+bx+\frac{\pi}{2}\right)^4 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{\int \cos^4(a+bx)dx}{8b^2}+\frac{3}{4}\int x^2\cos^2(a+bx)dx+\frac{x\cos^4(a+bx)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{8b^2}+\frac{3}{4}\int x^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{x\cos^4(a+bx)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{x\cos^4(a+bx)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{x\cos^4(a+bx)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{x\cos^4(a+bx)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$\Big\downarrow$ 24

$$\frac{3}{4}\int x^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{x\cos^4(a+bx)}{8b^2}-\frac{\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$\Big\downarrow$ 3792

$$\frac{3}{4}\left(-\frac{\int\cos^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{x\cos^4(a+bx)}{8b^2}-\frac{\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$\Big\downarrow$ 15

$$\frac{3}{4}\left(-\frac{\int\cos^2(a+bx)dx}{2b^2}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}\right)+\frac{x\cos^4(a+bx)}{8b^2}-\frac{\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$\Big\downarrow$ 3042

$$\frac{3}{4}\left(-\frac{\int\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}\right)+\frac{x\cos^4(a+bx)}{8b^2}-\frac{\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$\Big\downarrow$ 3115

$$\frac{3}{4}\left(-\frac{\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}}{2b^2}+\frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}\right)+\frac{x\cos^4(a+bx)}{8b^2}-\frac{\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

$\Big\downarrow$ 24

$$\frac{3}{4}\left(\frac{x\cos^2(a+bx)}{2b^2}-\frac{\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^3}{6}\right)+\frac{x\cos^4(a+bx)}{8b^2}-\frac{\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sin(a+bx)\cos^3(a+bx)}{4b}$$

3.24. $\int x^2\cos^4(a+bx)\,dx$

input
```
Int[x^2*Cos[a + b*x]^4,x]
```

output
```
(x*Cos[a + b*x]^4)/(8*b^2) + (x^2*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) - ((C
os[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/
(2*b)))/4)/(8*b^2) + (3*(x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) + (x^2*Cos[a +
 b*x]*Sin[a + b*x])/(2*b) - (x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b))/(2*b
^2)))/4
```

3.24.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.24.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{(64x^2b^2-32)\sin(2bx+2a)+(8x^2b^2-1)\sin(4bx+4a)+32b\left(x^2b^2+2\cos(2bx+2a)+\frac{\cos(4bx+4a)}{8}\right)x}{256b^3}$
risch	$\frac{x^3}{8}+\frac{x\cos(4bx+4a)}{64b^2}+\frac{(8x^2b^2-1)\sin(4bx+4a)}{256b^3}+\frac{x\cos(2bx+2a)}{4b^2}+\frac{(2x^2b^2-1)\sin(2bx+2a)}{8b^3}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-2a\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-3\right.}{}$
default	$\frac{a^2\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-2a\left((bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-3\right.}{}$
norman	$\frac{\frac{x^3}{8}-\frac{17\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{32b^3}-\frac{9\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{9\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{17\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{32b^3}+\frac{17x}{64b^2}+\frac{x^3\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{3x^3\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{4}}{}$

input
```
int(x^2*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/256*((64*b^2*x^2-32)*sin(2*b*x+2*a)+(8*b^2*x^2-1)*sin(4*b*x+4*a)+32*b*(x
^2*b^2+2*cos(2*b*x+2*a)+1/8*cos(4*b*x+4*a))*x)/b^3
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.66

$$\int x^2\cos^4(a+bx)\,dx$$
$$=\frac{8\,b^3x^3+8\,bx\cos\left(bx+a\right)^4+24\,bx\cos\left(bx+a\right)^2-15\,bx+\left(2\left(8\,b^2x^2-1\right)\cos\left(bx+a\right)^3+3\left(8\,b^2x^2-5\right)\cos\right.}{64\,b^3}$$

input
```
integrate(x^2*cos(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/64*(8*b^3*x^3 + 8*b*x*cos(b*x + a)^4 + 24*b*x*cos(b*x + a)^2 - 15*b*x +
(2*(8*b^2*x^2 - 1)*cos(b*x + a)^3 + 3*(8*b^2*x^2 - 5)*cos(b*x + a))*sin(b*
x + a))/b^3
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.56

$$\int x^2 \cos^4(a + bx)\, dx$$

$$= \begin{cases} \frac{x^3 \sin^4(a+bx)}{8} + \frac{x^3 \sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{x^3 \cos^4(a+bx)}{8} + \frac{3x^2 \sin^3(a+bx)\cos(a+bx)}{8b} + \frac{5x^2 \sin(a+bx)\cos^3(a+bx)}{8b} - \frac{15x \sin^4}{64\ldots} \\ \frac{x^3 \cos^4(a)}{3} \end{cases}$$

input
```
integrate(x**2*cos(b*x+a)**4,x)
```

output
```
Piecewise((x**3*sin(a + b*x)**4/8 + x**3*sin(a + b*x)**2*cos(a + b*x)**2/4
 + x**3*cos(a + b*x)**4/8 + 3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 5*
x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 15*x*sin(a + b*x)**4/(64*b**2) -
 3*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) + 17*x*cos(a + b*x)**4/(64*
b**2) - 15*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) - 17*sin(a + b*x)*cos(a
+ b*x)**3/(64*b**3), Ne(b, 0)), (x**3*cos(a)**4/3, True))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.40

$$\int x^2 \cos^4(a + bx)\, dx$$

$$= \frac{32\,(bx+a)^3 + 8\,(12\,bx + 12\,a + \sin(4\,bx + 4\,a) + 8\,\sin(2\,bx + 2\,a))a^2 - 4\left(24\,(bx+a)^2 + 4\,(bx+a)\sin(\ldots\right.}{\ldots}$$

input
```
integrate(x^2*cos(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/256*(32*(b*x + a)^3 + 8*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x
+ 2*a))*a^2 - 4*(24*(b*x + a)^2 + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x +
 a)*sin(2*b*x + 2*a) + cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))*a + 4*(b*x
+ a)*cos(4*b*x + 4*a) + 64*(b*x + a)*cos(2*b*x + 2*a) + (8*(b*x + a)^2 - 1
)*sin(4*b*x + 4*a) + 32*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

$$\int x^2\cos^4(a+bx)\,dx = \frac{1}{8}\,x^3 + \frac{x\cos\left(4\,bx+4\,a\right)}{64\,b^2} + \frac{x\cos\left(2\,bx+2\,a\right)}{4\,b^2} + \frac{\left(8\,b^2x^2-1\right)\sin\left(4\,bx+4\,a\right)}{256\,b^3} + \frac{\left(2\,b^2x^2-1\right)\sin\left(2\,bx+2\,a\right)}{8\,b^3}$$

input
```
integrate(x^2*cos(b*x+a)^4,x, algorithm="giac")
```

output
```
1/8*x^3 + 1/64*x*cos(4*b*x + 4*a)/b^2 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/256
*(8*b^2*x^2 - 1)*sin(4*b*x + 4*a)/b^3 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*
a)/b^3
```

3.24.9 Mupad [B] (verification not implemented)

Time = 14.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

$$\int x^2\cos^4(a+bx)\,dx = \frac{x^3}{8} - \frac{\frac{\sin(2\,a+2\,b\,x)}{8} + \frac{\sin(4\,a+4\,b\,x)}{256} + b\left(\frac{x\left(2\sin(a+b\,x)^2-1\right)}{4} + \frac{x\left(2\sin(2\,a+2\,b\,x)^2-1\right)}{64}\right) - b^2\left(\frac{x^2\,\sin(2\,a+2\,b\,x)}{4} + \frac{x^2\,\sin(4\,a+4\,b\,x)}{32}\right.}{b^3}$$

input
```
int(x^2*cos(a + b*x)^4,x)
```

output
```
x^3/8 - (sin(2*a + 2*b*x)/8 + sin(4*a + 4*b*x)/256 + b*((x*(2*sin(a + b*x)
^2 - 1))/4 + (x*(2*sin(2*a + 2*b*x)^2 - 1))/64) - b^2*((x^2*sin(2*a + 2*b*
x))/4 + (x^2*sin(4*a + 4*b*x))/32))/b^3
```

3.25 $\int x \cos^4(a+bx)\,dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x \cos^4(a+bx)\,dx = \frac{3x^2}{16}+\frac{3\cos^2(a+bx)}{16b^2}+\frac{\cos^4(a+bx)}{16b^2}+\frac{3x\cos(a+bx)\sin(a+bx)}{8b}+\frac{x\cos^3(a+bx)\sin(a+bx)}{4b}$$

output
```
3/16*x^2+3/16*cos(b*x+a)^2/b^2+1/16*cos(b*x+a)^4/b^2+3/8*x*cos(b*x+a)*sin(
b*x+a)/b+1/4*x*cos(b*x+a)^3*sin(b*x+a)/b
```

3.25.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int x \cos^4(a+bx)\,dx = \frac{16\cos(2(a+bx))+\cos(4(a+bx))+4bx(6bx+8\sin(2(a+bx))+\sin(4(a+bx)))}{128b^2}$$

input
```
Integrate[x*Cos[a + b*x]^4,x]
```

output
```
(16*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 4*b*x*(6*b*x + 8*Sin[2*(a + b*x)
] + Sin[4*(a + b*x)]))/(128*b^2)
```

3.25.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3791, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(a+bx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{3}{4}\int x\cos^2(a+bx)dx+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int x\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{3}{4}\left(\frac{\int xdx}{2}+\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\left(\frac{\cos^2(a+bx)}{4b^2}+\frac{x\sin(a+bx)\cos(a+bx)}{2b}+\frac{x^2}{4}\right)+\frac{\cos^4(a+bx)}{16b^2}+\frac{x\sin(a+bx)\cos^3(a+bx)}{4b}$$

input
```
Int[x*Cos[a + b*x]^4,x]
```

output
```
Cos[a + b*x]^4/(16*b^2) + (x*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*(x^2/
4 + Cos[a + b*x]^2/(4*b^2) + (x*Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4
```

3.25.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

3.25.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{24x^2b^2+4x\sin(4bx+4a)b+32x\sin(2bx+2a)b+\cos(4bx+4a)+16\cos(2bx+2a)-17}{128b^2}$
risch	$\frac{3x^2}{16}+\frac{\cos(4bx+4a)}{128b^2}+\frac{x\sin(4bx+4a)}{32b}+\frac{\cos(2bx+2a)}{8b^2}+\frac{x\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{(bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{3(bx+a)^2}{16}+\frac{\left(2\left(\cos^2(bx+a)\right)+3\right)^2}{64}-a\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a}{2}}{4}\right.}{b^2}$
default	$\frac{(bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{3(bx+a)^2}{16}+\frac{\left(2\left(\cos^2(bx+a)\right)+3\right)^2}{64}-a\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a}{2}}{4}\right.}{b^2}$
norman	$\frac{\frac{3x^2}{16}+\frac{3x^2\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{9x^2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8}+\frac{3x^2\left(\tan^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+\frac{3x^2\left(\tan^8\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{16}+\frac{5x\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{3x\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right.\right.}{4b}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^4}$

input
```
int(x*cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/128*(24*x^2*b^2+4*x*sin(4*b*x+4*a)*b+32*x*sin(2*b*x+2*a)*b+cos(4*b*x+4*a
)+16*cos(2*b*x+2*a)-17)/b^2
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int x\cos^4(a+bx)\,dx$$
$$=\frac{3\,b^2x^2+\cos\left(bx+a\right)^4+3\,\cos\left(bx+a\right)^2+2\,\left(2\,bx\cos\left(bx+a\right)^3+3\,bx\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{16\,b^2}$$

input
```
integrate(x*cos(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/16*(3*b^2*x^2 + cos(b*x + a)^4 + 3*cos(b*x + a)^2 + 2*(2*b*x*cos(b*x + a
)^3 + 3*b*x*cos(b*x + a))*sin(b*x + a))/b^2
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.72

$$\int x\cos^4(a+bx)\,dx$$
$$=\begin{cases}\frac{3x^2\sin^4(a+bx)}{16}+\frac{3x^2\sin^2(a+bx)\cos^2(a+bx)}{8}+\frac{3x^2\cos^4(a+bx)}{16}+\frac{3x\sin^3(a+bx)\cos(a+bx)}{8b}+\frac{5x\sin(a+bx)\cos^3(a+bx)}{8b}-\frac{3\sin^4(\cdots}{32\cdots}\\ \frac{x^2\cos^4(a)}{2}\end{cases}$$

input
```
integrate(x*cos(b*x+a)**4,x)
```

output
```
Piecewise((3*x**2*sin(a + b*x)**4/16 + 3*x**2*sin(a + b*x)**2*cos(a + b*x)
**2/8 + 3*x**2*cos(a + b*x)**4/16 + 3*x*sin(a + b*x)**3*cos(a + b*x)/(8*b)
 + 5*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - 3*sin(a + b*x)**4/(32*b**2) +
5*cos(a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*cos(a)**4/2, True))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22

$$\int x\cos^4(a+bx)\,dx$$
$$= \frac{24\,(bx+a)^2 - 4\,(12\,bx+12\,a+\sin\left(4\,bx+4\,a\right)+8\,\sin\left(2\,bx+2\,a\right))a+4\,(bx+a)\sin\left(4\,bx+4\,a\right)+32\,(b}{128\,b^2}$$

input
```
integrate(x*cos(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/128*(24*(b*x + a)^2 - 4*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x
+ 2*a))*a + 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a) +
 cos(4*b*x + 4*a) + 16*cos(2*b*x + 2*a))/b^2
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x\cos^4(a+bx)\,dx = \frac{3}{16}\,x^2 + \frac{x\sin\left(4\,bx+4\,a\right)}{32\,b} + \frac{x\sin\left(2\,bx+2\,a\right)}{4\,b} + \frac{\cos\left(4\,bx+4\,a\right)}{128\,b^2} + \frac{\cos\left(2\,bx+2\,a\right)}{8\,b^2}$$

input
```
integrate(x*cos(b*x+a)^4,x, algorithm="giac")
```

output
```
3/16*x^2 + 1/32*x*sin(4*b*x + 4*a)/b + 1/4*x*sin(2*b*x + 2*a)/b + 1/128*co
s(4*b*x + 4*a)/b^2 + 1/8*cos(2*b*x + 2*a)/b^2
```

3.25.9 Mupad [B] (verification not implemented)

Time = 14.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int x\cos^4(a+bx)\,dx = \frac{3\,x^2}{16} - \frac{\frac{\sin(2\,a+2\,b\,x)^2}{64} - b\left(\frac{x\,\sin(2\,a+2\,b\,x)}{4} + \frac{x\,\sin(4\,a+4\,b\,x)}{32}\right) + \frac{\sin(a+b\,x)^2}{4}}{b^2}$$

input
```
int(x*cos(a + b*x)^4,x)
```

output
```
(3*x^2)/16 - (sin(2*a + 2*b*x)^2/64 - b*((x*sin(2*a + 2*b*x))/4 + (x*sin(4
*a + 4*b*x))/32) + sin(a + b*x)^2/4)/b^2
```

3.25. $\int x \cos^4(a+bx)\,dx$

3.26 $\int \frac{\cos^4(a+bx)}{x} \, dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{\cos^4(a+bx)}{x} \, dx = \frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx) + \frac{1}{8}\cos(4a)\,\text{CosIntegral}(4bx) + \frac{3\log(x)}{8} - \frac{1}{2}\sin(2a)\text{Si}(2bx) - \frac{1}{8}\sin(4a)\text{Si}(4bx)$$

output
```
1/2*Ci(2*b*x)*cos(2*a)+1/8*Ci(4*b*x)*cos(4*a)+3/8*ln(x)-1/2*Si(2*b*x)*sin(
2*a)-1/8*Si(4*b*x)*sin(4*a)
```

3.26.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{\cos^4(a+bx)}{x} \, dx = \frac{1}{8}(4\cos(2a)\,\text{CosIntegral}(2bx) + \cos(4a)\,\text{CosIntegral}(4bx) + 3\log(x) - 4\sin(2a)\text{Si}(2bx) - \sin(4a)\text{Si}(4bx))$$

input
```
Integrate[Cos[a + b*x]^4/x,x]
```

output
```
(4*Cos[2*a]*CosIntegral[2*b*x] + Cos[4*a]*CosIntegral[4*b*x] + 3*Log[x] -
4*Sin[2*a]*SinIntegral[2*b*x] - Sin[4*a]*SinIntegral[4*b*x])/8
```

3.26.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(a+bx)}{x} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^4}{x}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{\cos(2a+2bx)}{2x}+\frac{\cos(4a+4bx)}{8x}+\frac{3}{8x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx)+\frac{1}{8}\cos(4a)\,\text{CosIntegral}(4bx)-\frac{1}{2}\sin(2a)\text{Si}(2bx)-\frac{1}{8}\sin(4a)\text{Si}(4bx)+\frac{3\log(x)}{8}$$

input
```
Int[Cos[a + b*x]^4/x,x]
```

output
```
(Cos[2*a]*CosIntegral[2*b*x])/2 + (Cos[4*a]*CosIntegral[4*b*x])/8 + (3*Log
[x])/8 - (Sin[2*a]*SinIntegral[2*b*x])/2 - (Sin[4*a]*SinIntegral[4*b*x])/8
```

3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.26.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{\mathrm{Si}(4bx)\sin(4a)}{8}+\frac{\mathrm{Ci}(4bx)\cos(4a)}{8}-\frac{\mathrm{Si}(2bx)\sin(2a)}{2}+\frac{\mathrm{Ci}(2bx)\cos(2a)}{2}+\frac{3\ln(bx)}{8}$
default	$-\frac{\mathrm{Si}(4bx)\sin(4a)}{8}+\frac{\mathrm{Ci}(4bx)\cos(4a)}{8}-\frac{\mathrm{Si}(2bx)\sin(2a)}{2}+\frac{\mathrm{Ci}(2bx)\cos(2a)}{2}+\frac{3\ln(bx)}{8}$
risch	$\frac{3\ln(x)}{8}+\frac{i\pi\ \mathrm{csgn}(bx)\mathrm{e}^{-4ia}}{16}-\frac{i\,\mathrm{Si}(4bx)\mathrm{e}^{-4ia}}{8}-\frac{\mathrm{e}^{-4ia}\,\mathrm{Ei}_1(-4ixb)}{16}+\frac{i\pi\ \mathrm{csgn}(bx)\mathrm{e}^{-2ia}}{4}-\frac{i\,\mathrm{Si}(2bx)\mathrm{e}^{-2ia}}{2}-\mathrm{e}^{-2}$

input
```
int(cos(b*x+a)^4/x,x,method=_RETURNVERBOSE)
```

output
```
-1/8*Si(4*b*x)*sin(4*a)+1/8*Ci(4*b*x)*cos(4*a)-1/2*Si(2*b*x)*sin(2*a)+1/2*
Ci(2*b*x)*cos(2*a)+3/8*ln(b*x)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \frac{1}{8}\cos(4a)\,\mathrm{Ci}(4bx)+\frac{1}{2}\cos(2a)\,\mathrm{Ci}(2bx) - \frac{1}{8}\sin(4a)\,\mathrm{Si}(4bx)-\frac{1}{2}\sin(2a)\,\mathrm{Si}(2bx)+\frac{3}{8}\log(x)$$

input
```
integrate(cos(b*x+a)^4/x,x, algorithm="fricas")
```

output
```
1/8*cos(4*a)*cos_integral(4*b*x) + 1/2*cos(2*a)*cos_integral(2*b*x) - 1/8*
sin(4*a)*sin_integral(4*b*x) - 1/2*sin(2*a)*sin_integral(2*b*x) + 3/8*log(
x)
```

3.26.6 Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \frac{3\log(x)}{8} - \frac{\sin(2a)\,\mathrm{Si}(2bx)}{2} - \frac{\sin(4a)\,\mathrm{Si}(4bx)}{8} + \frac{\cos(2a)\,\mathrm{Ci}(2bx)}{2} + \frac{\cos(4a)\,\mathrm{Ci}(4bx)}{8}$$

input
```
integrate(cos(b*x+a)**4/x,x)
```

output
```
3*log(x)/8 - sin(2*a)*Si(2*b*x)/2 - sin(4*a)*Si(4*b*x)/8 + cos(2*a)*Ci(2*b
*x)/2 + cos(4*a)*Ci(4*b*x)/8
```

3.26.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.54

$$\int \frac{\cos^4(a+bx)}{x}\,dx = -\frac{1}{16}\left(E_1(4i\,bx) + E_1(-4i\,bx)\right)\cos(4\,a) - \frac{1}{4}\left(E_1(2i\,bx) + E_1(-2i\,bx)\right)\cos(2\,a) + \frac{1}{16}\left(i\,E_1(4i\,bx) - i\,E_1(-4i\,bx)\right)\sin(4\,a) - \frac{1}{4}\left(-i\,E_1(2i\,bx) + i\,E_1(-2i\,bx)\right)\sin(2\,a) + \frac{3}{8}\log(bx)$$

input
```
integrate(cos(b*x+a)^4/x,x, algorithm="maxima")
```

output
```
-1/16*(exp_integral_e(1, 4*I*b*x) + exp_integral_e(1, -4*I*b*x))*cos(4*a)
- 1/4*(exp_integral_e(1, 2*I*b*x) + exp_integral_e(1, -2*I*b*x))*cos(2*a)
+ 1/16*(I*exp_integral_e(1, 4*I*b*x) - I*exp_integral_e(1, -4*I*b*x))*sin(
4*a) - 1/4*(-I*exp_integral_e(1, 2*I*b*x) + I*exp_integral_e(1, -2*I*b*x))
*sin(2*a) + 3/8*log(b*x)
```

3.26.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 428, normalized size of antiderivative = 7.25

$$\int \frac{\cos^4(a+bx)}{x}\,dx$$
$$= \frac{6\log(|x|)\tan(2a)^2\tan(a)^2 - \Re(\text{Ci}(4bx))\tan(2a)^2\tan(a)^2 - 4\Re(\text{Ci}(2bx))\tan(2a)^2\tan(a)^2 - 4\Re(\text{C}}{}$$

input
```
integrate(cos(b*x+a)^4/x,x, algorithm="giac")
```

output
```
1/16*(6*log(abs(x))*tan(2*a)^2*tan(a)^2 - real_part(cos_integral(4*b*x))*t
an(2*a)^2*tan(a)^2 - 4*real_part(cos_integral(2*b*x))*tan(2*a)^2*tan(a)^2
- 4*real_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a)^2 - real_part(cos_in
tegral(-4*b*x))*tan(2*a)^2*tan(a)^2 - 8*imag_part(cos_integral(2*b*x))*tan
(2*a)^2*tan(a) + 8*imag_part(cos_integral(-2*b*x))*tan(2*a)^2*tan(a) - 16*
sin_integral(2*b*x)*tan(2*a)^2*tan(a) - 2*imag_part(cos_integral(4*b*x))*t
an(2*a)*tan(a)^2 + 2*imag_part(cos_integral(-4*b*x))*tan(2*a)*tan(a)^2 - 4
*sin_integral(4*b*x)*tan(2*a)*tan(a)^2 + 6*log(abs(x))*tan(2*a)^2 - real_p
art(cos_integral(4*b*x))*tan(2*a)^2 + 4*real_part(cos_integral(2*b*x))*tan
(2*a)^2 + 4*real_part(cos_integral(-2*b*x))*tan(2*a)^2 - real_part(cos_int
egral(-4*b*x))*tan(2*a)^2 + 6*log(abs(x))*tan(a)^2 + real_part(cos_integra
l(4*b*x))*tan(a)^2 - 4*real_part(cos_integral(2*b*x))*tan(a)^2 - 4*real_pa
rt(cos_integral(-2*b*x))*tan(a)^2 + real_part(cos_integral(-4*b*x))*tan(a)
^2 - 2*imag_part(cos_integral(4*b*x))*tan(2*a) + 2*imag_part(cos_integral(
-4*b*x))*tan(2*a) - 4*sin_integral(4*b*x)*tan(2*a) - 8*imag_part(cos_integ
ral(2*b*x))*tan(a) + 8*imag_part(cos_integral(-2*b*x))*tan(a) - 16*sin_int
egral(2*b*x)*tan(a) + 6*log(abs(x)) + real_part(cos_integral(4*b*x)) + 4*r
eal_part(cos_integral(2*b*x)) + 4*real_part(cos_integral(-2*b*x)) + real_p
art(cos_integral(-4*b*x)))/(tan(2*a)^2*tan(a)^2 + tan(2*a)^2 + tan(a)^2 +
1)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(a+bx)}{x}\,dx = \int \frac{\cos\left(a+b\,x\right)^4}{x}\,dx$$

input
```
int(cos(a + b*x)^4/x,x)
```

output
```
int(cos(a + b*x)^4/x, x)
```

3.27 $\int \frac{\cos^4(a+bx)}{x^2} \, dx$

3.27.1 Optimal result

Integrand size = 12, antiderivative size = 66

$$\int \frac{\cos^4(a+bx)}{x^2} \, dx = -\frac{\cos^4(a+bx)}{x} - b \operatorname{CosIntegral}(2bx) \sin(2a) - \frac{1}{2} b \operatorname{CosIntegral}(4bx) \sin(4a) - b \cos(2a) \operatorname{Si}(2bx) - \frac{1}{2} b \cos(4a) \operatorname{Si}(4bx)$$

output
```
-cos(b*x+a)^4/x-b*cos(2*a)*Si(2*b*x)-1/2*b*cos(4*a)*Si(4*b*x)-b*Ci(2*b*x)*
sin(2*a)-1/2*b*Ci(4*b*x)*sin(4*a)
```

3.27.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \frac{\cos^4(a+bx)}{x^2} \, dx = -\frac{3 + 4\cos(2(a+bx)) + \cos(4(a+bx)) + 8bx \operatorname{CosIntegral}(2bx) \sin(2a) + 4bx \operatorname{CosIntegral}(4bx) \sin(4a) +}{8x}$$

input
```
Integrate[Cos[a + b*x]^4/x^2,x]
```

output
```
-1/8*(3 + 4*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 8*b*x*CosIntegral[2*b*x]
*Sin[2*a] + 4*b*x*CosIntegral[4*b*x]*Sin[4*a] + 8*b*x*Cos[2*a]*SinIntegral
[2*b*x] + 4*b*x*Cos[4*a]*SinIntegral[4*b*x])/x
```

3.27.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(a+bx)}{x^2} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin \left(a+bx+\frac{\pi}{2}\right)^4}{x^2}dx$$
$$\Big\downarrow \text{ 3794}$$
$$4b\int \left(-\frac{\sin(2a+2bx)}{4x}-\frac{\sin(4a+4bx)}{8x}\right)dx-\frac{\cos^4(a+bx)}{x}$$
$$\Big\downarrow \text{ 2009}$$
$$4b\left(-\frac{1}{4}\sin(2a)\text{CosIntegral}(2bx)-\frac{1}{8}\sin(4a)\text{CosIntegral}(4bx)-\frac{1}{4}\cos(2a)\text{Si}(2bx)-\frac{1}{8}\cos(4a)\text{Si}(4bx)\right)-\frac{\cos^4(a+bx)}{x}$$

input
```
Int[Cos[a + b*x]^4/x^2,x]
```

output
```
-(Cos[a + b*x]^4/x) + 4*b*(-1/4*(CosIntegral[2*b*x]*Sin[2*a]) - (CosIntegr
al[4*b*x]*Sin[4*a])/8 - (Cos[2*a]*SinIntegral[2*b*x])/4 - (Cos[4*a]*SinInt
egral[4*b*x])/8)
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.27.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

method	result
derivativedivides	$b\left(-\frac{\cos(4bx+4a)}{8bx}-\frac{\mathrm{Si}(4bx)\cos(4a)}{2}-\frac{\mathrm{Ci}(4bx)\sin(4a)}{2}-\frac{\cos(2bx+2a)}{2bx}-\mathrm{Si}\left(2bx\right)\cos\left(2a\right)-\mathrm{Ci}\left(2bx\right)\mathrm{s}\right.$
default	$b\left(-\frac{\cos(4bx+4a)}{8bx}-\frac{\mathrm{Si}(4bx)\cos(4a)}{2}-\frac{\mathrm{Ci}(4bx)\sin(4a)}{2}-\frac{\cos(2bx+2a)}{2bx}-\mathrm{Si}\left(2bx\right)\cos\left(2a\right)-\mathrm{Ci}\left(2bx\right)\mathrm{s}\right.$
risch	$\frac{4\,\mathrm{e}^{-2ia}\pi\ \mathrm{csgn}(bx)bx+4i\mathrm{e}^{-2ia}\ \mathrm{Ei}_1(-2ixb)bx-2ib\ \mathrm{Ei}_1(-4ixb)\mathrm{e}^{4ia}x+2i\,\mathrm{Ei}_1(-4ixb)\mathrm{e}^{-4ia}bx+2\,\mathrm{e}^{-4ia}\pi\ \mathrm{csgn}(bx)bx-4ib\ \mathrm{E}}{8x}$

input
```
int(cos(b*x+a)^4/x^2,x,method=_RETURNVERBOSE)
```

output
```
b*(-1/8*cos(4*b*x+4*a)/b/x-1/2*Si(4*b*x)*cos(4*a)-1/2*Ci(4*b*x)*sin(4*a)-1
/2*cos(2*b*x+2*a)/b/x-Si(2*b*x)*cos(2*a)-Ci(2*b*x)*sin(2*a)-3/8/b/x)
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx =$$
$$-\frac{2\,\cos\left(bx+a\right)^4+bx\,\mathrm{Ci}\left(4\,bx\right)\sin\left(4\,a\right)+2\,bx\,\mathrm{Ci}\left(2\,bx\right)\sin\left(2\,a\right)+bx\cos\left(4\,a\right)\mathrm{Si}\left(4\,bx\right)+2\,bx\cos\left(2\,a\right)\mathrm{Si}\left(\right.}{2\,x}$$

input
```
integrate(cos(b*x+a)^4/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*cos(b*x + a)^4 + b*x*cos_integral(4*b*x)*sin(4*a) + 2*b*x*cos_inte
gral(2*b*x)*sin(2*a) + b*x*cos(4*a)*sin_integral(4*b*x) + 2*b*x*cos(2*a)*s
in_integral(2*b*x))/x
```

3.27.6 Sympy [F]

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = \int \frac{\cos^4(a+bx)}{x^2}\,dx$$

input
```
integrate(cos(b*x+a)**4/x**2,x)
```

output
```
Integral(cos(a + b*x)**4/x**2, x)
```

3.27.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 717, normalized size of antiderivative = 10.86

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^4/x^2,x, algorithm="maxima")
```

output
```
1/32*(((exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*cos(2*a)
^2 + (exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*I*b*x))*sin(2*a)^2
)*cos(4*a)^3 + ((-I*exp_integral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4*I*
b*x))*cos(2*a)^2 + (-I*exp_integral_e(2, 4*I*b*x) + I*exp_integral_e(2, -4
*I*b*x))*sin(2*a)^2)*sin(4*a)^3 + 4*((exp_integral_e(2, 2*I*b*x) + exp_int
egral_e(2, -2*I*b*x))*cos(2*a)^3 + (-I*exp_integral_e(2, 2*I*b*x) + I*exp_
integral_e(2, -2*I*b*x))*sin(2*a)^3 + ((exp_integral_e(2, 2*I*b*x) + exp_i
ntegral_e(2, -2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + (exp_integral_e(2, 2*I*
b*x) + exp_integral_e(2, -2*I*b*x))*cos(2*a) + 3*cos(2*a)^2 + ((-I*exp_int
egral_e(2, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*cos(2*a)^2 - I*exp_in
tegral_e(2, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*sin(2*a))*cos(4*a)^2
 + (4*(exp_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*cos(2*a)^
3 + 4*(-I*exp_integral_e(2, 2*I*b*x) + I*exp_integral_e(2, -2*I*b*x))*sin(
2*a)^3 + 4*((exp_integral_e(2, 2*I*b*x) + exp_integral_e(2, -2*I*b*x))*cos
(2*a) + 3)*sin(2*a)^2 + ((exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -
4*I*b*x))*cos(2*a)^2 + (exp_integral_e(2, 4*I*b*x) + exp_integral_e(2, -4*
I*b*x))*sin(2*a)^2)*cos(4*a) + 4*(exp_integral_e(2, 2*I*b*x) + exp_integra
l_e(2, -2*I*b*x))*cos(2*a) + 12*cos(2*a)^2 + 4*((-I*exp_integral_e(2, 2*I*
b*x) + I*exp_integral_e(2, -2*I*b*x))*cos(2*a)^2 - I*exp_integral_e(2, 2*I
*b*x) + I*exp_integral_e(2, -2*I*b*x))*sin(2*a))*sin(4*a)^2 + ((exp_int...
```

3.27.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.38 (sec) , antiderivative size = 3220, normalized size of antiderivative = 48.79

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^4/x^2,x, algorithm="giac")
```

output
```
1/4*(b*x*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2
*tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*t
an(2*a)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*ta
n(b*x)^2*tan(2*a)^2*tan(a)^2 - b*x*imag_part(cos_integral(-4*b*x))*tan(2*b
*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 2*b*x*sin_integral(4*b*x)*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b*x*sin_integral(2*b*x)*tan(2*b*x)^
2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 - 4*b*x*real_part(cos_integral(2*b*x))*ta
n(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 4*b*x*real_part(cos_integral(-2*
b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 2*b*x*real_part(cos_inte
gral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 - 2*b*x*real_part(c
os_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)*tan(a)^2 + b*x*imag_
part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 2*b*x*imag_
part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 2*b*x*imag_
part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - b*x*imag_p
art(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 2*b*x*sin_i
ntegral(4*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b*x*sin_integral(2*b
*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - b*x*imag_part(cos_integral(4*b*x)
)*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 2*b*x*imag_part(cos_integral(2*b*x))*
tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 - 2*b*x*imag_part(cos_integral(-2*b*x))*t
an(2*b*x)^2*tan(b*x)^2*tan(a)^2 + b*x*imag_part(cos_integral(-4*b*x))*t...
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(a+bx)}{x^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^4}{x^2}\,dx$$

input
```
int(cos(a + b*x)^4/x^2,x)
```

output
```
int(cos(a + b*x)^4/x^2, x)
```

3.28 $\int \frac{\cos^4(a+bx)}{x^3}\,dx$

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = -\frac{\cos^4(a+bx)}{2x^2} - b^2\cos(2a)\,\text{CosIntegral}(2bx) - b^2\cos(4a)\,\text{CosIntegral}(4bx) + \frac{2b\cos^3(a+bx)\sin(a+bx)}{x} + b^2\sin(2a)\text{Si}(2bx) + b^2\sin(4a)\text{Si}(4bx)$$

output
```
-b^2*Ci(2*b*x)*cos(2*a)-b^2*Ci(4*b*x)*cos(4*a)-1/2*cos(b*x+a)^4/x^2+b^2*Si
(2*b*x)*sin(2*a)+b^2*Si(4*b*x)*sin(4*a)+2*b*cos(b*x+a)^3*sin(b*x+a)/x
```

3.28.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.32

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = -\frac{3 + 4\cos(2(a+bx)) + \cos(4(a+bx)) + 16b^2x^2\cos(2a)\,\text{CosIntegral}(2bx) + 16b^2x^2\cos(4a)\,\text{CosIntegral}(}{16x^2}$$

input
```
Integrate[Cos[a + b*x]^4/x^3,x]
```

output
```
-1/16*(3 + 4*Cos[2*(a + b*x)] + Cos[4*(a + b*x)] + 16*b^2*x^2*Cos[2*a]*Cos
Integral[2*b*x] + 16*b^2*x^2*Cos[4*a]*CosIntegral[4*b*x] - 8*b*x*Sin[2*(a
+ b*x)] - 4*b*x*Sin[4*(a + b*x)] - 16*b^2*x^2*Sin[2*a]*SinIntegral[2*b*x]
- 16*b^2*x^2*Sin[4*a]*SinIntegral[4*b*x])/x^2
```

3.28.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3795, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(a+bx)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^4}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$-8b^2\int \frac{\cos^4(a+bx)}{x}dx+6b^2\int \frac{\cos^2(a+bx)}{x}dx-\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\sin(a+bx)\cos^3(a+bx)}{x}$$

$$\Big\downarrow \text{3042}$$

$$6b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{x}dx-8b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^4}{x}dx-\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\sin(a+bx)\cos^3(a+bx)}{x}$$

$$\Big\downarrow \text{3793}$$

$$6b^2\int \left(\frac{\cos(2a+2bx)}{2x}+\frac{1}{2x}\right)dx-8b^2\int \left(\frac{\cos(2a+2bx)}{2x}+\frac{\cos(4a+4bx)}{8x}+\frac{3}{8x}\right)dx-\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\sin(a+bx)\cos^3(a+bx)}{x}$$

$$\Big\downarrow \text{2009}$$

$$6b^2\left(\frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx)-\frac{1}{2}\sin(2a)\text{Si}(2bx)+\frac{\log(x)}{2}\right)-8b^2\left(\frac{1}{2}\cos(2a)\,\text{CosIntegral}(2bx)+\frac{1}{8}\cos(4a)\,\text{CosIntegral}(4bx)-\frac{1}{2}\sin(2a)\text{Si}(2bx)-\frac{1}{8}\sin(4a)\text{Si}(4bx)+\frac{3\log(x)}{8}\right)-\frac{\cos^4(a+bx)}{2x^2}+\frac{2b\sin(a+bx)\cos^3(a+bx)}{x}$$

input
```
Int[Cos[a + b*x]^4/x^3,x]
```

output
```
-1/2*Cos[a + b*x]^4/x^2 + (2*b*Cos[a + b*x]^3*Sin[a + b*x])/x + 6*b^2*((Co
s[2*a]*CosIntegral[2*b*x])/2 + Log[x]/2 - (Sin[2*a]*SinIntegral[2*b*x])/2)
 - 8*b^2*((Cos[2*a]*CosIntegral[2*b*x])/2 + (Cos[4*a]*CosIntegral[4*b*x])/
8 + (3*Log[x])/8 - (Sin[2*a]*SinIntegral[2*b*x])/2 - (Sin[4*a]*SinIntegral
[4*b*x])/8)
```

3.28.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.28.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38

method	result
derivativedivides	$b^2\left(-\frac{\cos(4bx+4a)}{16b^2x^2}+\frac{\sin(4bx+4a)}{4bx}+\mathrm{Si}\left(4bx\right)\sin\left(4a\right)-\mathrm{Ci}\left(4bx\right)\cos\left(4a\right)-\frac{\cos(2bx+2a)}{4b^2x^2}+\frac{\sin(2bx}{2bx}\right.$
default	$b^2\left(-\frac{\cos(4bx+4a)}{16b^2x^2}+\frac{\sin(4bx+4a)}{4bx}+\mathrm{Si}\left(4bx\right)\sin\left(4a\right)-\mathrm{Ci}\left(4bx\right)\cos\left(4a\right)-\frac{\cos(2bx+2a)}{4b^2x^2}+\frac{\sin(2bx}{2bx}\right.$
risch	$-\frac{8ie^{-2ia}\pi\ \mathrm{csgn}(bx)b^2x^2+8ie^{-4ia}\pi\ \mathrm{csgn}(bx)b^2x^2-16ie^{-4ia}\ \mathrm{Si}(4bx)b^2x^2-16ie^{-2ia}\ \mathrm{Si}(2bx)b^2x^2-8\,\mathrm{e}^{4ia}\ \mathrm{Ei}_1(-4ixb)x^2b}{}$

input
```
int(cos(b*x+a)^4/x^3,x,method=_RETURNVERBOSE)
```

output
```
b^2*(-1/16*cos(4*b*x+4*a)/b^2/x^2+1/4*sin(4*b*x+4*a)/b/x+Si(4*b*x)*sin(4*a
)-Ci(4*b*x)*cos(4*a)-1/4*cos(2*b*x+2*a)/b^2/x^2+1/2*sin(2*b*x+2*a)/b/x+Si(
2*b*x)*sin(2*a)-Ci(2*b*x)*cos(2*a)-3/16/x^2/b^2)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = -\frac{2\,b^2x^2\cos\left(4\,a\right)\mathrm{Ci}\left(4\,bx\right)+2\,b^2x^2\cos\left(2\,a\right)\mathrm{Ci}\left(2\,bx\right)-4\,bx\cos\left(bx+a\right)^3\sin\left(bx+a\right)-2\,b^2x^2\sin\left(4\,a\right)\mathrm{Si}\left(\right.}{2\,x^2}$$

input
```
integrate(cos(b*x+a)^4/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*x^2*cos(4*a)*cos_integral(4*b*x) + 2*b^2*x^2*cos(2*a)*cos_inte
gral(2*b*x) - 4*b*x*cos(b*x + a)^3*sin(b*x + a) - 2*b^2*x^2*sin(4*a)*sin_i
ntegral(4*b*x) - 2*b^2*x^2*sin(2*a)*sin_integral(2*b*x) + cos(b*x + a)^4)/
x^2
```

3.28.6 Sympy [F]

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = \int \frac{\cos^4{(a+bx)}}{x^3}\,dx$$

input
```
integrate(cos(b*x+a)**4/x**3,x)
```

output
```
Integral(cos(a + b*x)**4/x**3, x)
```

3.28.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 790, normalized size of antiderivative = 8.78

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^4/x^3,x, algorithm="maxima")
```

output
```
-1/32*(((exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*cos(2*a
)^2 + (exp_integral_e(3, 4*I*b*x) + exp_integral_e(3, -4*I*b*x))*sin(2*a)^
2)*cos(4*a)^3 + ((-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -4*I
*b*x))*cos(2*a)^2 + (-I*exp_integral_e(3, 4*I*b*x) + I*exp_integral_e(3, -
4*I*b*x))*sin(2*a)^2)*sin(4*a)^3 + 2*(2*(exp_integral_e(3, 2*I*b*x) + exp_
integral_e(3, -2*I*b*x))*cos(2*a)^3 + 2*(-I*exp_integral_e(3, 2*I*b*x) + I
*exp_integral_e(3, -2*I*b*x))*sin(2*a)^3 + (2*(exp_integral_e(3, 2*I*b*x)
+ exp_integral_e(3, -2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + 2*(exp_integral_
e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x))*cos(2*a) + 3*cos(2*a)^2 + 2*(
(-I*exp_integral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*cos(2*a)^2
 - I*exp_integral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*sin(2*a))
*cos(4*a)^2 + (4*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -2*I*b*x)
)*cos(2*a)^3 + 4*(-I*exp_integral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I
*b*x))*sin(2*a)^3 + 2*(2*(exp_integral_e(3, 2*I*b*x) + exp_integral_e(3, -
2*I*b*x))*cos(2*a) + 3)*sin(2*a)^2 + ((exp_integral_e(3, 4*I*b*x) + exp_in
tegral_e(3, -4*I*b*x))*cos(2*a)^2 + (exp_integral_e(3, 4*I*b*x) + exp_inte
gral_e(3, -4*I*b*x))*sin(2*a)^2)*cos(4*a) + 4*(exp_integral_e(3, 2*I*b*x)
+ exp_integral_e(3, -2*I*b*x))*cos(2*a) + 6*cos(2*a)^2 + 4*((-I*exp_integr
al_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*cos(2*a)^2 - I*exp_integ
ral_e(3, 2*I*b*x) + I*exp_integral_e(3, -2*I*b*x))*sin(2*a))*sin(4*a)^2...
```

3.28.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 3920, normalized size of antiderivative = 43.56

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^4/x^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*x^2*real_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(
2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(2*b*x))*tan(2*b*x)^2*ta
n(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(-2*b*x))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 4*b^2*x^2*real_part(cos_integ
ral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2 + 8*b^2*x^2*imag_
part(cos_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a) - 8*b^
2*x^2*imag_part(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*t
an(a) + 16*b^2*x^2*sin_integral(2*b*x)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*
tan(a) + 8*b^2*x^2*imag_part(cos_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*
tan(2*a)*tan(a)^2 - 8*b^2*x^2*imag_part(cos_integral(-4*b*x))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)*tan(a)^2 + 16*b^2*x^2*sin_integral(4*b*x)*tan(2*b*x)^
2*tan(b*x)^2*tan(2*a)*tan(a)^2 + 4*b^2*x^2*real_part(cos_integral(4*b*x))*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(2*b*
x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos_integral(
-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 + 4*b^2*x^2*real_part(cos_inte
gral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2 - 4*b^2*x^2*real_part(cos
_integral(4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part(c
os_integral(2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*real_part
(cos_integral(-2*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 - 4*b^2*x^2*real_p
art(cos_integral(-4*b*x))*tan(2*b*x)^2*tan(b*x)^2*tan(a)^2 + 4*b^2*x^2*...
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(a+bx)}{x^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^4}{x^3}\,dx$$

input
```
int(cos(a + b*x)^4/x^3,x)
```

output
```
int(cos(a + b*x)^4/x^3, x)
```

3.29 $\int (c+dx)^3 \sec(a+bx)\,dx$

3.29.1 Optimal result

Integrand size = 14, antiderivative size = 205

$$\begin{aligned}\int (c+dx)^3 \sec(a+bx)\,dx = &-\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} \\ &+\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ &-\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\ &-\frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\ &+\frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &-\frac{6id^3 \operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4}+\frac{6id^3 \operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4}\end{aligned}$$

output
```
-2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I
*(b*x+a)))/b^2-3*I*d*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-6*d^2*(d*x+
c)*polylog(3,-I*exp(I*(b*x+a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a
)))/b^3-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I
*(b*x+a)))/b^4
```

3.29.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.96

$$\int (c+dx)^3 \sec(a+bx)\,dx = -\frac{i\left(2b^3(c+dx)^3 \arctan\left(e^{i(a+bx)}\right) - 3d\left(b^2(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right) + 2ibd(c+dx) \operatorname{PolyLog}\left(3,-i\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x],x]
```

output
```
((-I)*(2*b^3*(c + d*x)^3*ArcTan[E^(I*(a + b*x))] - 3*d*(b^2*(c + d*x)^2*Po
lyLog[2, (-I)*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, (-I)*E^(I*
(a + b*x))] - 2*d^2*PolyLog[4, (-I)*E^(I*(a + b*x))]) + 3*d*(b^2*(c + d*x)
^2*PolyLog[2, I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, I*E^(I*(
a + b*x))] - 2*d^2*PolyLog[4, I*E^(I*(a + b*x))])))/b^4
```

3.29.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$-\frac{3d\int (c+dx)^2 \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2 \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 7143}$$

$$-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}$$

3.29. $\int(c+dx)^3\sec(a+bx)\,dx$

input
```
Int[(c + d*x)^3*Sec[a + b*x],x]
```

output
```
((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)^2*Poly
Log[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (-I
)*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^2))/b))/b -
 (3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(
c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4, I*E^(I*(a + b*x)
)])/b^2))/b))/b
```

3.29.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.29.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(180) = 360$.

Time = 1.73 (sec) , antiderivative size = 685, normalized size of antiderivative = 3.34

method	result
risch	$\frac{3c^2 d \ln(1-ie^{i(bx+a)})x}{b} + \frac{3c^2 d \ln(1-ie^{i(bx+a)})a}{b^2} - \frac{3ic^2 d \operatorname{Li}_2(ie^{i(bx+a)})}{b^2} + \frac{3id^3 \operatorname{Li}_2(-ie^{i(bx+a)})x^2}{b^2} - \frac{3id^3 \operatorname{Li}_2(ie^{i(bx+a)})x^2}{b^2} -$

input
```
int((d*x+c)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x-6*I/b^3*c*d^2*a^2*arctan(exp(
I*(b*x+a)))+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))-6*I/b^2*d^2*c*polylog(2
,I*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+3/b*c^2*d*ln(1-I
*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3*I/b^2*c^2*d*poly
log(2,I*exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2-3*I/b
^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a
)))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-6/b^3*d^3*polylog(3,-I*exp(
I*(b*x+a)))*x+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+6/b^3*d^3*polylog(3,I*exp
(I*(b*x+a)))*x-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*e
xp(I*(b*x+a)))-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+6/b^3*d^2*c*polylog(3,I*
exp(I*(b*x+a)))-6/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))-2*I/b*c^3*arctan(
exp(I*(b*x+a)))-3/b^3*a^2*d^2*c*ln(1-I*exp(I*(b*x+a)))-3/b*d^2*c*ln(1+I*ex
p(I*(b*x+a)))*x^2+3/b^3*a^2*d^2*c*ln(1+I*exp(I*(b*x+a)))+3/b*d^2*c*ln(1-I*
exp(I*(b*x+a)))*x^2-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,I
*exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4
```

3.29.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(167) = 334$.

Time = 0.33 (sec) , antiderivative size = 970, normalized size of antiderivative = 4.73

$$\int (c+dx)^3 \sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4
, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + si
n(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^
2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x
+ a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x
+ a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*
dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*
x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I)
- (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I
*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1
) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^
3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*
x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*...
```

3.29.6 Sympy [F]

$$\int (c+dx)^3 \sec(a+bx)\,dx = \int (c+dx)^3 \sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**3*sec(a + b*x), x)
```

3.29.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(167) = 334$.

Time = 0.48 (sec) , antiderivative size = 722, normalized size of antiderivative = 3.52

$$\int (c + dx)^3 \sec(a + bx)\, dx$$

$$= \frac{2\,c^3 \log\left(\sec\left(bx + a\right) + \tan\left(bx + a\right)\right) - \frac{6\,ac^2 d \log(\sec(bx+a)+\tan(bx+a))}{b} + \frac{6\,a^2 cd^2 \log(\sec(bx+a)+\tan(bx+a))}{b^2} - \frac{2\,a^3 d^3 \log(s}{}}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*c^3*log(sec(b*x + a) + tan(b*x + a)) - 6*a*c^2*d*log(sec(b*x + a) +
 tan(b*x + a))/b + 6*a^2*c*d^2*log(sec(b*x + a) + tan(b*x + a))/b^2 - 2*a^
3*d^3*log(sec(b*x + a) + tan(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b
*x + I*a)) - 12*I*d^3*polylog(4, -I*e^(I*b*x + I*a)) - 2*(I*(b*x + a)^3*d^
3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 +
 I*a^2*d^3)*(b*x + a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x
 + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a
*b*c*d^2 + I*a^2*d^3)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1)
- 6*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*
c*d^2 - I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d + 2
*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b
*x + a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b
*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b
*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
 a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12*(b*c*d
^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x + I*a)) - 12*(b*c*d^2 +
(b*x + a)*d^3 - a*d^3)*polylog(3, -I*e^(I*b*x + I*a)))/b^3)/b
```

3.29.8 Giac [F]

$$\int (c+dx)^3 \sec(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\cos(a+b\,x)}\,dx$$

input
```
int((c + d*x)^3/cos(a + b*x),x)
```

output
```
int((c + d*x)^3/cos(a + b*x), x)
```

3.30 $\int (c+dx)^2 \sec(a+bx) \, dx$

3.30.1 Optimal result

Integrand size = 14, antiderivative size = 137

$$\int (c+dx)^2 \sec(a+bx) \, dx = -\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx) \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx) \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{2d^2 \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{2d^2 \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}$$

output
```
-2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+2*I*d*(d*x+c)*polylog(2,-I*exp(I*(
b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*d^2*polylog(3
,-I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3
```

3.30.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.95

$$\int (c+dx)^2 \sec(a+bx) \, dx = -\frac{2i\left(b^2(c+dx)^2 \arctan\left(e^{i(a+bx)}\right) - d\left(b(c+dx) \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right) + id \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)\right) + d}{b^3}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x],x]
```

output
```
((-2*I)*(b^2*(c + d*x)^2*ArcTan[E^(I*(a + b*x))] - d*(b*(c + d*x)*PolyLog[
2, (-I)*E^(I*(a + b*x))] + I*d*PolyLog[3, (-I)*E^(I*(a + b*x))]) + d*(b*(c
 + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + I*d*PolyLog[3, I*E^(I*(a + b*x))])
))/b^3
```

3.30.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$-\frac{2d\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{id\int \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{id\int \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^2}\right)}{b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x],x]
```

output
```
((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLo
g[2, (-I)*E^(I*(a + b*x))])/b - (d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2))
/b - (2*d*((I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b - (d*PolyLog[3, I
*E^(I*(a + b*x))])/b^2))/b
```

3.30.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.30.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(120) = 240.

Time = 1.47 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.86

method	result
risch	$\frac{4icda\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}-\frac{2ic^2\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{a^2d^2\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)}{b^3}-\frac{d^2\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)x^2}{b}+\frac{d^2\ln\left(1-i\mathrm{e}^{i(bx+a)}\right)x^2}{b}-$

input
```
int((d*x+c)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2*I/b*c^2*arctan(exp(I*(b*x+a)))+1/b^
3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+1/b*d^
2*ln(1-I*exp(I*(b*x+a)))*x^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*d^2*
polylog(3,I*exp(I*(b*x+a)))/b^3-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+2/b*c*d*l
n(1-I*exp(I*(b*x+a)))*x-2/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a-2*I/b^2*c*d*pol
ylog(2,I*exp(I*(b*x+a)))-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))+2/b^2*c*d*ln
(1-I*exp(I*(b*x+a)))*a-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-2*I/b^2*d^2*
polylog(2,I*exp(I*(b*x+a)))*x+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x+2
*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))
```

3.30.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 598 vs. 2(111) = 222.

Time = 0.32 (sec) , antiderivative size = 598, normalized size of antiderivative = 4.36

$$\int (c+dx)^2\sec(a+bx)\,dx =$$

$$-\frac{2\,d^2\mathrm{polylog}(3,i\,\cos(bx+a)+\sin(bx+a))-2\,d^2\mathrm{polylog}(3,i\,\cos(bx+a)-\sin(bx+a))+2\,d^2\mathrm{polylo}}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x
 + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(I*b*d^2*x +
 I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*d
ilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*co
s(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a
) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*s
in(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*si
n(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*
cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c
*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2
*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*
x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*
x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*
x + a) + I))/b^3
```

3.30.6 Sympy [F]

$$\int (c+dx)^2 \sec(a+bx)\,dx = \int (c+dx)^2 \sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**2*sec(a + b*x), x)
```

3.30.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 402 vs. $2(111) = 222$.

Time = 0.42 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.93

$$\int (c+dx)^2 \sec(a+bx)\,dx$$

$$= \frac{2c^2 \log(\sec(bx+a)+\tan(bx+a)) - \frac{4acd\log(\sec(bx+a)+\tan(bx+a))}{b} + \frac{2a^2d^2\log(\sec(bx+a)+\tan(bx+a))}{b^2} + \frac{4d^2\mathrm{Li}_3(ie^{(ib\cdots}}{\cdots}}{\cdots}$$

input
```
integrate((d*x+c)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*log(sec(b*x + a) + tan(b*x + a)) - 4*a*c*d*log(sec(b*x + a) + t
an(b*x + a))/b + 2*a^2*d^2*log(sec(b*x + a) + tan(b*x + a))/b^2 + (4*d^2*p
olylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*b*x + I*a)) - 2*(I
*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a),
sin(b*x + a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a)
)*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(I*b*c*d + I*(b*x + a)*d^2
- I*a*d^2)*dilog(I*e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*
d^2)*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x +
 a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 - 2*sin(b*x + a) + 1))/b^2)/b
```

3.30.8 Giac [F]

$$\int (c+dx)^2 \sec(a+bx)\, dx = \int (dx+c)^2 \sec(bx+a)\, dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a), x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec(a+bx)\, dx = \int \frac{(c+d\,x)^2}{\cos(a+b\,x)}\, dx$$

input
```
int((c + d*x)^2/cos(a + b*x),x)
```

output
```
int((c + d*x)^2/cos(a + b*x), x)
```

3.31 $\int (c+dx)\sec(a+bx)\,dx$

3.31.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (c+dx)\sec(a+bx)\,dx = -\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

output
```
-2*I*(d*x+c)*arctan(exp(I*(b*x+a)))/b+I*d*polylog(2,-I*exp(I*(b*x+a)))/b^2
-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2
```

3.31.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int (c+dx)\sec(a+bx)\,dx = -\frac{2idx\arctan\left(e^{ia+ibx}\right)}{b} + \frac{c\,\text{arctanh}(\sin(a+bx))}{b} + \frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x],x]
```

output
```
((-2*I)*d*x*ArcTan[E^(I*a + I*b*x)])/b + (c*ArcTanh[Sin[a + b*x]])/b + (I*
d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x)
)])/b^2
```

3.31.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4669}$$

$$-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

input
```
Int[(c + d*x)*Sec[a + b*x],x]
```

output
```
((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(
a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2
```

3.31.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{-\frac{da\ln(\sec(bx+a)+\tan(bx+a))}{b}+c\ln(\sec(bx+a)+\tan(bx+a))+\frac{d(-(bx+a)\ln(1+ie^{i(bx+a)})+(bx+a)\ln(1-ie^{i(bx+a)})+i\operatorname{dilog}(1 \cdots}{b}}{b}$
default	$\frac{-\frac{da\ln(\sec(bx+a)+\tan(bx+a))}{b}+c\ln(\sec(bx+a)+\tan(bx+a))+\frac{d(-(bx+a)\ln(1+ie^{i(bx+a)})+(bx+a)\ln(1-ie^{i(bx+a)})+i\operatorname{dilog}(1 \cdots}{b}}{b}$
risch	$-\frac{2ic\arctan(e^{i(bx+a)})}{b}-\frac{d\ln(1+ie^{i(bx+a)})x}{b}-\frac{d\ln(1+ie^{i(bx+a)})a}{b^2}+\frac{d\ln(1-ie^{i(bx+a)})x}{b}+\frac{d\ln(1-ie^{i(bx+a)})a}{b^2}$

input
```
int((d*x+c)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*d*a*ln(sec(b*x+a)+tan(b*x+a))+c*ln(sec(b*x+a)+tan(b*x+a))+1/b*d*
(-(b*x+a)*ln(1+I*exp(I*(b*x+a)))+(b*x+a)*ln(1-I*exp(I*(b*x+a)))+I*dilog(1+
I*exp(I*(b*x+a)))-I*dilog(1-I*exp(I*(b*x+a)))))
```

3.31.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(60) = 120$.

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 4.08

$$\int (c + dx) \sec(a + bx) \, dx$$
$$= \frac{-i\, d\mathrm{Li}_2(i \cos(bx + a) + \sin(bx + a)) - i\, d\mathrm{Li}_2(i \cos(bx + a) - \sin(bx + a)) + i\, d\mathrm{Li}_2(-i \cos(bx + a) + \sin}{}$$

input
```
integrate((d*x+c)*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a)
- sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I
*cos(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x +
 a) + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a
*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a)
 + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d
)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a)
- I*sin(b*x + a) + I))/b^2
```

3.31.6 Sympy [F]

$$\int (c + dx) \sec(a + bx) \, dx = \int (c + dx) \sec(a + bx) \, dx$$

input
```
integrate((d*x+c)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)*sec(a + b*x), x)
```

3.31.7 Maxima [F]

$$\int (c+dx)\sec(a+bx)\,dx = \int (dx+c)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(4*b*d*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*a)
*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1), x) + c*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*s
in(b*x + a) + 1) - c*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a)
+ 1))/b
```

3.31.8 Giac [F]

$$\int (c+dx)\sec(a+bx)\,dx = \int (dx+c)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*sec(b*x + a), x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sec(a+bx)\,dx = \int \frac{c+d\,x}{\cos(a+b\,x)}\,dx$$

input
```
int((c + d*x)/cos(a + b*x),x)
```

output
```
int((c + d*x)/cos(a + b*x), x)
```

3.32 $\int \frac{\sec(a+bx)}{c+dx}\,dx$

3.32.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(sec(b*x+a)/(d*x+c),x)
```

3.32.2 Mathematica [N/A]

Not integrable

Time = 4.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Sec[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Sec[a + b*x]/(c + d*x), x]
```

3.32.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \; 4680$$

$$\int \frac{\sec(a+bx)}{c+dx}dx$$

input
```
Int[Sec[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

3.32.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.32.4 Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sec(bx+a)}{dx+c}dx$$

input
```
int(sec(b*x+a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)/(d*x+c),x)
```

3.32.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)/(d*x + c), x)
```

3.32.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)/(d*x+c),x)
```

output
```
Integral(sec(a + b*x)/(c + d*x), x)
```

3.32.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)/(d*x + c), x)
```

3.32.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)/(d*x + c), x)
```

3.32.9 Mupad [N/A]

Not integrable

Time = 13.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sec(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*(c + d*x)), x)
```

3.33 $\int (c+dx)^3 \sec^2(a+bx)\,dx$

3.33.1 Optimal result

Integrand size = 16, antiderivative size = 114

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = -\frac{i(c+dx)^3}{b}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}+\frac{(c+dx)^3\tan(a+bx)}{b}$$

output
```
-I*(d*x+c)^3/b+3*d*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*I*d^2*(d*x+c)*po
lylog(2,-exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+(d
*x+c)^3*tan(b*x+a)/b
```

3.33.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = \frac{-6ibd^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)+3d^3\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)+2b^2(c+dx)^2\left(-ib(c+dx)+3d\,\mathrm{lo}\right.}{2b^4}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]^2,x]
```

output
```
((-6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] + 3*d^3*PolyLog[3
, -E^((2*I)*(a + b*x))] + 2*b^2*(c + d*x)^2*((-I)*b*(c + d*x) + 3*d*Log[1
+ E^((2*I)*(a + b*x))] + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

3.33.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 4672, 25, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{3d\int -(c+dx)^2\tan(a+bx)dx}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\int (c+dx)^2\tan(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\int (c+dx)^2\tan(a+bx)dx}{b}$$

$$\Big\downarrow \text{4202}$$

$$\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{(c+dx)^3\tan(a+bx)}{b}-3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\frac{(c+dx)^3\tan(a+bx)}{b}-3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\frac{(c+dx)^3\tan(a+bx)}{b}-3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}$$

input
```
Int[(c + d*x)^3*Sec[a + b*x]^2,x]
```

output
```
(-3*d*(((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*
I)*(a + b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))
])/b - (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/b)))/b + ((c + d*x)^
3*Tan[a + b*x])/b
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.33.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(104) = 208$.

Time = 2.14 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.77

method	result
risch	$-\frac{6id^2cx^2}{b}+\frac{4id^3a^3}{b^4}-\frac{6id^2ca^2}{b^3}-\frac{3id^2c\,\mathrm{Li}_2\left(-\mathrm{e}^{2i(bx+a)}\right)}{b^3}+\frac{12d^2ca\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}+\frac{6d^2c\ln\left(\mathrm{e}^{2i(bx+a)}+1\right)x}{b^2}+\frac{2i\left(d^3x^3+3c\right.}{b\left(\mathrm{e}^{2i(\right.}}$

input
```
int((d*x+c)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-6*I/b*d^2*c*x^2+4*I/b^4*d^3*a^3-6*I/b^3*d^2*c*a^2-3*I/b^3*d^2*c*polylog(2
,-exp(2*I*(b*x+a)))+12/b^3*d^2*c*a*ln(exp(I*(b*x+a)))+6/b^2*d^2*c*ln(exp(2
*I*(b*x+a))+1)*x+2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a
))+1)-2*I/b*d^3*x^3+6*I/b^3*d^3*a^2*x-12*I/b^2*d^2*c*a*x-6/b^4*d^3*a^2*ln(
exp(I*(b*x+a)))-3*I/b^3*d^3*polylog(2,-exp(2*I*(b*x+a)))*x+3/b^2*d^3*ln(ex
p(2*I*(b*x+a))+1)*x^2+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4-6/b^2*d*c^2
*ln(exp(I*(b*x+a)))+3/b^2*d*c^2*ln(exp(2*I*(b*x+a))+1)
```

3.33.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(101) = 202.

Time = 0.32 (sec) , antiderivative size = 790, normalized size of antiderivative = 6.93

$$\int (c+dx)^3\sec^2(a+bx)\,dx$$
$$=\frac{6\,d^3\cos(bx+a)\,\mathrm{polylog}(3,i\,\cos(bx+a)+\sin(bx+a))+6\,d^3\cos(bx+a)\,\mathrm{polylog}(3,i\,\cos(bx+a)-\sin}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*d^3*
cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a
)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(
3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x +
a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*
x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*co
s(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d
^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*s
in(b*x + a) + I) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)
*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*
b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin
(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*c
os(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b
^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin
(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-c
os(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a...
```

3.33.6 Sympy [F]

$$\int (c+dx)^3\sec^2(a+bx)\,dx=\int (c+dx)^3\sec^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*sec(a + b*x)**2, x)
```

3.33.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(101) = 202.

Time = 0.43 (sec) , antiderivative size = 1059, normalized size of antiderivative = 9.29

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*c^3*tan(b*x + a) - 6*a*c^2*d*tan(b*x + a)/b + 6*a^2*c*d^2*tan(b*x +
 a)/b^2 - 2*a^3*d^3*tan(b*x + a)/b^3 + 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) - 6*((co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*
sin(2*b*x + 2*a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*co
s(2*b*x + 2*a) + 1)*b^2) + 3*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos
(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3) + 2*(6*((b*x + a
)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + ((b*x + a)^2*d^3 + 2*(b*c*d^2 -
a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^3 + 2*(-I*b*c*d^2 +
 I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2
*b*x + 2*a) - 6*(b*c*d^2 + (b*x + a)*d^3 - a*d^3 + (b*c*d^2 + (b*x + a)*d^
3 - a*d^3)*cos(2*b*x + 2*a) + (I*b*c*d^2 + I*(b*x + a)*d^3 - I*a*d^3)*sin(
2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - 3*(I*(b*x + a)^2*d^3 + 2*(I*b*
c*d^2 - I*a*d^3)*(b*x + a) + (I*(b*x + a)^2*d^3 + 2*(I*b*c*d^2 - I*a*d^...
```

3.33.8 Giac [F]

$$\int (c+dx)^3 \sec^2(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)^2, x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec^2(a+bx)\, dx = \int \frac{(c+d\,x)^3}{\cos{(a+b\,x)}^2}\, dx$$

input
```
int((c + d*x)^3/cos(a + b*x)^2,x)
```

output
```
int((c + d*x)^3/cos(a + b*x)^2, x)
```

3.34 $\int (c+dx)^2 \sec^2(a+bx)\,dx$

3.34.1 Optimal result

Integrand size = 16, antiderivative size = 82

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = -\frac{i(c+dx)^2}{b} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\tan(a+bx)}{b}$$

output
```
-I*(d*x+c)^2/b+2*d*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^2-I*d^2*polylog(2,-exp
(2*I*(b*x+a)))/b^3+(d*x+c)^2*tan(b*x+a)/b
```

3.34.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = \frac{-id^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right) + b(c+dx)\left(-ib(c+dx)+2d\log\left(1+e^{2i(a+bx)}\right)+b(c+dx)\tan(a+bx)\right)}{b^3}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]^2,x]
```

output
```
((-I)*d^2*PolyLog[2, -E^((2*I)*(a + b*x))] + b*(c + d*x)*((-I)*b*(c + d*x)
 + 2*d*Log[1 + E^((2*I)*(a + b*x))] + b*(c + d*x)*Tan[a + b*x]))/b^3
```

3.34.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sec^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \ 4672$$

$$\frac{2d\int -((c+dx)\tan(a+bx))dx}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\int (c+dx)\tan(a+bx)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\int (c+dx)\tan(a+bx)dx}{b}$$

$$\Big\downarrow \ 4202$$

$$\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx\right)}{b}$$

$$\Big\downarrow \ 2620$$

$$\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}$$

$$\Big\downarrow \ 2715$$

$$\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}$$

$$\Big\downarrow \ 2838$$

$$\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]^2,x]
```

output
```
(-2*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)
*(a + b*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])/(4*b^2))))/b + ((c
+ d*x)^2*Tan[a + b*x])/b
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.34.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(76) = 152.

Time = 1.88 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.07

method	result
risch	$\frac{2i(x^2d^2+2cdx+c^2)}{b(\mathrm{e}^{2i(bx+a)}+1)}-\frac{4dc\ln(\mathrm{e}^{i(bx+a)})}{b^2}+\frac{2dc\ln(\mathrm{e}^{2i(bx+a)}+1)}{b^2}-\frac{2id^2x^2}{b}-\frac{4id^2ax}{b^2}-\frac{2id^2a^2}{b^3}+\frac{2d^2\ln(\mathrm{e}^{2i(bx+a)}+1)x}{b^2}-i$

input
```
int((d*x+c)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)-4/b^2*d*c*ln(exp(I*(b*x+a
)))+2/b^2*d*c*ln(exp(2*I*(b*x+a))+1)-2*I/b*d^2*x^2-4*I/b^2*d^2*a*x-2*I/b^3
*d^2*a^2+2/b^2*d^2*ln(exp(2*I*(b*x+a))+1)*x-I*d^2*polylog(2,-exp(2*I*(b*x+
a)))/b^3+4/b^3*d^2*a*ln(exp(I*(b*x+a)))
```

3.34.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(73) = 146.

Time = 0.29 (sec) , antiderivative size = 450, normalized size of antiderivative = 5.49

$$\int (c+dx)^2\sec^2(a+bx)\,dx$$
$$=\frac{i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))-i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos(bx+a)-\sin(bx+a))-i}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
(I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b*x +
 a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d^2*cos(b*x + a)*dilog(-I*cos
(b*x + a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin
(b*x + a)) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a
) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) +
I) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)
 + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) +
 (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) +
(b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (
b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c
*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a))
```

3.34.6 Sympy [F]

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = \int (c+dx)^2 \sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*sec(a + b*x)**2, x)
```

3.34.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(73) = 146.

Time = 0.49 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.95

$$\int (c+dx)^2 \sec^2(a+bx)\,dx$$
$$= \frac{2\,b^2c^2 + 2\,(bd^2x + bcd + (bd^2x + bcd)\cos(2\,bx + 2\,a) - (-i\,bd^2x - i\,bcd)\sin(2\,bx + 2\,a))\arctan(\sin(2\,bx}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
(2*b^2*c^2 + 2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (-I
*b*d^2*x - I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
+ 2*a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(2*b*x + 2*a) - (d^2*cos(2*
b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + (
-I*b*d^2*x - I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x
+ b*c*d)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1) - 2*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x)*sin(2*b*x + 2*a
))/(-I*b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) - I*b^3)
```

3.34.8 Giac [F]

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = \int (dx+c)^2 \sec{(bx+a)}^2\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)^2, x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\cos{(a+b\,x)}^2}\,dx$$

input
```
int((c + d*x)^2/cos(a + b*x)^2,x)
```

output
```
int((c + d*x)^2/cos(a + b*x)^2, x)
```

3.35 $\int (c+dx)\sec^2(a+bx)\,dx$

3.35.1 Optimal result

Integrand size = 14, antiderivative size = 28

$$\int (c+dx)\sec^2(a+bx)\,dx=\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}$$

output
```
d*ln(cos(b*x+a))/b^2+(d*x+c)*tan(b*x+a)/b
```

3.35.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int (c+dx)\sec^2(a+bx)\,dx=\frac{d\log(\cos(a+bx))}{b^2}+\frac{c\tan(a+bx)}{b}+\frac{dx\tan(a+bx)}{b}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]^2,x]
```

output
```
(d*Log[Cos[a + b*x]])/b^2 + (c*Tan[a + b*x])/b + (d*x*Tan[a + b*x])/b
```

3.35.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{d\int -\tan(a+bx)dx}{b}+\frac{(c+dx)\tan(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)\tan(a+bx)}{b}-\frac{d\int \tan(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\tan(a+bx)}{b}-\frac{d\int \tan(a+bx)dx}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}$$

input
```
Int[(c + d*x)*Sec[a + b*x]^2,x]
```

output
```
(d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.35.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{-\frac{da\tan(bx+a)}{b}+c\tan(bx+a)+\frac{d((bx+a)\tan(bx+a)+\ln(\cos(bx+a)))}{b}}{b}$
default	$\frac{-\frac{da\tan(bx+a)}{b}+c\tan(bx+a)+\frac{d((bx+a)\tan(bx+a)+\ln(\cos(bx+a)))}{b}}{b}$
risch	$-\frac{2idx}{b}-\frac{2ida}{b^2}+\frac{2i(dx+c)}{b(\mathrm{e}^{2i(bx+a)}+1)}+\frac{d\ln(\mathrm{e}^{2i(bx+a)}+1)}{b^2}$
parallelrisch	$\frac{-d\ln\left(\sec^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos(bx+a)+d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\cos(bx+a)+d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)\cos(bx+a)+(dx+c)b\sin(bx-}{b^2\cos(bx+a)}$
norman	$\frac{-\frac{2c\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{2dx\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}}{\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}{b^2}+\frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{b^2}-\frac{d\ln\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2}$

input
```
int((d*x+c)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*d*a*tan(b*x+a)+c*tan(b*x+a)+1/b*d*((b*x+a)*tan(b*x+a)+ln(cos(b*x
+a))))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int (c+dx)\sec^2(a+bx)\,dx = \frac{d\cos(bx+a)\log(-\cos(bx+a)) + (bdx+bc)\sin(bx+a)}{b^2\cos(bx+a)}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
(d*cos(b*x + a)*log(-cos(b*x + a)) + (b*d*x + b*c)*sin(b*x + a))/(b^2*cos(
b*x + a))
```

3.35.6 Sympy [F]

$$\int (c+dx)\sec^2(a+bx)\,dx = \int (c+dx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*sec(a + b*x)**2, x)
```

3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(28) = 56.

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.68

$$\int (c+dx)\sec^2(a+bx)\,dx$$

$$= \frac{2c\tan(bx+a) - \frac{2ad\tan(bx+a)}{b} + \frac{\left(\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)\log\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2\right.}{\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)b}}{2b}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*c*tan(b*x + a) - 2*a*d*tan(b*x + a)/b + ((cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*d/((c
os(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(28) = 56$.

Time = 0.64 (sec) , antiderivative size = 1404, normalized size of antiderivative = 50.14

$$\int (c+dx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2*a)^
2 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan(1/2*a)^2 -
 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b
*x)^4 + 8*tan(1/2*b*x)^3*tan(1/2*a) + 20*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*t
an(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 2*tan(1/2*b*x)^2 - 8*tan(1/2*b*x
)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/
2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4
*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2
*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*c*tan(1/2*b*x)^2*tan(1/2*a)
+ 4*b*c*tan(1/2*b*x)*tan(1/2*a)^2 - 4*b*d*x*tan(1/2*b*x) + d*log(4*(tan(1/
2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*tan(1/2*b*x)^3*t
an(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 8*tan(1/2*b
*x)^3*tan(1/2*a) + 20*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2
*a)^3 + tan(1/2*a)^4 - 2*tan(1/2*b*x)^2 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*ta
n(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b*x)^2*tan
(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1))*tan(1/2
*b*x)^2 - 4*b*d*x*tan(1/2*a) + 4*d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*
tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*
x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 8*tan(1/2*b*x)^3*tan(1/2*a) + 20*t...
```

3.35.9 Mupad [B] (verification not implemented)

Time = 14.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int (c+dx)\sec^2(a+bx)\,dx = \frac{d\ln\left(e^{a\,2i}\,e^{b\,x\,2i}+1\right)}{b^2} + \frac{(c+d\,x)\ 2i}{b\ \left(e^{a\,2i+b\,x\,2i}+1\right)} - \frac{d\,x\,2i}{b}$$

input
```
int((c + d*x)/cos(a + b*x)^2,x)
```

output
```
(d*log(exp(a*2i)*exp(b*x*2i) + 1))/b^2 + ((c + d*x)*2i)/(b*(exp(a*2i + b*x
*2i) + 1)) - (d*x*2i)/b
```

3.35. $\int (c+dx)\sec^2(a+bx)\,dx$

3.36 $\int \frac{\sec^2(a+bx)}{c+dx}\,dx$

3.36.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec^2(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(sec(b*x+a)^2/(d*x+c),x)
```

3.36.2 Mathematica [N/A]

Not integrable

Time = 6.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Sec[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Sec[a + b*x]^2/(c + d*x), x]
```

3.36.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)^2}{c+dx}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\sec^2(a+bx)}{c+dx}dx$$

input
```
Int[Sec[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

3.36.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.36.4 Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(bx+a)}{dx+c}dx$$

input
```
int(sec(b*x+a)^2/(d*x+c),x)
```

output
```
int(sec(b*x+a)^2/(d*x+c),x)
```

3.36.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec{(bx+a)}^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2/(d*x + c), x)
```

3.36.6 Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sec(a + b*x)**2/(c + d*x), x)
```

3.36.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 290, normalized size of antiderivative = 18.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
2*((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*
c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(
sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2
*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + sin(2*b*
x + 2*a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*
b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

3.36.8 Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2/(d*x + c), x)
```

3.36.9 Mupad [N/A]

Not integrable

Time = 13.70 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*(c + d*x)), x)
```

3.37 $\int (c+dx)^3 \sec^3(a+bx)\,dx$

3.37.1 Optimal result

Integrand size = 16, antiderivative size = 337

$$\begin{aligned}\int (c+dx)^3 \sec^3(a+bx)\,dx = &-\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} \\ &-\frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3id^3\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4} \\ &+\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} \\ &-\frac{3id^3\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4} \\ &-\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} \\ &-\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\ &+\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &-\frac{3id^3\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4}+\frac{3id^3\operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4} \\ &-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

output
```
-6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3-I*(d*x+c)^3*arctan(exp(I*(b*x+
a)))/b+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(
2,-I*exp(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3/2*I*d*(
d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-I*exp(I*
(b*x+a)))/b^3+3*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3-3*I*d^3*polylo
g(4,-I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/2*d*(
d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*sec(b*x+a)*tan(b*x+a)/b
```

3.37.2 Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.92

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \frac{-2ib^3(c+dx)^3\arctan\left(e^{i(a+bx)}\right)-6id^2\left(2b(c+dx)\arctan\left(e^{i(a+bx)}\right)-d\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)+d\,\text{PolyL}\right.}{}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]^3,x]
```

output
```
((-2*I)*b^3*(c + d*x)^3*ArcTan[E^(I*(a + b*x))] - (6*I)*d^2*(2*b*(c + d*x)
*ArcTan[E^(I*(a + b*x))] - d*PolyLog[2, (-I)*E^(I*(a + b*x))] + d*PolyLog[
2, I*E^(I*(a + b*x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a
+ b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))] - 2*d^2*Po
lyLog[4, (-I)*E^(I*(a + b*x))]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, I*E^
(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))] - 2*d^2
*PolyLog[4, I*E^(I*(a + b*x))]) - 3*b^2*d*(c + d*x)^2*Sec[a + b*x] + b^3*(
c + d*x)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b^4)
```

3.37.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4674, 3042, 4669, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sec^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \; 4674$$

$$\frac{3d^2\int (c+dx)\sec(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^3\sec(a+bx)dx-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{3d^2\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b^2}+\frac{1}{2}\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \; 4669$$

$$\frac{3d^2\left(-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \; 2715$$

$$\frac{3d^2\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \; 2838$$

$$\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)+\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \; 3011$$

3.37. $\int (c+dx)^3\sec^3(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\right.}{b}\right.$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}-$$

$$\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$\downarrow$ 7163

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}}{b}\right.}{}\right.$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}-$$

$$\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$\downarrow$ 2720

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i}{}\right.}{}\right.$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}-$$

$$\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$\downarrow$ 7143

3.37. $\int (c+dx)^3\sec^3(a+bx)\,dx$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan(e^{i(a+bx)})}{b}+\frac{id\,\text{PolyLog}(2,-ie^{i(a+bx)})}{b^2}-\frac{id\,\text{PolyLog}(2,ie^{i(a+bx)})}{b^2}\right)}{b^2}+$$

$$\frac{1}{2}\left(-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(3,-ie^{i(a}\right.}{b}\right.}{b}\right.}{b}\right.$$

$$\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)^3*Sec[a + b*x]^3,x]
```

output
```
(3*d^2*(((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I
)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2))/b^2 +
(((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)^2*Pol
yLog[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (-
I)*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^2))/b))/b
- (3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*
(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4, I*E^(I*(a + b*x
))])/b^2))/b))/b)/2 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^
3*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

3.37.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.37. $\int (c+dx)^3 \sec^3(a+bx) \, dx$

3.37.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. 2(293) = 586.

Time = 1.95 (sec) , antiderivative size = 1127, normalized size of antiderivative = 3.34

method	result	size
risch	Expression too large to display	1127

input
```
int((d*x+c)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2/(exp(2*I*(b*x+a))+1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*ex
p(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))-d^3*x^3*b*exp(I*(b*x+a))+b*c^3
*exp(3*I*(b*x+a))-3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*x+a)
)-3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a))-b*c^3*exp(I*(b*x
+a))-3*I*c^2*d*exp(3*I*(b*x+a))-3*I*d^3*x^2*exp(I*(b*x+a))-6*I*c*d^2*x*exp
(I*(b*x+a))-3*I*c^2*d*exp(I*(b*x+a)))+I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))
+3/2/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))
*a+3/2/b^3*a^2*d^2*c*ln(1+I*exp(I*(b*x+a)))-3/2/b^3*a^2*d^2*c*ln(1-I*exp(I
*(b*x+a)))-3/2/b*c*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+3/2/b*c*d^2*ln(1-I*exp(I
*(b*x+a)))*x^2-3/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-3/2/b*c^2*d*ln(1+I*e
xp(I*(b*x+a)))*x-3/2*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3/2*I/b^2*d
^3*polylog(2,-I*exp(I*(b*x+a)))*x^2+6*I/b^4*d^3*a*arctan(exp(I*(b*x+a)))+3
/2*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-3/2*I/b^2*c^2*d*polylog(2,I*ex
p(I*(b*x+a)))-6*I/b^3*d^2*c*arctan(exp(I*(b*x+a)))-3*I/b^3*c*d^2*a^2*arcta
n(exp(I*(b*x+a)))+3*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-3*I/b^2*c*d
^2*polylog(2,I*exp(I*(b*x+a)))*x+3*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+3*
I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3/b^3*d^3*polylog(3,-I*exp(I*(b*x+a
)))*x-1/2/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+1/2/b*d^3*ln(1-I*exp(I*(b*x+a))
)*x^3+3/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x-3/b^3*d^3*ln(1+I*exp(I*(b*x+
a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a+3/b^3*d^3*ln(1-I*exp(I*(b*x+a...
```

3.37.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1315 vs. 2(273) = 546.

Time = 0.37 (sec) , antiderivative size = 1315, normalized size of antiderivative = 3.90

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*
I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*c
os(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x
 + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*
I*b^2*c*d^2*x + I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a)
 + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + 2*I*
d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x
^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(
b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2
*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*
c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^
2*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a
^2 + 2)*b*c*d^2 - (a^3 + 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin
(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*
c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I*
cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d + 2*b*d^3)*x)*cos
(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 6*a)*d^3 + 3*(b^3*c^2*d
 + 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) -...
```

3.37.6 Sympy [F]

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \int (c+dx)^3 \sec^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*sec(a + b*x)**3, x)
```

3.37.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3831 vs. 2(273) = 546.

Time = 1.56 (sec) , antiderivative size = 3831, normalized size of antiderivative = 11.37

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + l
og(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - l
og(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x +
 a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/
b^2 - a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1)
 + log(sin(b*x + a) - 1))/b^3 + 4*(2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 +
2)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2
 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + (I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(
I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2
 + 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 + 6*I*b*c*
d^2 - 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2
*I*a*b*c*d^2 + (I*a^2 + 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos
(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 +
3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d
^3)*(b*x + a) + ((b*x + a)^3*d^3 + 6*b*c*d^2 - 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 2)*d^3)*(b*x + a))...
```

3.37.8 Giac [F]

$$\int (c+dx)^3 \sec^3(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)^3, x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int (c + dx)^3 \sec^3(a + bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^3/cos(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.38 $\int (c+dx)^2 \sec^3(a+bx)\,dx$

3.38.1 Optimal result

Integrand size = 16, antiderivative size = 193

$$\begin{aligned}\int (c+dx)^2 \sec^3(a+bx)\,dx = & -\frac{i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2 \text{arctanh}(\sin(a+bx))}{b^3} \\ & +\frac{id(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ & -\frac{id(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\ & -\frac{d^2\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ & -\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

output
```
-I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+d^2*arctanh(sin(b*x+a))/b^3+I*d*(d*x
+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)
))/b^2-d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+d^2*polylog(3,I*exp(I*(b*x+a))
)/b^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*sec(b*x+a)*tan(b*x+a)/b
```

3.38.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95

$$\int (c+dx)^2 \sec^3(a+bx)\,dx$$
$$=\frac{-2ib^2(c+dx)^2\arctan\left(e^{i(a+bx)}\right)+2d^2\text{arctanh}(\sin(a+bx))+2ibd(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)-2ibd}{}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]^3,x]
```

output
```
((-2*I)*b^2*(c + d*x)^2*ArcTan[E^(I*(a + b*x))] + 2*d^2*ArcTanh[Sin[a + b*
x]] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c
+ d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x
))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 2*b*d*(c + d*x)*Sec[a + b*x] +
 b^2*(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b^3)
```

3.38.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4674, 3042, 4257, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sec^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx$$
$$\Big\downarrow \text{4674}$$
$$\frac{d^2\int \sec(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^2\sec(a+bx)dx-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \csc\left(a+bx+\frac{\pi}{2}\right)dx}{b^2}+\frac{1}{2}\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 4257

$$\frac{1}{2}\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d^2\text{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 4669

$$\frac{1}{2}\left(-\frac{2d\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}\right)+\frac{d^2\text{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 3011

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{id\int \text{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{id\int \text{PolyLog}(2,ie^{i(a+bx}}{b}\right.}{b}\right.$$
$$\frac{d^2\text{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 2720

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}(2,-ie^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{d\int e^{-i}}{b}\right.}{b}\right.$$
$$\frac{d^2\text{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 7143

$$\frac{1}{2}\left(-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{d\,\text{PolyLog}(3,-ie^{i(a+bx)})}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}(2,}{b}\right.}{}\right.$$
$$\frac{d^2\text{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]^3,x]
```

3.38. $\int (c+dx)^2\sec^3(a+bx)\,dx$

output
```
(d^2*ArcTanh[Sin[a + b*x]])/b^3 + (((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*
x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - (d*Poly
Log[3, (-I)*E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, I*E^
(I*(a + b*x))])/b - (d*PolyLog[3, I*E^(I*(a + b*x))])/b^2))/b)/2 - (d*(c +
 d*x)*Sec[a + b*x])/b^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

3.38.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.38.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(174) = 348.

Time = 1.78 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.03

method	result
risch	$-\frac{2id^2\arctan(e^{i(bx+a)})}{b^3}+\frac{d^2\,\mathrm{Li}_3(ie^{i(bx+a)})}{b^3}-\frac{d^2\,\mathrm{Li}_3(-ie^{i(bx+a)})}{b^3}-\frac{i(x^2d^2b\,e^{3i(bx+a)}+2cdxb\,e^{3i(bx+a)}+b\,c^2e^{3i(bx+a)}-x^2d^2b}{}$

input
```
int((d*x+c)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*I/b^3*d^2*arctan(exp(I*(b*x+a)))+d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-d^
2*polylog(3,-I*exp(I*(b*x+a)))/b^3-I/b^2/(exp(2*I*(b*x+a))+1)^2*(x^2*d^2*b
*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+a))+b*c^2*exp(3*I*(b*x+a))-x^2*d^
2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))-b*c
^2*exp(I*(b*x+a))-2*I*c*d*exp(3*I*(b*x+a))-2*I*d^2*x*exp(I*(b*x+a))-2*I*c*
d*exp(I*(b*x+a)))-1/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2-1/b^2*c*d*ln(1+I*ex
p(I*(b*x+a)))*a+1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+1/b*c*d*ln(1-I*exp(I*
(b*x+a)))*x+2*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-I/b^2*d^2*polylog(2,I*exp
(I*(b*x+a)))*x+I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-I/b^3*d^2*a^2*arct
an(exp(I*(b*x+a)))+1/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a-1/b*c*d*ln(1+I*exp(I
*(b*x+a)))*x+1/2/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-1/2/b^3*a^2*d^2*ln(1-I
*exp(I*(b*x+a)))-I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))+I/b^2*c*d*polylog(2
,-I*exp(I*(b*x+a)))-I/b*c^2*arctan(exp(I*(b*x+a)))
```

3.38.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(165) = 330$.

Time = 0.34 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.12

$$\int (c+dx)^2 \sec^3(a+bx)\,dx =$$
$$-\frac{2\,d^2\cos{(bx+a)}^2\,\mathrm{polylog}(3, i\,\cos{(bx+a)}+\sin{(bx+a)}) - 2\,d^2\cos{(bx+a)}^2\,\mathrm{polylog}(3, i\,\cos{(bx+a)}}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d
^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*
x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2
*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*cos(
b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*
cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*
c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^2*x
 - I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2
 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x
 + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b
*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a
^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x
^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a)
 - sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^
2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*
x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*cos(b*x + a)^2*log(-co
s(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 + 2)*d^2)*c
os(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b*d^2*x + b*c*d
)*cos(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/...
```

3.38.6 Sympy [F]

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = \int (c+dx)^2 \sec^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*sec(a + b*x)**3, x)
```

3.38.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1891 vs. $2(165) = 330$.

Time = 0.66 (sec) , antiderivative size = 1891, normalized size of antiderivative = 9.80

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + l
og(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log
(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(s
in(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/b^2 +
4*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + ((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x
+ a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) +
 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*b
*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^2*d^2 +
2*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d
- I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 +
2*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b
*x + a), -sin(b*x + a) + 1) + 4*((b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*a*d^2 +
 2*(b*c*d - (a + I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) - 4*((b*x + a)^2*d^2
+ 2*I*b*c*d - 2*I*a*d^2 + 2*(b*c*d - (a - I)*d^2)*(b*x + a))*cos(b*x + a)
+ 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4
*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b...
```

3.38.8 Giac [F]

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = \int (dx+c)^2 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)^3, x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^2/cos(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.39 $\int (c+dx) \sec^3(a+bx) \, dx$

3.39.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\int (c+dx) \sec^3(a+bx) \, dx = -\frac{i(c+dx) \arctan\left(e^{i(a+bx)}\right)}{b} + \frac{id \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} - \frac{id \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{d \sec(a+bx)}{2b^2} + \frac{(c+dx) \sec(a+bx) \tan(a+bx)}{2b}$$

output
```
-I*(d*x+c)*arctan(exp(I*(b*x+a)))/b+1/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/b
^2-1/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c
)*sec(b*x+a)*tan(b*x+a)/b
```

3.39.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 389 vs. $2(117) = 234$.

Time = 4.17 (sec) , antiderivative size = 389, normalized size of antiderivative = 3.32

$$\int (c+dx)\sec^3(a+bx)\,dx = \frac{c\operatorname{arctanh}(\sin(a+bx))}{2b}$$
$$+\frac{d\Big(\left(-a+\frac{\pi}{2}-bx\right)\left(\log\left(1-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\log\left(1+e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)-\left(-a+\frac{\pi}{2}\right)\log\left(\tan\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-\right.\right.\right.}{2b^2}$$
$$+\frac{dx}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}$$
$$-\frac{d\sin\left(\frac{bx}{2}\right)}{2b^2\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$
$$-\frac{dx}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}$$
$$+\frac{d\sin\left(\frac{bx}{2}\right)}{2b^2\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}+\frac{c\sec(a+bx)\tan(a+bx)}{2b}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]^3,x]
```

output
```
(c*ArcTanh[Sin[a + b*x]])/(2*b) + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a
+ Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[
(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[
2, E^(I*(-a + Pi/2 - b*x))])))/(2*b^2) + (d*x)/(4*b*(Cos[a/2 + (b*x)/2] -
Sin[a/2 + (b*x)/2])^2) - (d*Sin[(b*x)/2])/(2*b^2*(Cos[a/2] - Sin[a/2])*(Co
s[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (d*x)/(4*b*(Cos[a/2 + (b*x)/2] +
 Sin[a/2 + (b*x)/2])^2) + (d*Sin[(b*x)/2])/(2*b^2*(Cos[a/2] + Sin[a/2])*(C
os[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(
2*b)
```

3.39.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4673, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4673}$$

$$\frac{1}{2}\int (c+dx)\sec(a+bx)dx-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{4669}$$

$$\frac{1}{2}\left(-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{2}\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Sec[a + b*x]^3,x]
```

output
```
(((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*
(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2)/2 - (d*Sec[a +
 b*x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

3.39.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.39.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(98) = 196.

Time = 1.38 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.28

method	result
risch	$-\frac{i\left(bxd\,\mathrm{e}^{3i(bx+a)}+bc\,\mathrm{e}^{3i(bx+a)}-bxd\,\mathrm{e}^{i(bx+a)}-bc\,\mathrm{e}^{i(bx+a)}-id\,\mathrm{e}^{3i(bx+a)}-id\,\mathrm{e}^{i(bx+a)}\right)}{b^{2}\left(\mathrm{e}^{2i(bx+a)}+1\right)^{2}}-\frac{ic\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{d\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)x}{2b}$

input
```
int((d*x+c)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2/(exp(2*I*(b*x+a))+1)^2*(b*x*d*exp(3*I*(b*x+a))+b*c*exp(3*I*(b*x+a))
-b*x*d*exp(I*(b*x+a))-b*c*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))-I*d*exp(I*(b
*x+a)))-I/b*c*arctan(exp(I*(b*x+a)))-1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/2/
b^2*d*ln(1+I*exp(I*(b*x+a)))*a+1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/2/b^2*d*
ln(1-I*exp(I*(b*x+a)))*a+1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-1/2*I/b^2*d
*dilog(1-I*exp(I*(b*x+a)))+I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

3.39.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(93) = 186.

Time = 0.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 3.72

$$\int (c+dx)\sec^3(a+bx)\,dx$$
$$=\frac{-i\,d\cos\left(bx+a\right)^2\operatorname{Li}_2(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right))-i\,d\cos\left(bx+a\right)^2\operatorname{Li}_2(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right))+}{}$$

input
```
integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(-I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*cos(b*
x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-
I*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a)
- sin(b*x + a)) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x
+ a) + I) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) +
 I) + (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)
- (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b
*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*
x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a
*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*c
os(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) +
 2*(b*d*x + b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

3.39.6 Sympy [F]

$$\int (c+dx)\sec^3(a+bx)\,dx = \int (c+dx)\sec^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*sec(a + b*x)**3, x)
```

3.39.7 Maxima [F]

$$\int (c+dx)\sec^3(a+bx)\,dx = \int (dx+c)\sec{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(4*(d*cos(3*b*x + 3*a) + d*cos(b*x + a) - (b*d*x + b*c)*sin(3*b*x + 3
*a) + (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 4*(2*d*cos(2*b*x + 2*
a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 8*(d*cos(b*x
 + a) + (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 4*d*cos(b*x + a) -
4*(b^2*d*cos(4*b*x + 4*a)^2 + 4*b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(4*b*x
 + 4*a)^2 + 4*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*d*sin(2*b*x
+ 2*a)^2 + 4*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(2*b^2*d*cos(2*b*x + 2*a)
+ b^2*d)*cos(4*b*x + 4*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*
sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - (b*c*cos(4*b*x + 4*a)^2 + 4
*b*c*cos(2*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c
 + 2*(2*b*c*cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*co
s(2*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b
*x + 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2
*b*c*cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d*
x + b*c)*cos(b*x + a) + d*sin(3*b*x + 3*a) + d*sin(b*x + a))*sin(4*b*x + 4
*a) - 4*(b*d*x + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x...
```

3.39.8 Giac [F]

$$\int (c+dx)\sec^3(a+bx)\,dx = \int (dx+c)\sec{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*sec(b*x + a)^3, x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/cos(a + b*x)^3,x)
```

output
```
\text{Hanged}
```

3.40 $\int \frac{\sec^2(a+bx)}{c+dx}\,dx$

3.40.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec^2(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(sec(b*x+a)^2/(d*x+c),x)
```

3.40.2 Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Sec[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Sec[a + b*x]^2/(c + d*x), x]
```

3.40.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)^2}{c+dx}dx$$

$$\Big\downarrow \; 4680$$

$$\int \frac{\sec^2(a+bx)}{c+dx}dx$$

input
```
Int[Sec[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

3.40.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.40.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(bx+a)}{dx+c}dx$$

input
```
int(sec(b*x+a)^2/(d*x+c),x)
```

output
```
int(sec(b*x+a)^2/(d*x+c),x)
```

3.40.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2/(d*x + c), x)
```

3.40.6 Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sec(a + b*x)**2/(c + d*x), x)
```

3.40.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 290, normalized size of antiderivative = 18.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx=\int \frac{\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
2*((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*
c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(
sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2
*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + sin(2*b*
x + 2*a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*
b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

3.40.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx=\int \frac{\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2/(d*x + c), x)
```

3.40.9 Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*(c + d*x)), x)
```

3.41 $\int (c+dx)^{5/2}\cos(a+bx)\,dx$

3.41.1 Optimal result

Integrand size = 16, antiderivative size = 194

$$\begin{aligned}\int (c+dx)^{5/2}\cos(a+bx)\,dx &= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{2b^2} \\ &+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{7/2}} \\ &- \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{b}\end{aligned}$$

output
```
5/2*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2+(d*x+c)^(5/2)*sin(b*x+a)/b+15/8*d^(5/2)
*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(
1/2)*Pi^(1/2)/b^(7/2)+15/8*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/4*d^2*sin(b*x+a
)*(d*x+c)^(1/2)/b^3
```

3.41.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.64

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx =$$
$$-\frac{d^3e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x],x]
```

output
```
-1/2*(d^3*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((-I)*b*(c +
d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, (I*b*(c +
d*x))/d]))/(b^4*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

3.41.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.938, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^{5/2}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{3777}$$
$$\frac{5d\int -(c+dx)^{3/2}\sin(a+bx)dx}{2b}+\frac{(c+dx)^{5/2}\sin(a+bx)}{b}$$
$$\Big\downarrow \text{25}$$
$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\int (c+dx)^{3/2}\sin(a+bx)dx}{2b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\int(c+dx)^{3/2}\sin(a+bx)dx}{2b}$$

$\Big\downarrow$ 3777

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\int\sqrt{c+dx}\cos(a+bx)dx}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$\Big\downarrow$ 3777

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{d\int-\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}+\frac{\sqrt{c+dx}\sin(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$\Big\downarrow$ 25

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$\Big\downarrow$ 3787

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

3.41. $\int(c+dx)^{5/2}\cos(a+bx)\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$$\Big\downarrow \text{3785}$$

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$$\Big\downarrow \text{3786}$$

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$$\Big\downarrow \text{3832}$$

3.41. $\int(c+dx)^{5/2}\cos(a+bx)\,dx$

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

$\big\downarrow$ 3833

$$\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x],x]
```

output
```
((c + d*x)^(5/2)*Sin[a + b*x])/b - (5*d*(-(((c + d*x)^(3/2)*Cos[a + b*x])/
b) + (3*d*(-1/2*(d*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d]
)))/b + (Sqrt[c + d*x]*Sin[a + b*x])/b))/(2*b)))/(2*b)
```

3.41. $\int (c+dx)^{5/2}\cos(a+bx)\,dx$

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.41. $\int (c+dx)^{5/2} \cos(a+bx)\,dx$

3.41.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\right.}{}\right.}{}\right)}{b}}{d}$
default	$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\right.}{}\right.}{}\right)}{b}}{d}$

input

```
int((d*x+c)^(5/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2/d*(1/2/b*d*(d*x+c)^(5/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-5/2/b*d*(-1/2/b*d*
(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2)*
sin(b*(d*x+c)/d+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*
d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*
d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-\ldots}{}\right.}{}$$

input

```
integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresn
el_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 2*sqrt(
d*x + c)*(10*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) + (4*b^3*d^2*x^2 + 8*b^3*c
*d*x + 4*b^3*c^2 - 15*b*d^2)*sin(b*x + a)))/b^4
```

3.41.6 Sympy [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*cos(a + b*x), x)
```

3.41.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.36

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \frac{\sqrt{2}\left(40\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-15\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="maxima")
```

output
```
1/32*sqrt(2)*(40*sqrt(2)*(d*x + c)^(3/2)*b^2*d*cos(((d*x + c)*b - b*c + a*
d)/d) - 15*(-(I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I
 - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(I*b/d)) - 15*((I - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d
) - (I + 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-I*b/d)) + 4*(4*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d
*x + c)*b*d^2)*sin(((d*x + c)*b - b*c + a*d)/d))/b^4
```

3.41.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 1246, normalized size of antiderivative = 6.42

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a),x, algorithm="giac")
```

output
```
-1/16*(8*(I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)))*c^3 + 6*c*d^2*((I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4
*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(
d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2
)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(
d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d
)/b^2)/d^2) + d^3*((-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b
*c*d^2 - 15*I*d^3)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^3) + 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d -
 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c
)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b^3)/d^3 + (I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^...
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\,dx = \int \cos(a+b\,x)\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*(c + d*x)^(5/2), x)
```

3.42 $\int (c+dx)^{3/2} \cos(a+bx)\, dx$

3.42.1 Optimal result

Integrand size = 16, antiderivative size = 169

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \frac{3d\sqrt{c+dx}\cos(a+bx)}{2b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{2b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{b}$$

output
```
(d*x+c)^(3/2)*sin(b*x+a)/b-3/4*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/4*d^(3/2)*Fr
esnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2
)*Pi^(1/2)/b^(5/2)+3/2*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2
```

3.42.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.72

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \frac{de^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} + \frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x],x]
```

output
```
(d*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[5/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I
)*b*(c + d*x))/d] + (E^(((2*I)*b*c)/d)*Gamma[5/2, (I*b*(c + d*x))/d])/Sqrt
[(I*b*(c + d*x))/d]))/(2*b^2*E^((I*(b*c + a*d))/d))
```

3.42.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{3/2}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{3d\int -\sqrt{c+dx}\sin(a+bx)dx}{2b}+\frac{(c+dx)^{3/2}\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\int \sqrt{c+dx}\sin(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\int \sqrt{c+dx}\sin(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\int \frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}$$

$\downarrow$ 3787

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}$$

$\downarrow$ 3785

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}$$

$\downarrow$ 3786

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}$$

$\downarrow$ 3832

3.42. $\int(c+dx)^{3/2}\cos(a+bx)\,dx$

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}$$

$\downarrow$ 3833

$$\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x],x]
```

output
```
(-3*d*(-((Sqrt[c + d*x]*Cos[a + b*x])/b) + (d*((Sqrt[2*Pi]*Cos[a - (b*c)/d
]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])
- (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
 (b*c)/d])/(Sqrt[b]*Sqrt[d])))/(2*b)))/(2*b) + ((c + d*x)^(3/2)*Sin[a + b*
x])/b
```

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.42.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{b}}{d}$
default	$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{b}}{d}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/2/b*d*(d*x+c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*
(d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx =$$

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\,(3}{4\,b^3}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresne
l_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*(3*b*d
*cos(b*x + a) + 2*(b^2*d*x + b^2*c)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

3.42.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*cos(a + b*x), x)
```

3.42.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx = \frac{\sqrt{2}\left(8\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+12\sqrt{2}\sqrt{dx+c}bd\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-3\left(-(i-1)\sqrt{\cdots}\right.\right.}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*(8*sqrt(2)*(d*x + c)^(3/2)*b^2*sin(((d*x + c)*b - b*c + a*d)/
d) + 12*sqrt(2)*sqrt(d*x + c)*b*d*cos(((d*x + c)*b - b*c + a*d)/d) - 3*(-(
I - 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)
*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) -
 3*((I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqr
t(pi)*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)))/b^3
```

3.42.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 779, normalized size of antiderivative = 4.61

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx =$$

$$-\frac{4\left(\frac{i\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{-i\,bc+i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+d^2\left(\cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(4*(I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*
c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x +
 c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqr
t(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x +
 c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2
)/d^2) + 4*(-I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*er
f(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*I*sqrt(d*x
+ c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 2*I*sqrt(d*x + c)*d*e^((-
I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)*c)/d
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*(c + d*x)^(3/2), x)
```

3.43 $\int \sqrt{c+dx}\cos(a+bx)\,dx$

3.43.1 Optimal result

Integrand size = 16, antiderivative size = 142

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{b}$$

output
```
-1/2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+sin(b*x
+a)*(d*x+c)^(1/2)/b
```

3.43.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b^2\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x],x]
```

output
```
(d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d
] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d
]))/(2*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

3.43.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{d\int -\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}+\frac{\sqrt{c+dx}\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}$$

$$\Big\downarrow \text{3787}$$

$$\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}$$

$$\Big\downarrow \text{3785}$$

$$\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}$$

$$\Big\downarrow \text{3786}$$

$$\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x],x]
```

output
```
-1/2*(d*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/b + (Sq
rt[c + d*x]*Sin[a + b*x])/b
```

3.43. $\int\sqrt{c+dx}\cos(a+bx)\,dx$

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.43.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2b\sqrt{\frac{b}{d}}}}{d}$	144
default	$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2b\sqrt{\frac{b}{d}}}}{d}$	144

input
```
int((d*x+c)^(1/2)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/2/b*d*(d*x+c)^(1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi
^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d)))
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.89

$$\int \sqrt{c+dx}\cos(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)-2\sqrt{dx}}{2b^2}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 2*sqrt(d*x + c)*
b*sin(b*x + a))/b^2
```

3.43.6 Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = \int \sqrt{c+dx}\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*cos(a + b*x), x)
```

3.43.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.38

$$\int \sqrt{c+dx}\cos(a+bx)\,dx$$
$$= \frac{\sqrt{2}\left(4\sqrt{2}\sqrt{dx+c}b\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left(-(i+1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i-1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*(4*sqrt(2)*sqrt(d*x + c)*b*sin(((d*x + c)*b - b*c + a*d)/d) +
(-(I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi
)*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) +
((I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(pi)
*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)))/b
^2
```

3.43.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.00

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = -\frac{i\sqrt{2}\sqrt{\pi}(2bc+id)d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} + \frac{i\sqrt{2}\sqrt{\pi}(2bc-id)d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b} -$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(-I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2
*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*(I*sqrt(2)*sqrt(p
i)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2
)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)))*c
 + 2*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 2*I*sqrt(
d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\,dx = \int \cos(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*(c + d*x)^(1/2), x)
```

3.44 $\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx$

3.44.1 Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}} - \frac{\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1
/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

input
```
Integrate[Cos[a + b*x]/Sqrt[c + d*x],x]
```

output
```
((I/2)*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c + d
*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, (I*b*(c +
d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x])
```

3.44.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3787}$$

$$\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3042}$$

$$\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3785}$$

$$\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3786}$$

$$\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int \sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

$$\Big\downarrow \; 3833$$

$$\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[Cos[a + b*x]/Sqrt[c + d*x],x]
```

output
```
(Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/S
qrt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])
```

3.44.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.44.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$	100
default	$\frac{\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}$	100

input
```
int(cos(b*x+a)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/
2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/
2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{bc-ad}{d}\right)}{b}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*
x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d))/b
```

3.44.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(cos(a + b*x)/sqrt(c + d*x), x)
```

3.44.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.35

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\sqrt{2}\left(\left((i-1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{bc-ad}{d}\right)+(i+1)\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{ib}{d}}\right)+\left(-(i+\right.\right.}{4b}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*(((I - 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I +
 1)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I
*b/d)) + (-(I + 1)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*
sqrt(pi)*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/
d)))/b
```

3.44.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.42

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\frac{i\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{ibc-iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{2d}\right)e^{\left(\frac{-ibc+iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}}{2d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)))/d
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(1/2), x)
```

3.45 $\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx$

3.45.1 Optimal result

Integrand size = 16, antiderivative size = 139

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cos(a+bx)}{d\sqrt{c+dx}} - \frac{2\sqrt{b}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}}$$

output
```
-2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b
^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b*x+a
)/d/(d*x+c)^(1/2)
```

3.45.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-ia}\left(e^{2ia-\frac{ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+e^{-ibx}\left(-1-e^{2i(a+bx)}+e^{\frac{ib(c+dx)}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2}\right.\right.\right.}{d\sqrt{c+dx}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(E^((2*I)*a - (I*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-I)*b*(c
+ d*x))/d] + (-1 - E^((2*I)*(a + b*x)) + E^((I*b*(c + d*x))/d)*Sqrt[(I*b*(
c + d*x))/d]*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(d*E^(I*a)*Sqrt[c +
 d*x])
```

3.45.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 3778, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{2b\int -\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2b\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2b\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3787}$$

$$-\frac{2b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{2b\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(-2*Cos[a + b*x])/(d*Sqrt[c + d*x]) - (2*b*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (
Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b
*c)/d])/(Sqrt[b]*Sqrt[d])))/d
```

3.45. $\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx$

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.45. $\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx$

3.45.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	140
default	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	140

input
```
int(cos(b*x+a)/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/(d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d)))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx =$$

$$-\frac{2\left(\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(\right.\right.}{d^2x+cd}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin
(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(p
i*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d
) + sqrt(d*x + c)*cos(b*x + a))/(d^2*x + c*d)
```

3.45.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(3/2),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**(3/2), x)
```

3.45.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\Big(\Big(-(i+1)\sqrt{2}\Gamma\Big(-\frac{1}{2},\frac{i(dx+c)b}{d}\Big)+(i-1)\sqrt{2}\Gamma\Big(-\frac{1}{2},-\frac{i(dx+c)b}{d}\Big)\Big)\cos\Big(-\frac{bc-ad}{d}\Big)+\Big((i-}{4\sqrt{dx+c}}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/4*((-(I + 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamm
a(-1/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-1
/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(
-(b*c - a*d)/d))*sqrt((d*x + c)*b/d)/(sqrt(d*x + c)*d)
```

3.45.8 Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x + c)^(3/2), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{3/2}} \,d x$$

input
```
int(cos(a + b*x)/(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(3/2), x)
```

3.46 $\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx$

3.46.1 Optimal result

Integrand size = 16, antiderivative size = 168

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx=-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}-\frac{4b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{4b^{3/2}\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{3d^{5/2}}+\frac{4b\sin(a+bx)}{3d^2\sqrt{c+dx}}$$

output
```
-2/3*cos(b*x+a)/d/(d*x+c)^(3/2)-4/3*b^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+4/3*b^(3/
2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2
^(1/2)*Pi^(1/2)/d^(5/2)+4/3*b*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

3.46.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.13

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx=\frac{e^{-ia}\left(-2ie^{2ia-\frac{ibc}{d}}\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)\right)+\cdots\right.}{6d^2(c+dx)^{3/2}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
((-2*I)*E^((2*I)*a - (I*b*c)/d)*(E^((I*b*(c + d*x))/d)*((-I)*d + 2*b*(c +
d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-I)*b*(c + d*x))
/d]) + (-2*d + (4*I)*b*(c + d*x) - 4*d*E^((I*b*(c + d*x))/d)*((I*b*(c + d*
x))/d)^(3/2)*Gamma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(6*d^2*E^(I*a)*(c +
 d*x)^(3/2))
```

3.46.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3778, 25, 3042, 3778, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \; 3778$$

$$\frac{2b\int -\frac{\sin(a+bx)}{(c+dx)^{3/2}}dx}{3d} - \frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2b\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}dx}{3d} - \frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\int \frac{\sin(a+bx)}{(c+dx)^{3/2}}dx}{3d} - \frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \; 3778$$

$$-\frac{2b\left(\frac{2b\int \frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{d} - \frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d} - \frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\left(\frac{2b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{2b\left(\frac{2b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\left(\frac{2b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{2b\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{2b\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int \sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{2b\left(\frac{2b\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

3.46. $\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx$

$$\Big\downarrow \text{3833}$$

$$-\frac{2b\left(\frac{2b\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\sin(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\cos(a+bx)}{3d(c+dx)^{3/2}}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(-2*Cos[a + b*x])/(3*d*(c + d*x)^(3/2)) - (2*b*((2*b*((Sqrt[2*Pi]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sq
rt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/d - (2*Sin[a + b*x])/(d*Sqrt[c + d*x
])))/(3*d)
```

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.46.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}\right)}{3d}}{d}$
default	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}+\frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}\right)}{3d}}{d}$

input
```
int(cos(b*x+a)/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/3/(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-2/3*b/d*(-1/(d*x+c)^(
1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.24

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{2\left(2\sqrt{2}(\pi bd^2x^2+2\pi bcdx+\pi bc^2)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-2\sqrt{2}(\pi bd^2x^2+2\pi bcdx+\pi b}{3\left(d^4x^2+2\,cd^3x\right.}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(2*sqrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 2*sq
rt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*
(d*cos(b*x + a) - 2*(b*d*x + b*c)*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^
2*d^2)
```

3.46.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(5/2),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**(5/2), x)
```

3.46.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.77

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{i(dx+c)b}{d}\right.\right.}{4\,(dx+c)^{\frac{3}{2}}d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/4*(((I - 1)*sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamm
a(-3/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I + 1)*sqrt(2)*gamma(-3
/2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(
-(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2)/((d*x + c)^(3/2)*d)
```

3.46.8 Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x + c)^(5/2), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(5/2), x)
```

3.47 $\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx$

3.47.1 Optimal result

Integrand size = 16, antiderivative size = 193

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}+\frac{8b^2\cos(a+bx)}{15d^3\sqrt{c+dx}} + \frac{8b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{15d^{7/2}} + \frac{4b\sin(a+bx)}{15d^2(c+dx)^{3/2}}$$

output
```
-2/5*cos(b*x+a)/d/(d*x+c)^(5/2)+4/15*b*sin(b*x+a)/d^2/(d*x+c)^(3/2)+8/15*b
^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^2*cos
(b*x+a)/d^3/(d*x+c)^(1/2)
```

3.47.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.18

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{e^{-ia}\left(2e^{2ia}\left(-3d^2e^{ibx}+2be^{-\frac{ibc}{d}}(c+dx)\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right)^3\right.\right.\right.}{}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
(2*E^((2*I)*a)*(-3*d^2*E^(I*b*x) + (2*b*(c + d*x)*(E^((I*b*(c + d*x))/d)*(
(-I)*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2,
((-I)*b*(c + d*x))/d]))/E^((I*b*c)/d)) + (-6*d^2 + (4*I)*b*d*(c + d*x) + 8
*b^2*(c + d*x)^2 + 8*d^2*E^((I*b*(c + d*x))/d)*((I*b*(c + d*x))/d)^(5/2)*G
amma[1/2, (I*b*(c + d*x))/d])/E^(I*b*x))/(30*d^3*E^(I*a)*(c + d*x)^(5/2))
```

3.47.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {3042, 3778, 25, 3042, 3778, 3042, 3778, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{2b\int -\frac{\sin(a+bx)}{(c+dx)^{5/2}}dx}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}dx}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\int \frac{\sin(a+bx)}{(c+dx)^{5/2}}dx}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2b\left(\frac{2b\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\left(\frac{2b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2b\left(\frac{2b\left(\frac{2b\int -\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b\left(\frac{2b\left(-\frac{2b\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\left(\frac{2b\left(-\frac{2b\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3787}$$

3.47. $\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx$

$$-\frac{2b\left(\frac{2b\left(-\frac{2b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\left(\frac{2b\left(-\frac{2b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{2b\left(\frac{2b\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3786}$$

3.47. $\int\frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx$

$$-\frac{2b\left(\frac{2b\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{2b\left(\frac{2b\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3833}$$

3.47. $\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx$

$$-\frac{2b\left(\frac{2b\left(-\frac{2b\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2\sin(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}-\frac{2\cos(a+bx)}{5d(c+dx)^{5/2}}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
(-2*Cos[a + b*x])/(5*d*(c + d*x)^(5/2)) - (2*b*((2*b*((-2*Cos[a + b*x])/(d
*Sqrt[c + d*x]) - (2*b*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[
(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqr
t[d])))/d))/(3*d) - (2*Sin[a + b*x])/(3*d*(c + d*x)^(3/2))))/(5*d)
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.47.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\cdots\right.\right.}{d\sqrt{\cdots}}\right.}{3d}\right.}{5d}}{d}$
default	$\frac{-\frac{2\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\cdots\right.\right.}{d\sqrt{\cdots}}\right.}{3d}\right.}{5d}}{d}$

input
```
int(cos(b*x+a)/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/5/(d*x+c)^(5/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-2/5*b/d*(-1/3/(d*x+c)
^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(1/2)*cos(b*(d*x+c
)/d+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.53

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(4\sqrt{2}(\pi b^2d^3x^3+3\pi b^2cd^2x^2+3\pi b^2c^2dx+\pi b^2c^3)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x +
pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*
x + c)*sqrt(b/(pi*d))) + 4*sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 +
3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(d*x + c)*((4*b^2*d^2*x^2
+ 8*b^2*c*d*x + 4*b^2*c^2 - 3*d^2)*cos(b*x + a) + 2*(b*d^2*x + b*c*d)*sin(
b*x + a)))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
```

3.47.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(7/2),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**(7/2), x)
```

3.47.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.67

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{i(dx+c}{d}\right.\right.}{4(dx+c)^{\frac{5}{2}}d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/4*((-(I + 1)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gam
ma(-5/2, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((I - 1)*sqrt(2)*gamma(-
5/2, I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin
(-(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2)/((d*x + c)^(5/2)*d)
```

3.47.8 Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x + c)^(7/2), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^(7/2),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(7/2), x)
```

3.48 $\int (c+dx)^{5/2}\cos^2(a+bx)\,dx$

3.48.1 Optimal result

Integrand size = 18, antiderivative size = 231

$$\begin{aligned}\int (c+dx)^{5/2}\cos^2(a+bx)\,dx &= -\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}\\&+\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}+\frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}}\\&+\frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{7/2}}\\&+\frac{(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)}{2b}-\frac{15d^2\sqrt{c+dx}\sin(2a+2bx)}{64b^3}\end{aligned}$$

output
```
-5/16*d*(d*x+c)^(3/2)/b^2+1/7*(d*x+c)^(7/2)/d+5/8*d*(d*x+c)^(3/2)*cos(b*x+
a)^2/b^2+1/2*(d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)/b+15/128*d^(5/2)*cos(2*a-
2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/
2)+15/128*d^(5/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2
*a-2*b*c/d)*Pi^(1/2)/b^(7/2)-15/64*d^2*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^3
```

3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.65

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \frac{64(c+dx)^4-\frac{7\sqrt{2}d^4e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)}{b^4}-\frac{7\sqrt{2}d^4e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(c+dx)}{d}\right)}{b^4}}{448d\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2,x]
```

output
```
(64*(c + d*x)^4 - (7*Sqrt[2]*d^4*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c +
 d*x))/d]*Gamma[7/2, ((-2*I)*b*(c + d*x))/d])/b^4 - (7*Sqrt[2]*d^4*Sqrt[(I
*b*(c + d*x))/d]*Gamma[7/2, ((2*I)*b*(c + d*x))/d])/(b^4*E^((2*I)*(a - (b*
c)/d))))/(448*d*Sqrt[c + d*x])
```

3.48.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3792, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^{5/2}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \; 3792$$

$$-\frac{15d^2\int \sqrt{c+dx}\cos^2(a+bx)dx}{16b^2}+\frac{1}{2}\int (c+dx)^{5/2}dx+\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \; 17$$

$$-\frac{15d^2\int\sqrt{c+dx}\cos^2(a+bx)dx}{16b^2}+\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow 3042$$

$$-\frac{15d^2\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{16b^2}+\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow 3793$$

$$-\frac{15d^2\int\left(\frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)+\frac{1}{2}\sqrt{c+dx}\right)dx}{16b^2}+\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow 2009$$

$$\frac{5d(c+dx)^{3/2}\cos^2(a+bx)}{8b^2}-\frac{15d^2\left(-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}\right)}{16b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2,x]
```

output
```
(c + d*x)^(7/2)/(7*d) + (5*d*(c + d*x)^(3/2)*Cos[a + b*x]^2)/(8*b^2) + ((c
 + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (15*d^2*((c + d*x)^(3/2)/
(3*d) - (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c
+ d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelC[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(8*b^(3/2)
) + (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)))/(16*b^2)
```

3.48. $\int(c+dx)^{5/2}\cos^2(a+bx)\,dx$

3.48.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.48.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{7}+\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad}{}\right.}{4b}\right.}{}\right.}{}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{7}{2}}}{7}+\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad}{}\right.}{4b}\right.}{}\right.}{}}{d}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/14*(d*x+c)^(7/2)+1/8/b*d*(d*x+c)^(5/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c
)/d)-5/8/b*d*(-1/4/b*d*(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+3/4/
b*d*(1/4/b*d*(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/
2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+
c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(
1/2)/d))))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.12

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \frac{105\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\,\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-a}{d}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/896*(105*pi*d^4*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) + 105*pi*d^4*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(32*b^4*d^3*x^3 + 96*b^4*
c*d^2*x^2 + 32*b^4*c^3 - 70*b^2*c*d^2 + 140*(b^2*d^3*x + b^2*c*d^2)*cos(b*
x + a)^2 + 7*(16*b^3*d^3*x^2 + 32*b^3*c*d^2*x + 16*b^3*c^2*d - 15*b*d^3)*c
os(b*x + a)*sin(b*x + a) + 2*(48*b^4*c^2*d - 35*b^2*d^3)*x)*sqrt(d*x + c))
/(b^4*d)
```

3.48.6 Sympy [F]

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx=\int (c+dx)^{\frac{5}{2}}\cos^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(5/2)*cos(a + b*x)**2, x)
```

3.48.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.28

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx=\frac{\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d}+1120\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-105\left(-(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/7168*sqrt(2)*(512*sqrt(2)*(d*x + c)^(7/2)*b^4/d + 1120*sqrt(2)*(d*x + c)
^(3/2)*b^2*d*cos(2*((d*x + c)*b - b*c + a*d)/d) - 105*(-(I + 1)*4^(1/4)*sq
rt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/
d)) - 105*((I - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)
/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) + 56*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 1
5*sqrt(2)*sqrt(d*x + c)*b*d^2)*sin(2*((d*x + c)*b - b*c + a*d)/d))/b^4
```

3.48.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 1331, normalized size of antiderivative = 5.76

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/8960*(2240*(-I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)) + I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*
sqrt(d*x + c))*c^3 - 28*c*d^2*(64*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*
c + 15*sqrt(d*x + c)*c^2)/d^2 - 15*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d -
3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(4*I*(d*
x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-
I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - 15*(I*sqrt(pi)*(16*b^2*c^2 +
8*I*b*c*d - 3*d^2)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2
) + 2*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c
)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*(256*(5*(d
*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)/d^3 - 35*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2
 + 15*I*d^3)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 2*(1
6*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x +
 c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 1...
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\,dx = \int \cos(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.49 $\int (c+dx)^{3/2}\cos^2(a+bx)\,dx$

3.49.1 Optimal result

Integrand size = 18, antiderivative size = 203

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = -\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d} + \frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2}-\frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{32b^{5/2}}+\frac{(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)}{2b}$$

output
```
1/5*(d*x+c)^(5/2)/d+1/2*(d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)/b-3/32*d^(3/2)
*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1
/2)/b^(5/2)+3/32*d^(3/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)-3/16*d*(d*x+c)^(1/2)/b^2+3/8*d*cos(b*x
+a)^2*(d*x+c)^(1/2)/b^2
```

3.49.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.74

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \frac{\sqrt{c+dx}\left(32(c+dx)^2+\frac{5\sqrt{2}d^2e^{2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)}{b^2\sqrt{-\frac{ib(c+dx)}{d}}}+\frac{5\sqrt{2}d^2e^{-2i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},\frac{2ib(c+dx)}{d}\right)}{b^2\sqrt{\frac{ib(c+dx)}{d}}}\right)}{160d}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(32*(c + d*x)^2 + (5*Sqrt[2]*d^2*E^((2*I)*(a - (b*c)/d))*Ga
mma[5/2, ((-2*I)*b*(c + d*x))/d])/(b^2*Sqrt[((-I)*b*(c + d*x))/d]) + (5*Sq
rt[2]*d^2*Gamma[5/2, ((2*I)*b*(c + d*x))/d])/(b^2*E^((2*I)*(a - (b*c)/d))*
Sqrt[(I*b*(c + d*x))/d])))/(160*d)
```

3.49.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3792, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{3/2}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{3d^2\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}+\frac{1}{2}\int (c+dx)^{3/2}dx+\frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{3d^2\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}+\frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{\sqrt{c+dx}}dx}{16b^2}+\frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3d^2\int\left(\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}+\frac{1}{2\sqrt{c+dx}}\right)dx}{16b^2}+\frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d^2\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{16b^2}+\frac{3d\sqrt{c+dx}\cos^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2,x]
```

output
```
(c + d*x)^(5/2)/(5*d) + (3*d*Sqrt[c + d*x]*Cos[a + b*x]^2)/(8*b^2) - (3*d^
2*(Sqrt[c + d*x]/d + (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sq
rt[b]*Sqrt[d])))/(16*b^2) + ((c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x])/(2
*b)
```

3.49.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.49.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{5}+\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{8b}\right.}{4b}}{d}$
default	$\frac{\frac{(dx+c)^{\frac{5}{2}}}{5}+\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{8b}\right.}{4b}}{d}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/10*(d*x+c)^(5/2)+1/8/b*d*(d*x+c)^(3/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c
)/d)-3/8/b*d*(-1/4/b*d*(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+1/8/
b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.96

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx =$$

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,(16\ldots}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/160*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c
*d*x + 16*b^3*c^2 + 30*b*d^2*cos(b*x + a)^2 - 15*b*d^2 + 40*(b^2*d^2*x + b
^2*c*d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(b^3*d)
```

3.49.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*cos(a + b*x)**2, x)
```

3.49.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.35

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{128\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d} + 160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right) + 120\sqrt{2}\sqrt{dx+c}bd\cos\left(\frac{2((dx+c)b-}{d}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/1280*sqrt(2)*(128*sqrt(2)*(d*x + c)^(5/2)*b^3/d + 160*sqrt(2)*(d*x + c)^
(3/2)*b^2*sin(2*((d*x + c)*b - b*c + a*d)/d) + 120*sqrt(2)*sqrt(d*x + c)*b
*d*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*
(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/
d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*((
I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1
)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-2*I*b/d)))/b^3
```

3.49.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 821, normalized size of antiderivative = 4.04

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/960*(240*(-I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
) + I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/
d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sq
rt(d*x + c))*c^2 - d^2*(64*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*
sqrt(d*x + c)*c^2)/d^2 - 15*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*
d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(4*I*(d*x + c)^
(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x
+ c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 - 15*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c
*d - 3*d^2)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(
-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*
e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 40*(3*I*sqrt(pi)*(4*b
*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 3*I*sqrt(
pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) -
 16*(d*x + c)^(3/2) + 48*sqrt(d*x + c)*c - 6*I*sqrt(d*x + c)*d*e^(-2*(I*(d
*x + c)*b - I*b*c + I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + ...
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.50 $\int \sqrt{c+dx}\cos^2(a+bx)\,dx$

3.50.1 Optimal result

Integrand size = 18, antiderivative size = 158

$$\begin{aligned}\int \sqrt{c+dx}\cos^2(a+bx)\,dx &= \frac{(c+dx)^{3/2}}{3d}-\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}\\&\quad-\frac{\sqrt{d}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}\\&\quad+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}\end{aligned}$$

output
```
1/3*(d*x+c)^(3/2)/d-1/8*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/
d^(1/2)/Pi^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(2*b^(1/2)*(d*x+c)^
(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(
2*b*x+2*a)*(d*x+c)^(1/2)/b
```

3.50.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx$$
$$=\frac{16(c+dx)^2+\frac{3\sqrt{2}d^2e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{b^2}+\frac{3\sqrt{2}d^2e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{b^2}}{48d\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2,x]
```

output
```
(16*(c + d*x)^2 + (3*Sqrt[2]*d^2*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c +
 d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/b^2 + (3*Sqrt[2]*d^2*Sqrt[(I
*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d])/(b^2*E^((2*I)*(a - (b*
c)/d))))/(48*d*Sqrt[c + d*x])
```

3.50.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2}\sqrt{c+dx}\cos(2a+2bx)+\frac{1}{2}\sqrt{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2,x]
```

output
```
(c + d*x)^(3/2)/(3*d) - (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2
*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) - (Sqrt[d]*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(4*b)
```

3.50.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.50.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$	150
default	$\frac{\frac{(dx+c)^{\frac{3}{2}}}{3}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$	150

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/6*(d*x+c)^(3/2)+1/8/b*d*(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)
/d)-1/16/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/
(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d
)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx =$$

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,(2}{24\,b^2d}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/24*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x +
a)*sin(b*x + a) + 2*b^2*c)*sqrt(d*x + c))/(b^2*d)
```

3.50.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx = \int \sqrt{c+dx}\cos^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*cos(a + b*x)**2, x)
```

3.50.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.45

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx$$

$$=\frac{\sqrt{2}\left(\frac{32\,\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d}+24\,\sqrt{2}\sqrt{dx+c}b\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/192*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2/d + 24*sqrt(2)*sqrt(d*x + c)
*b*sin(2*((d*x + c)*b - b*c + a*d)/d) - 3*((I + 1)*4^(1/4)*sqrt(pi)*d*(b^2
/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(
1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 3*(-(I - 1)
*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4
)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(-2*I*b/d)))/b^2
```

3.50.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx = -\frac{12\left(-\frac{i\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{i\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-4\sqrt{dx}\cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/48*(12*(-I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1))
+ I*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt
(d*x + c))*c + 3*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b) - 3*I*sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*(d*x + c)^(3/2) + 48*sqrt(d*x + c)*c -
 6*I*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*I*sqrt
(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\,dx = \int \cos(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.51 $\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$

3.51.1 Optimal result

Integrand size = 18, antiderivative size = 130

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{c+dx}}{d} + \frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

output
```
1/2*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi
^(1/2)/b^(1/2)/d^(1/2)-1/2*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(1/2)/d^(1/2)+(d*x+c)^(1/2)/d
```

3.51.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{8\left(\frac{c}{d}+x\right) - \frac{i\sqrt{2}e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{i\sqrt{2}e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2ib(c+dx)}{d}\right)}{b}}{8\sqrt{c+dx}}$$

input
```
Integrate[Cos[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
(8*(c/d + x) - (I*Sqrt[2]*E^((2*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[1/2, ((-2*I)*b*(c + d*x))/d])/b + (I*Sqrt[2]*Sqrt[(I*b*(c + d*x))
/d]*Gamma[1/2, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))))/(8*Sqr
t[c + d*x])
```

3.51.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}+\frac{1}{2\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

input
```
Int[Cos[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
Sqrt[c + d*x]/d + (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*FresnelS[(2
*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2*Sqrt[
b]*Sqrt[d])
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.51.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\sqrt{dx+c}+\frac{\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$	108
default	$\frac{\sqrt{dx+c}+\frac{\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{2\sqrt{\frac{b}{d}}}}{d}$	108

input
```
int(cos(b*x+a)^2/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/2*(d*x+c)^(1/2)+1/4*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*Fresne
lC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2
/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) + 2\sqrt{dx+c}b}{2\,bd}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-2*(b*c - a*d)/d) + 2*sqrt(d*x + c)*b)/(b*d)
```

3.51.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos^2{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*x+c)**(1/2),x)
```

output
```
Integral(cos(a + b*x)**2/sqrt(c + d*x), x)
```

3.51.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.44

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx =$$
$$-\frac{\sqrt{2}\left(\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right) + (i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{2\,(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{2\ldots}\right.}{}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*(((I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d
)/d) + (I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)
*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*(b^2/d^2)^(1/4)*sin(-2*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) - 8*sqrt(2)*sqrt(d*x + c)
*b/d)/b
```

3.51.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{-\frac{i\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{i\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-4\sqrt{dx+c}}{4\,d}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(-I*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*s
qrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(d*x
+ c))/d
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\sqrt{c+d\,x}}\,dx$$

input
```
int(cos(a + b*x)^2/(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2/(c + d*x)^(1/2), x)
```

3.51. $\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}\,dx$

3.52 $\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx$

3.52.1 Optimal result

Integrand size = 18, antiderivative size = 135

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}-\frac{2\sqrt{b}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{d^{3/2}} - \frac{2\sqrt{b}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^{3/2}}$$

output
```
-2*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*b^(
1/2)*Pi^(1/2)/d^(3/2)-2*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))
*sin(2*a-2*b*c/d)*b^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b*x+a)^2/d/(d*x+c)^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.30

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-\frac{2i(ad+b(c+dx))}{d}}\left(\sqrt{2}e^{2i(2a+bx)}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(-\left(1+e^{2i(a+bx)}\right)^2+\sqrt{\quad}\right.\right.}{2d\sqrt{c+dx}}$$

input
```
Integrate[Cos[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[2]*E^((2*I)*(2*a + b*x))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-2*
I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*(-(1 + E^((2*I)*(a + b*x)))^2 + Sqr
t[2]*E^(((2*I)*b*(c + d*x))/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, ((2*I)*b
*(c + d*x))/d]))/(2*d*E^(((2*I)*(a*d + b*(c + d*x)))/d)*Sqrt[c + d*x])
```

3.52.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3794, 27, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3794}$$

$$\frac{4b\int -\frac{\sin(2a+2bx)}{2\sqrt{c+dx}}dx}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \; 3785$$

$$-\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \; 3786$$

$$-\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \; 3832$$

$$-\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \; 3833$$

$$-\frac{2b\left(\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}$$

input
```
Int[Cos[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
(-2*Cos[a + b*x]^2)/(d*Sqrt[c + d*x]) - (2*b*((Sqrt[Pi]*Cos[2*a - (2*b*c)/
d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[d
]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[
2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/d
```

3.52. $\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.52.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{-\frac{1}{\sqrt{dx+c}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	146
default	$\frac{-\frac{1}{\sqrt{dx+c}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{S}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{d\sqrt{\frac{b}{d}}}}{d}$	146

input
```
int(cos(b*x+a)^2/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/2/(d*x+c)^(1/2)-1/2/(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)
-b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx =$$
$$-\frac{2\left((\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-}{d}\right.\right.}{d^2x+cd}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-2*((pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqr
t(d*x + c)*sqrt(b/(pi*d))) + (pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(2*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + sqrt(d*x + c)*cos(b*
x + a)^2)/(d^2*x + c*d)
```

3.52.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*x+c)**(3/2),x)
```

output
```
Integral(cos(a + b*x)**2/(c + d*x)**(3/2), x)
```

3.52.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{2}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{2i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)}{8}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(sqrt(2)*((-(I + 1)*sqrt(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) + (I - 1)*s
qrt(2)*gamma(-1/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + ((I - 1)*s
qrt(2)*gamma(-1/2, 2*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -2*I*(
d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) - 8)/(sqrt(d*x +
 c)*d)
```

3.52.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos(bx+a)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/(d*x + c)^(3/2), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^2/(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2/(c + d*x)^(3/2), x)
```

3.53 $\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx$

3.53.1 Optimal result

Integrand size = 18, antiderivative size = 170

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx=-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{8b^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{3d^{5/2}}+\frac{8b^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^{5/2}}+\frac{8b\cos(a+bx)\sin(a+bx)}{3d^2\sqrt{c+dx}}$$

output
```
-2/3*cos(b*x+a)^2/d/(d*x+c)^(3/2)-8/3*b^(3/2)*cos(2*a-2*b*c/d)*FresnelC(2*
b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/d^(5/2)+8/3*b^(3/2)*Fresn
elS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^
(5/2)+8/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

3.53.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx=\frac{e^{-\frac{2i(bc+ad)}{d}}\left(-2\sqrt{2}de^{4ia}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2ib(c+dx)}{d}\right)-2\sqrt{2}de^{\frac{4ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{2}{}\right.\right.}{3d^2(c+dx)^{3/2}}$$

input
```
Integrate[Cos[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(-2*Sqrt[2]*d*E^((4*I)*a)*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*
b*(c + d*x))/d] - 2*Sqrt[2]*d*E^(((4*I)*b*c)/d)*((I*b*(c + d*x))/d)^(3/2)*
Gamma[1/2, ((2*I)*b*(c + d*x))/d] + 2*E^(((2*I)*(b*c + a*d))/d)*(-(d*Cos[a
 + b*x]^2) + 2*b*(c + d*x)*Sin[2*(a + b*x)]))/(3*d^2*E^(((2*I)*(b*c + a*d)
)/d)*(c + d*x)^(3/2))
```

3.53.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3795, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3795}$$

$$-\frac{16b^2\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{16b^2\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{16b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{16b^2\int \left(\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}+\frac{1}{2\sqrt{c+dx}}\right)dx}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{16b^2\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}$$

input
```
Int[Cos[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(16*b^2*Sqrt[c + d*x])/(3*d^3) - (2*Cos[a + b*x]^2)/(3*d*(c + d*x)^(3/2))
- (16*b^2*(Sqrt[c + d*x]/d + (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*
FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d
])/(2*Sqrt[b]*Sqrt[d])))/(3*d^2) + (8*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*
Sqrt[c + d*x])
```

3.53.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.53.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{1}{3(dx+c)^{\frac{3}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2b\cdots}{d}\right)\cdots\right)}{d\sqrt{\frac{b}{d}}}\cdots\right)}{3d}}{d}$
default	$\frac{-\frac{1}{3(dx+c)^{\frac{3}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}+\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad-2bc}{d}\right)\mathrm{C}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2b\cdots}{d}\right)\cdots\right)}{d\sqrt{\frac{b}{d}}}\cdots\right)}{3d}}{d}$

input
```
int(cos(b*x+a)^2/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/6/(d*x+c)^(3/2)-1/6/(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)
-2/3*b/d*(-1/(d*x+c)^(1/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)
/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.21

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx =$$
$$-\frac{2\left(4\left(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-4\left(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2\right)\sqrt{\cdots}\cdots\right)}{3\left(d^4x^2+2\cdots\right)}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(4*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-2*(b*
c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 4*(pi*b*d^2*x^2
+ 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + (d*cos(b*x + a)^2 - 4*(b*d*x + b*c)*co
s(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

3.53.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*x+c)**(5/2),x)
```

output
```
Integral(cos(a + b*x)**2/(c + d*x)**(5/2), x)
```

3.53.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{3\sqrt{2}\left(\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{3}{2}, \frac{2i(dx+c)b}{d}\right) + (i+1)\sqrt{2}\Gamma\left(-\frac{3}{2}, -\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right.}{}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/12*(3*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I + 1
)*sqrt(2)*gamma(-3/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I +
1)*sqrt(2)*gamma(-3/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-3/2, -2
*I*(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) - 4)/((d*x
 + c)^(3/2)*d)
```

3.53.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos(bx+a)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/(d*x + c)^(5/2), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int(cos(a + b*x)^2/(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2/(c + d*x)^(5/2), x)
```

3.54 $\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx$

3.54.1 Optimal result

Integrand size = 18, antiderivative size = 216

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx &= -\frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}} \\ &\quad + \frac{32b^2\cos^2(a+bx)}{15d^3\sqrt{c+dx}} + \frac{32b^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{15d^{7/2}} \\ &\quad + \frac{32b^{5/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{15d^{7/2}} + \frac{8b\cos(a+bx)\sin(a+bx)}{15d^2(c+dx)^{3/2}}\end{aligned}$$

output
```
-2/5*cos(b*x+a)^2/d/(d*x+c)^(5/2)+8/15*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)
^(3/2)+32/15*b^(5/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/Pi^(1/2))*Pi^(1/2)/d^(7/2)+32/15*b^(5/2)*FresnelC(2*b^(1/2)*(d*x+c)^(
1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(7/2)-16/15*b^2/d^3/(d*
x+c)^(1/2)+32/15*b^2*cos(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

3.54.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.10

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{-6d^2 + e^{2ia}\left(-3d^2e^{2ibx} + 4be^{-\frac{2ibc}{d}}(c+dx)\left(e^{\frac{2ib(c+dx)}{d}}(-id+4b(c+dx)) - 4i\sqrt{2}d\left(-\frac{ib}{}\right.\right.\right.}{}$$

input
```
Integrate[Cos[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(-6*d^2 + E^((2*I)*a)*(-3*d^2*E^((2*I)*b*x) + (4*b*(c + d*x)*(E^(((2*I)*b*
(c + d*x))/d)*((-I)*d + 4*b*(c + d*x)) - (4*I)*Sqrt[2]*d*(((-I)*b*(c + d*x
))/d)^(3/2)*Gamma[1/2, ((-2*I)*b*(c + d*x))/d]))/E^(((2*I)*b*c)/d)) + (-3*
d^2 - (2*I)*b*(c + d*x)*(-2*d + (8*I)*b*(c + d*x) - 8*Sqrt[2]*d*E^(((2*I)*
b*(c + d*x))/d)*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d
]))/E^((2*I)*(a + b*x)))/(30*d^3*(c + d*x)^(5/2))
```

3.54.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3042, 3795, 17, 3042, 3794, 27, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3795}$$

$$-\frac{16b^2\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2} + \frac{8b^2\int \frac{1}{(c+dx)^{3/2}}dx}{15d^2} + \frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{16b^2\int \frac{\cos^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{16b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^{3/2}}dx}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3794}$$

$$-\frac{16b^2\left(\frac{4b\int -\frac{\sin(2a+2bx)}{2\sqrt{c+dx}}dx}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{16b^2\left(-\frac{2b\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{16b^2\left(-\frac{2b\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3787}$$

$$-\frac{16b^2\left(-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

3.54. $\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx$

$$-\frac{16b^2\left(-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{16b^2\left(-\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{16b^2\left(-\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{16b^2\left(-\frac{2b\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{16b^2\left(-\frac{2b\left(\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cos^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{16b^2}{15d^3\sqrt{c+dx}}$$

3.54. $\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx$

input
```
Int[Cos[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(-16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cos[a + b*x]^2)/(5*d*(c + d*x)^(5/2)
) - (16*b^2*((-2*Cos[a + b*x]^2)/(d*Sqrt[c + d*x]) - (2*b*((Sqrt[Pi]*Cos[2
*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(S
qrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sq
rt[Pi])]*Sin[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/d))/(15*d^2) + (8*b*Cos
[a + b*x]*Sin[a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

3.54.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.54.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{1}{5(dx+c)^{\frac{5}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{5d}}{d}$
default	$\frac{-\frac{1}{5(dx+c)^{\frac{5}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{\sqrt{dx+c}}-\frac{2b\sqrt{\pi}\left(\cos\left(\frac{2ad}{\cdots}\right.\right.}{\cdots}\right.}{\cdots}\right.}{5d}}{d}$

input
```
int(cos(b*x+a)^2/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/10/(d*x+c)^(5/2)-1/10/(d*x+c)^(5/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/
d)-2/5*b/d*(-1/3/(d*x+c)^(3/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+4/3*b/d*(-
1/(d*x+c)^(1/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-2*b/d*Pi^(1/2)/(b/d)^(1/2
)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+s
in(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.50

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(16\left(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*
c^3)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt
(b/(pi*d))) + 16*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x +
 pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*si
n(-2*(b*c - a*d)/d) - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - (16*b^2*
d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 3*d^2)*cos(b*x + a)^2 - 4*(b*d^2*x +
 b*c*d)*cos(b*x + a)*sin(b*x + a))*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2 +
 3*c^2*d^4*x + c^3*d^3)
```

3.54.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos^2{(a+bx)}}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)**2/(d*x+c)**(7/2),x)
```

output
```
Integral(cos(a + b*x)**2/(c + d*x)**(7/2), x)
```

3.54.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.63

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{5\sqrt{2}\left(\left((i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{2i(dx+c)b}{d}\right)-(i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)\right.}{}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/10*(5*sqrt(2)*(((I + 1)*sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) - (I - 1)
*sqrt(2)*gamma(-5/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I - 1
)*sqrt(2)*gamma(-5/2, 2*I*(d*x + c)*b/d) + (I + 1)*sqrt(2)*gamma(-5/2, -2*
I*(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) - 2)/((d*x
+ c)^(5/2)*d)
```

3.54.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos(bx+a)^2}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/(d*x + c)^(7/2), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos(a+b\,x)^2}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(cos(a + b*x)^2/(c + d*x)^(7/2),x)
```

output
```
int(cos(a + b*x)^2/(c + d*x)^(7/2), x)
```

3.55 $\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx$

3.55.1 Optimal result

Integrand size = 18, antiderivative size = 247

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx &= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}} \\ &+ \frac{32b^2\cos^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{128b^{7/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{105d^{9/2}} \\ &- \frac{128b^{7/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{105d^{9/2}} \\ &+ \frac{8b\cos(a+bx)\sin(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cos(a+bx)\sin(a+bx)}{105d^4\sqrt{c+dx}}\end{aligned}$$

output
```
-16/105*b^2/d^3/(d*x+c)^(3/2)-2/7*cos(b*x+a)^2/d/(d*x+c)^(7/2)+32/105*b^2*
cos(b*x+a)^2/d^3/(d*x+c)^(3/2)+8/35*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^(5
/2)+128/105*b^(7/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*Pi^(1/2)/d^(9/2)-128/105*b^(7/2)*FresnelS(2*b^(1/2)*(d*x+c)^
(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/d^(9/2)-128/105*b^3*cos(
b*x+a)*sin(b*x+a)/d^4/(d*x+c)^(1/2)
```

3.55.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{2\Big(-8b^2d(c+dx)^2-15d^3\cos^2(a+bx)+16b^2d(c+dx)^2\cos^2(a+bx)+16\sqrt{2}b^2de^{2i\left(a-\right.}}{}$$

input
```
Integrate[Cos[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(2*(-8*b^2*d*(c + d*x)^2 - 15*d^3*Cos[a + b*x]^2 + 16*b^2*d*(c + d*x)^2*Co
s[a + b*x]^2 + 16*Sqrt[2]*b^2*d*E^((2*I)*(a - (b*c)/d))*(c + d*x)^2*(((-I)
*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-2*I)*b*(c + d*x))/d] + (16*Sqrt[2]*b^
2*d*(c + d*x)^2*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((2*I)*b*(c + d*x))/d
])/E^((2*I)*(a - (b*c)/d)) + 6*b*d^2*(c + d*x)*Sin[2*(a + b*x)] - 32*b^3*(
c + d*x)^3*Sin[2*(a + b*x)]))/(105*d^4*(c + d*x)^(7/2))
```

3.55.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3795, 17, 3042, 3795, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \ 3795$$

$$-\frac{16b^2\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}+\frac{8b^2\int \frac{1}{(c+dx)^{5/2}}dx}{35d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \ 17$$

$$-\frac{16b^2\int \frac{\cos^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{16b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^{5/2}}dx}{35d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3795}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}\right)}{35d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \frac{\cos^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^2}{\sqrt{c+dx}}dx}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{16b^2\left(-\frac{16b^2\int \left(\frac{\cos(2a+2bx)}{2\sqrt{c+dx}}+\frac{1}{2\sqrt{c+dx}}\right)dx}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

3.55. $\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx$

$$-\frac{16b^2\left(-\frac{16b^2\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}+\frac{8b\sin(a+bx)\cos(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cos^2(}{3d(c+}\right.}{35d^2}$$

$$\frac{8b\sin(a+bx)\cos(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cos^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

input
```
Int[Cos[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(-16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cos[a + b*x]^2)/(7*d*(c + d*x)^(7
/2)) + (8*b*Cos[a + b*x]*Sin[a + b*x])/(35*d^2*(c + d*x)^(5/2)) - (16*b^2*
((16*b^2*Sqrt[c + d*x])/(3*d^3) - (2*Cos[a + b*x]^2)/(3*d*(c + d*x)^(3/2))
 - (16*b^2*(Sqrt[c + d*x]/d + (Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]
*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/
d])/(2*Sqrt[b]*Sqrt[d])))/(3*d^2) + (8*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2
*Sqrt[c + d*x])))/(35*d^2)
```

3.55.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

3.55.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{7(dx+c)^{\frac{7}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx}{d}\right.}{}\right.}{}\right.}{}\right.}{}}{d}$
default	$\frac{-\frac{1}{7(dx+c)^{\frac{7}{2}}}-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{7(dx+c)^{\frac{7}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{5(dx+c)^{\frac{5}{2}}}+\frac{4b\left(-\frac{\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}-\frac{4b\left(-\frac{\sin\left(\frac{2b(dx}{d}\right.}{}\right.}{}\right.}{}\right.}{}}{d}$

input
```
int(cos(b*x+a)^2/(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/14/(d*x+c)^(7/2)-1/14/(d*x+c)^(7/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/
d)-2/7*b/d*(-1/5/(d*x+c)^(5/2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+4/5*b/d*(-
1/3/(d*x+c)^(3/2)*cos(2*b*(d*x+c)/d+2*(a*d-b*c)/d)-4/3*b/d*(-1/(d*x+c)^(1/
2)*sin(2*b*(d*x+c)/d+2*(a*d-b*c)/d)+2*b/d*Pi^(1/2)/(b/d)^(1/2)*(cos(2*(a*d
-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c
)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(195) = 390$.

Time = 0.33 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.69

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{2\left(64\left(\pi b^3d^4x^4+4\,\pi b^3cd^3x^3+6\,\pi b^3c^2d^2x^2+4\,\pi b^3c^3dx+\pi b^3c^4\right)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\,\mathrm{C}\right.}{}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
2/105*(64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4*
pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_
cos(2*sqrt(d*x + c)*sqrt(b/(pi*d))) - 64*(pi*b^3*d^4*x^4 + 4*pi*b^3*c*d^3*
x^3 + 6*pi*b^3*c^2*d^2*x^2 + 4*pi*b^3*c^3*d*x + pi*b^3*c^4)*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - (8*b^
2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - (16*b^2*d^3*x^2 + 32*b^2*c*d^2*
x + 16*b^2*c^2*d - 15*d^3)*cos(b*x + a)^2 + 4*(16*b^3*d^3*x^3 + 48*b^3*c*d
^2*x^2 + 16*b^3*c^3 - 3*b*c*d^2 + 3*(16*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)
*sin(b*x + a))*sqrt(d*x + c))/(d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c
^3*d^5*x + c^4*d^4)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**2/(d*x+c)**(9/2),x)
```

output
```
Timed out
```

3.55.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.55

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx =$$
$$-\frac{7\sqrt{2}\left(\left(-(i-1)\sqrt{2}\Gamma\left(-\frac{7}{2},\frac{2i(dx+c)b}{d}\right)+(i+1)\sqrt{2}\Gamma\left(-\frac{7}{2},-\frac{2i(dx+c)b}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+\left(-(i+1)\sqrt{2}\Gamma\right.\right.}{7(dx+c)^{\frac{7}{2}}d}$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
-1/7*(7*sqrt(2)*((-(I - 1)*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I + 1
)*sqrt(2)*gamma(-7/2, -2*I*(d*x + c)*b/d))*cos(-2*(b*c - a*d)/d) + (-(I +
1)*sqrt(2)*gamma(-7/2, 2*I*(d*x + c)*b/d) + (I - 1)*sqrt(2)*gamma(-7/2, -2
*I*(d*x + c)*b/d))*sin(-2*(b*c - a*d)/d))*((d*x + c)*b/d)^(7/2) + 1)/((d*x
 + c)^(7/2)*d)
```

3.55.8 Giac [F]

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\cos(bx+a)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2/(d*x + c)^(9/2), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^2}{\left(c+d\,x\right)^{9/2}}\,dx$$

input
```
int(cos(a + b*x)^2/(c + d*x)^(9/2),x)
```

output
```
int(cos(a + b*x)^2/(c + d*x)^(9/2), x)
```

3.56 $\int (c+dx)^{5/2}\cos^3(a+bx)\,dx$

3.56.1 Optimal result

Integrand size = 18, antiderivative size = 410

$$\begin{aligned}\int (c+dx)^{5/2}\cos^3(a+bx)\,dx &= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{3b^2} \\ &+ \frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} + \frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\ &+ \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} \\ &+ \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} \\ &+ \frac{45d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} \\ &- \frac{45d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} + \frac{2(c+dx)^{5/2}\sin(a+bx)}{3b} \\ &+ \frac{(c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)}{3b} - \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}\end{aligned}$$

output
```
5/3*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2+5/18*d*(d*x+c)^(3/2)*cos(b*x+a)^3/b^2+2
/3*(d*x+c)^(5/2)*sin(b*x+a)/b+1/3*(d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)/b+
5/864*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1
/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7
/2)+45/32*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+45/32*d^(5/2)*FresnelC(b^(1/2)*2^(1
/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-
45/16*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3-5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1
/2)/b^3
```

3.56.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.58

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx =$$

$$-\frac{d^3 e^{-\frac{3i(bc+ad)}{d}}\left(243 e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{ib(c+dx)}{d}\right)+243 e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e^{6ia}\sqrt{\cdots}\right.\right.}{648b^4\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3,x]
```

output
```
-1/648*(d^3*(243*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamm
a[7/2, ((-I)*b*(c + d*x))/d] + 243*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(
c + d*x))/d]*Gamma[7/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqrt[((-
I)*b*(c + d*x))/d]*Gamma[7/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*
Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((3*I)*b*(c + d*x))/d])))/(b^4*E^(((3*I
)*(b*c + a*d))/d)*Sqrt[c + d*x])
```

3.56.3 Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.40, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {3042, 3792, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3793, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{5/2}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{5d^2\int \sqrt{c+dx}\cos^3(a+bx)dx}{12b^2}+\frac{2}{3}\int (c+dx)^{5/2}\cos(a+bx)dx+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5d^2\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\int (c+dx)^{5/2}\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{5d^2\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{5d\int -(c+dx)^{3/2}\sin(a+bx)dx}{2b}+\frac{(c+dx)^{5/2}\sin(a+bx)}{b}\right)+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{5d^2\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\int (c+dx)^{3/2}\sin(a+bx)dx}{2b}\right)+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5d^2\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\int (c+dx)^{3/2}\sin(a+bx)dx}{2b}\right)+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{5d^2\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\int\sqrt{c+dx}\cos(a+bx)dx}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5d^2\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{5d^2\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{d\int-\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}+\frac{\sqrt{c+dx}\sin(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

3.56. $\int (c+dx)^{5/2}\cos^3(a+bx)\,dx$

$$-\frac{5d^2\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{5d^2\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\int\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3787

$$-\frac{5d^2\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cos}{b}\right.}{2b}\right.$$

$$\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

3.56. $\int(c+dx)^{5/2}\cos^3(a+bx)\,dx$

$\downarrow$ 3042

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{-\frac{5d^2\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+dx)^{3/2}\cdots}{\cdots}\right.}{2b}\cdots\right.$$

$$\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3785

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{-\frac{5d^2\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}-\frac{(c+d\cdots}{\cdots}\right.}{2b}\cdots\right.$$

$$\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3786

3.56. $\int(c+dx)^{5/2}\cos^3(a+bx)\,dx$

$$-\frac{5d^2\int\sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{12b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right.}{2b}\right.$$

$$\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3793

$$-\frac{5d^2\int\left(\frac{3}{4}\sqrt{c+dx}\cos(a+bx)+\frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx}{12b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right.}{2b}\right.$$

$$\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 2009

3.56. $\int(c+dx)^{5/2}\cos^3(a+bx)\,dx$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right.}{2b}\right.$$

$$\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}-$$

$$\frac{5d^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\sqrt{\right.}{4b^{3/2}}\right.}{12b^2}$$

$$\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3832

3.56. $\int(c+dx)^{5/2}\cos^3(a+bx)\,dx$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b}-\frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

$$\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2}-$$

$$\frac{5d^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{}}{}\right.}{4b^{3/2}}\right.}{12b^2}$$

$$\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\downarrow \text{3833}$$

3.56. $\int(c+dx)^{5/2}\cos^3(a+bx)\,dx$

$$\frac{5d(c+dx)^{3/2}\cos^3(a+bx)}{18b^2} -$$

$$\frac{5d^2\left(-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\cdots\right.}{4b^{3/2}}\right.}{12b^2}$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sin(a+bx)}{b} - \frac{5d\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(a+bx)}{b} - \frac{d\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}} + \frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+\cdots}}{\sqrt{d}}\right.}{\sqrt{b}\sqrt{d}}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

$$\frac{(c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3,x]
```

3.56. $\int (c+dx)^{5/2}\cos^3(a+bx)\,dx$

output
```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x]^3)/(18*b^2) + ((c + d*x)^(5/2)*Cos[a + b
*x]^2*Sin[a + b*x])/(3*b) + (2*(((c + d*x)^(5/2)*Sin[a + b*x])/b - (5*d*(-
(((c + d*x)^(3/2)*Cos[a + b*x])/b) + (3*d*(-1/2*(d*((Sqrt[2*Pi]*Cos[a - (b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt
[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Si
n[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sin[a + b*x])/b))/(
2*b)))/(2*b)))/3 - (5*d^2*((-3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fresnel
S[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt
[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sq
rt[d]])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/
d])/(4*b^(3/2)) + (3*Sqrt[c + d*x]*Sin[a + b*x])/(4*b) + (Sqrt[c + d*x]*Si
n[3*a + 3*b*x])/(12*b)))/(12*b^2)
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.56.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{3d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{15d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,(}{}\right.}{}\right.}{4b}}{}$
default	$\frac{\frac{3d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{15d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,(}{}\right.}{}\right.}{4b}}{}$

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(3/8/b*d*(d*x+c)^(5/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-15/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2)
*sin(b*(d*x+c)/d+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/2
4/b*d*(d*x+c)^(5/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-5/24/b*d*(-1/6/b*d*(d
*x+c)^(3/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.90

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+1215\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 1215*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 12
15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos
(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^3 + 60*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) +
 (24*b^3*d^2*x^2 + 48*b^3*c*d*x + 24*b^3*c^2 - 100*b*d^2 + (12*b^3*d^2*x^2
 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt
(d*x + c))/b^4
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3,x)
```

output
```
Timed out
```

3.56.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.33

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+6480\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-5\left(-(i+1)\cdot 9^{\frac{1}{4}}\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) + 6480*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^
(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(3*I*b/d)) - 1215*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^
2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 1215*((I - 1)*sq
rt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)
*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(-I*b/d)) - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos
(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4
)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x +
 c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d
) + 648*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*
b - b*c + a*d)/d))*d/b^5
```

3.56.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 2480, normalized size of antiderivative = 6.05

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/1728*(72*(9*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(27*(
I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(
sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d +
4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c
- I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)
*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(2
*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-
3*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(2)*sqrt(pi)*(
4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d...
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)^3\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.57 $\int (c+dx)^{3/2}\cos^3(a+bx)\,dx$

3.57.1 Optimal result

Integrand size = 18, antiderivative size = 354

$$\begin{aligned}\int (c+dx)^{3/2}\cos^3(a+bx)\,dx &= \frac{d\sqrt{c+dx}\cos(a+bx)}{b^2} \\ &+ \frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2} - \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} \\ &- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} \\ &+ \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} \\ &+ \frac{9d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} \\ &+ \frac{2(c+dx)^{3/2}\sin(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)}{3b}\end{aligned}$$

output
```
2/3*(d*x+c)^(3/2)*sin(b*x+a)/b+1/3*(d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)/b
-1/144*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)+1/144*d^(3/2)*FresnelS(b^(1/2)*6^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(
5/2)-9/16*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+9/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+d
*cos(b*x+a)*(d*x+c)^(1/2)/b^2+1/6*d*cos(b*x+a)^3*(d*x+c)^(1/2)/b^2
```

3.57.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.71

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^{5/2}\left(81e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{ib(c+dx)}{d}\right)+81e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},\frac{ib(c+dx)}{d}\right)\right.}{216d\left(\frac{b^2(c+dx)^2}{d^2}\right)^{3/2}}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3,x]
```

output
```
((c + d*x)^(5/2)*(81*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gam
ma[5/2, ((-I)*b*(c + d*x))/d] + 81*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((-I)*
b*(c + d*x))/d]*Gamma[5/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqrt[
(I*b*(c + d*x))/d]*Gamma[5/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*
Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((3*I)*b*(c + d*x))/d])))/(216*d*E^(
((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2))
```

3.57.3 Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.42, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3042, 3792, 3042, 3777, 25, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3793, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{3/2}\sin\left(a+bx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{d^2\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int (c+dx)^{3/2}\cos(a+bx)dx+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int (c+dx)^{3/2}\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{3d\int -\sqrt{c+dx}\sin(a+bx)dx}{2b}+\frac{(c+dx)^{3/2}\sin(a+bx)}{b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\int \sqrt{c+dx}\sin(a+bx)dx}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\int \sqrt{c+dx}\sin(a+bx)dx}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

3.57. $\int (c+dx)^{3/2}\cos^3(a+bx)\,dx$

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\int\frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3787

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3785

3.57. $\int (c+dx)^{3/2}\cos^3(a+bx)\,dx$

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3786

$$-\frac{d^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int \sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3793

$$-\frac{d^2\int \left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}}+\frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int \sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right)+\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 2009

3.57. $\int (c+dx)^{3/2}\cos^3(a+bx)\,dx$

$$\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right)}{2b}\right.$$

$$\frac{d^2\left(\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right.}{12b^2}$$

$$\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3832

$$\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos(a+bx)}{b}\right.}{2b}\right.$$

$$\frac{d^2\left(\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right.}{12b^2}$$

$$\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

$\downarrow$ 3833

3.57. $\int (c+dx)^{3/2}\cos^3(a+bx)\,dx$

$$-\frac{d^2\left(\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+\cdots}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\cdots\right)}{12b^2}$$

$$\frac{d\sqrt{c+dx}\cos^3(a+bx)}{6b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sin(a+bx)}{b}-\frac{3d\left(\frac{d\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{2b}-\frac{\sqrt{c+dx}\cos\cdots}{b}\right)}{2b}\cdots\right)$$

$$\frac{(c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3,x]
```

output
```
(d*Sqrt[c + d*x]*Cos[a + b*x]^3)/(6*b^2) - (d^2*((3*Sqrt[Pi/2]*Cos[a - (b*
c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqr
t[d]) + (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(2*Sqrt[b]*Sqrt[d]
) - (3*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin
[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/(12*b^2) + ((c + d*x)^(3/2)*Cos[a + b
*x]^2*Sin[a + b*x])/(3*b) + (2*((-3*d*(-((Sqrt[c + d*x]*Cos[a + b*x])/b) +
 (d*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*
x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/(2*b)))/(2*
b) + ((c + d*x)^(3/2)*Sin[a + b*x])/b))/3
```

3.57. $\int (c+dx)^{3/2}\cos^3(a+bx)\,dx$

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.57.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{3d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{9d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-b}{d}\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$
default	$\frac{3d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{9d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-b}{d}\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}$

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(3/8/b*d*(d*x+c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-9/8/b*d*(-1/2/b*d*
(d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d))+1/24/b*d*(d*x+c)^(3/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/8/b*d*(
-1/6/b*d*(d*x+c)^(1/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx =$$

$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+81\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 81*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 81*sq
rt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x +
a)^3 + 6*b*d*cos(b*x + a) + 2*(2*b^2*d*x + 2*b^2*c + (b^2*d*x + b^2*c)*cos
(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

3.57.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*cos(a + b*x)**3, x)
```

3.57.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.40

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \frac{\left(\frac{48(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{432(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+24\sqrt{dx+c}b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d + 432*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b
^2*cos(3*((d*x + c)*b - b*c + a*d)/d) + 648*sqrt(d*x + c)*b^2*cos(((d*x +
c)*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^
(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 81*(-(I -
1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - 81*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c -
a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*
b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d
)))*d/b^4
```

3.57.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 1549, normalized size of antiderivative = 4.38

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/288*(12*(9*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(27*(I*sqrt
(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sq
rt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf
(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(2*I*(d*
x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 27*(-I*sqrt(2)*sqrt(pi)*(4*b^2*
c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(...
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)^3\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.58 $\int \sqrt{c+dx}\cos^3(a+bx)\,dx$

3.58.1 Optimal result

Integrand size = 18, antiderivative size = 304

$$\begin{aligned}\int \sqrt{c+dx}\cos^3(a+bx)\,dx = &-\frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}\\ &-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}\\ &-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}}\\ &-\frac{3\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}}\\ &+\frac{3\sqrt{c+dx}\sin(a+bx)}{4b}+\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}\end{aligned}$$

output
```
-1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/
2)-3/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-3/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+3/4*s
in(b*x+a)*(d*x+c)^(1/2)/b+1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

3.58.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.77

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx$$
$$= \frac{de^{-\frac{3i(bc+ad)}{d}}\left(27e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+27e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e^{6ia}\sqrt{-\frac{ib}{}}\right.\right.}{72b^2\sqrt{c+dx}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3,x]
```

output
```
(d*(27*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-
I)*b*(c + d*x))/d] + 27*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d
]*Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqrt[((-I)*b*(c + d
*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*Sqrt[(I*b*(
c + d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])))/(72*b^2*E^(((3*I)*(b*c +
 a*d))/d)*Sqrt[c + d*x])
```

3.58.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{3793}$$
$$\int \left(\frac{3}{4}\sqrt{c+dx}\cos(a+bx)+\frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{3\sqrt{c+dx}\sin(a+bx)}{4b}+\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3,x]
```

output
```
(-3*Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[
c + d*x])/Sqrt[d]])/(4*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(12*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sq
rt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (3*Sqrt[c
 + d*x]*Sin[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.58.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{3d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{3d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{3b(}{}\right.}{1}}{d}$
default	$\frac{\frac{3d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{4b}-\frac{3d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\sin\left(\frac{3b(}{}\right.}{1}}{d}$

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(3/8/b*d*(d*x+c)^(1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)-3/16/b*d*2^(1/2)*P
i^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))+1/24/b*d*(d*x+c)^(1/2)*sin(3*b*(d*x+c)/d+3*(a*d-b*c)
/d)-1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fre
snelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b
*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx =$$

$$-\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*
c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)
*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-(b*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 + 2*b)
*sqrt(d*x + c)*sin(b*x + a))/b^2
```

3.58.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx = \int \sqrt{c+dx}\cos^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*cos(a + b*x)**3, x)
```

3.58.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx$$
$$= \frac{\left(\frac{24\sqrt{dx+c}b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d} + \frac{216\sqrt{dx+c}b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} + \left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d + 216*sqr
t(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqr
t(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(3*I*b/d)) - 27*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*s
in(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqr
t(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-3*I*b/d)))*d/b^3
```

3.58.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.79

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/144*(-27*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf
(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 27*I*sqrt(2)
*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*(9*I*sqrt(2)*sqrt(pi)*d*erf(1
/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)
*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt
(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1))
+ I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)))*c + 54*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d
)/b - 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - ...
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.59 $\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx$

3.59.1 Optimal result

Integrand size = 18, antiderivative size = 257

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{2\sqrt{b}\sqrt{d}} - \frac{3\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{2\sqrt{b}\sqrt{d}}$$

output
```
1/12*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-1/12*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2
)+3/4*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)-3/4*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)
```

3.59.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.92

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{ie^{-\frac{3i(bc+ad)}{d}}\left(-9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right)+9e^{2ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(-e^{6ia}\sqrt{-\right.\right.}{24b\sqrt{c+dx}}$$

input
```
Integrate[Cos[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
((I/24)*(-9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[1/2
, ((-I)*b*(c + d*x))/d] + 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x
))/d]*Gamma[1/2, (I*b*(c + d*x))/d] + Sqrt[3]*(-(E^((6*I)*a)*Sqrt[((-I)*b*
(c + d*x))/d]*Gamma[1/2, ((-3*I)*b*(c + d*x))/d]) + E^(((6*I)*b*c)/d)*Sqrt
[(I*b*(c + d*x))/d]*Gamma[1/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(((3*I)*(b*c
 + a*d))/d)*Sqrt[c + d*x])
```

3.59.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}}+\frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{3\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

input
```
Int[Cos[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
(3*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])
/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC
[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[
Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(2*Sqrt[b]*Sqrt[d]) - (3*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.59.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\left(\cos\left(\frac{3ad-3bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{3}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\right.\right.}{12\sqrt{\frac{b}{d}}}}{d}$
default	$\frac{\frac{3\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4\sqrt{\frac{b}{d}}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\left(\cos\left(\frac{3ad-3bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{3}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\right.\right.}{12\sqrt{\frac{b}{d}}}}{d}$

input
```
int(cos(b*x+a)^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(3/8*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/P
i^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))+1/24*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{6}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\,\sqrt{2}\pi\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\pi}{12\,b}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(6)*pi*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*
sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*sqrt(b/(pi*d))*cos(-(b*c - a*
d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*sqr
t(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c -
a*d)/d) - sqrt(6)*pi*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-3*(b*c - a*d)/d))/b
```

3.59.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos^3{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(cos(b*x+a)**3/(d*x+c)**(1/2),x)
```

output
```
Integral(cos(a + b*x)**3/sqrt(c + d*x), x)
```

3.59.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.47

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\left(\left(\frac{(i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)}{d} + \frac{(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\left(-\frac{3(bc-ad)}{d}\right)}{d}\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{3ib}{d}}\right) - 9\left(-\frac{(i-1 \cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/48*(((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a
*d)/d)/d + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c
- a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(p
i)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (I + 1)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((
I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c
)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos
(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*
sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)))*d/b^2
```

3.59.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.29

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\frac{9i\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} - \frac{i\sqrt{6}\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(-\frac{3\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)} - 9i\sqrt{2}\sqrt{\pi}d\,\mathrm{er}}{24\,d}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/24*(9*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))/d
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cos(a+b\,x)^3}{\sqrt{c+d\,x}}\,dx$$

input
```
int(cos(a + b*x)^3/(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3/(c + d*x)^(1/2), x)
```

3.60 $\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx$

3.60.1 Optimal result

Integrand size = 18, antiderivative size = 271

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}} - \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}\\
&\quad - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}\\
&\quad - \frac{\sqrt{b}\sqrt{\frac{3\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{d^{3/2}}\\
&\quad - \frac{3\sqrt{b}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{3/2}}\end{aligned}$$

output
```
-3/2*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-3/2*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-1/2*cos
(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(
1/2)*6^(1/2)*Pi^(1/2)/d^(3/2)-1/2*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*b^(1/2)*6^(1/2)*Pi^(1/2)/d^(3/2)-2*cos(b
*x+a)^3/d/(d*x+c)^(1/2)
```

3.60.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.08

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-3ia}\left(-e^{-3ibx}-3e^{2ia-ibx}-e^{3i(2a+bx)}-3e^{i(4a+bx)}+3e^{4ia-\frac{ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right.\right.}{}$$

input
```
Integrate[Cos[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
(-E^((-3*I)*b*x) - 3*E^((2*I)*a - I*b*x) - E^((3*I)*(2*a + b*x)) - 3*E^(I*
(4*a + b*x)) + 3*E^((4*I)*a - (I*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[
1/2, ((-I)*b*(c + d*x))/d] + 3*E^(I*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/
d]*Gamma[1/2, (I*b*(c + d*x))/d] + Sqrt[3]*E^((6*I)*a - ((3*I)*b*c)/d)*Sqr
t[((-I)*b*(c + d*x))/d]*Gamma[1/2, ((-3*I)*b*(c + d*x))/d] + Sqrt[3]*E^(((
3*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[1/2, ((3*I)*b*(c + d*x))/d])/(4
*d*E^((3*I)*a)*Sqrt[c + d*x])
```

3.60.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \ 3794$$

$$\frac{6b\int\left(-\frac{\sin(a+bx)}{4\sqrt{c+dx}}-\frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}-\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \ 2009$$

$$\frac{6b\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right.}{d}$$

$$\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}}$$

input
```
Int[Cos[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
(-2*Cos[a + b*x]^3)/(d*Sqrt[c + d*x]) + (6*b*(-1/2*(Sqrt[Pi/2]*Cos[a - (b*
c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[
d]) - (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
 + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(2*Sqrt[b]*Sqrt[d])
- (Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
 (b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/d
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.60.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2\sqrt{dx+c}}-\frac{3b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2d\sqrt{\frac{b}{d}}}-\frac{\cos\left(\frac{3b(dx+c)}{d}+\frac{3ad-3b}{d}\right.}{2\sqrt{dx+c}}}{d}$
default	$\frac{-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2\sqrt{dx+c}}-\frac{3b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{2d\sqrt{\frac{b}{d}}}-\frac{\cos\left(\frac{3b(dx+c)}{d}+\frac{3ad-3b}{d}\right.}{2\sqrt{dx+c}}}{d}$

input
```
int(cos(b*x+a)^3/(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/4/(d*x+c)^(1/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-3/4*b/d*2^(1/2)*Pi^(1
/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d))-1/4/(d*x+c)^(1/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/4*b
/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/
2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx =$$
$$-\frac{\sqrt{6}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\sqrt{2}(\pi dx+\pi c)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx\right.}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(6)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel
_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqr
t(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d))) + 3*sqrt(2)*(pi*d*x + pi*c)*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + sqrt(6)*(pi*d*x + pi*c)*s
qrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*
c - a*d)/d) + 4*sqrt(d*x + c)*cos(b*x + a)^3)/(d^2*x + c*d)
```

3.60.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos^3{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)**3/(d*x+c)**(3/2),x)
```

output
```
Integral(cos(a + b*x)**3/(c + d*x)**(3/2), x)
```

3.60.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\sqrt{3}\left(\left(-(i+1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},\frac{3i\,(dx+c)b}{d}\right)+(i-1)\,\sqrt{2}\Gamma\left(-\frac{1}{2},-\frac{3i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)}{}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
1/16*(sqrt(3)*((-(I + 1)*sqrt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) + (I - 1)*
sqrt(2)*gamma(-1/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I - 1)*
sqrt(2)*gamma(-1/2, 3*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-1/2, -3*I*
(d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*sqrt((d*x + c)*b/d) - 3*(((I + 1)*s
qrt(2)*gamma(-1/2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-1/2, -I*(d*x
+ c)*b/d))*cos(-(b*c - a*d)/d) + (-(I - 1)*sqrt(2)*gamma(-1/2, I*(d*x + c)
*b/d) + (I + 1)*sqrt(2)*gamma(-1/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d)
)*sqrt((d*x + c)*b/d))/(sqrt(d*x + c)*d)
```

3.60.8 Giac [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(bx+a\right)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/(d*x + c)^(3/2), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cos\left(a+b\,x\right)^3}{\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int(cos(a + b*x)^3/(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3/(c + d*x)^(3/2), x)
```

3.61 $\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx$

3.61.1 Optimal result

Integrand size = 18, antiderivative size = 292

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx &= -\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{b^{3/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} \\ &\quad - \frac{b^{3/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} \\ &\quad + \frac{b^{3/2}\sqrt{6\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{d^{5/2}} \\ &\quad + \frac{b^{3/2}\sqrt{2\pi}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{d^{5/2}} + \frac{4b\cos^2(a+bx)\sin(a+bx)}{d^2\sqrt{c+dx}}\end{aligned}$$

output
```
-2/3*cos(b*x+a)^3/d/(d*x+c)^(3/2)-b^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+b^(3/2)*Fre
snelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)
*Pi^(1/2)/d^(5/2)-b^(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/d^(5/2)+b^(3/2)*FresnelS(b^(1/2
)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2
)/d^(5/2)+4*b*cos(b*x+a)^2*sin(b*x+a)/d^2/(d*x+c)^(1/2)
```

3.61.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.92

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{-4d\cos^3(a+bx) - 3de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{ib(c+dx)}{d}\right) - 3de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{3}}{}$$

input
```
Integrate[Cos[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
(-4*d*Cos[a + b*x]^3 - 3*d*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(3/2
)*Gamma[1/2, ((-I)*b*(c + d*x))/d] - (3*d*((I*b*(c + d*x))/d)^(3/2)*Gamma[
1/2, (I*b*(c + d*x))/d])/E^(I*(a - (b*c)/d)) - 3*Sqrt[3]*d*E^((3*I)*(a - (
b*c)/d))*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-3*I)*b*(c + d*x))/d] -
 (3*Sqrt[3]*d*((I*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((3*I)*b*(c + d*x))/d])
/E^((3*I)*(a - (b*c)/d)) + 24*b*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(6*
d^2*(c + d*x)^(3/2))
```

3.61.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.52, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules used = {3042, 3795, 3042, 3787, 3042, 3785, 3786, 3793, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{ 3795}$$

$$-\frac{12b^2\int \frac{\cos^3(a+bx)}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\int \frac{\cos(a+bx)}{\sqrt{c+dx}}dx}{d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8b^2\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d^2}-\frac{12b^2\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{12b^2\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{12b^2\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{12b^2\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{12b^2\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}+\frac{8b^2\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{12b^2\int\left(\frac{3\cos(a+bx)}{4\sqrt{c+dx}}+\frac{\cos(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d^2}+\frac{8b^2\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

3.61. $\int\frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx$

$$\frac{8b^2\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d^2}-$$

$$\frac{12b^2\left(\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+\cdots}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\cdots}{d^2}$$

$$\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$\downarrow$ 3832

$$\frac{8b^2\left(\frac{2\cos\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d^2}-$$

$$\frac{12b^2\left(\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+\cdots}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\cdots}{d^2}$$

$$\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

$\downarrow$ 3833

$$-\frac{12b^2\left(\frac{3\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{\cdots}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\cdots}{d^2}$$

$$\frac{8b^2\left(\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d^2}+$$

$$\frac{4b\sin(a+bx)\cos^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{3d(c+dx)^{3/2}}$$

input
```
Int[Cos[a + b*x]^3/(c + d*x)^(5/2),x]
```

3.61. $\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx$

output
```
(-2*Cos[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) - (12*b^2*((3*Sqrt[Pi/2]*Cos[a -
 (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]
*Sqrt[d]) + (Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*FresnelS[(Sqrt[
b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(2*Sqrt[b]*Sqr
t[d]) - (3*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]
*Sin[a - (b*c)/d])/(2*Sqrt[b]*Sqrt[d])))/d^2 + (8*b^2*((Sqrt[2*Pi]*Cos[a -
 (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*S
qrt[d]) - (Sqrt[2*Pi]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]
*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/d^2 + (4*b*Cos[a + b*x]^2*Sin[a + b
*x])/(d^2*Sqrt[c + d*x])
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.61.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2(dx+c)^{\frac{3}{2}}} - \frac{b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}} + \frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{d}$
default	$-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{2(dx+c)^{\frac{3}{2}}} - \frac{b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}} + \frac{b\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{C}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{d\sqrt{\frac{b}{d}}}\right.}{d}$

input
```
int(cos(b*x+a)^3/(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/4/(d*x+c)^(3/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(
1/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/12
/(d*x+c)^(3/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-1/2*b/d*(-1/(d*x+c)^(1/2)*
sin(3*b*(d*x+c)/d+3*(a*d-b*c)/d)+b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx =$$
$$-\frac{3\sqrt{6}(\pi bd^2x^2+2\,\pi bcdx+\pi bc^2)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}(\pi bd^2x^2+2\,\pi bcdx+\pi b}{}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*sqrt(6)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*co
s(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*
sqrt(2)*(pi*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 3*sqrt(2)*(p
i*b*d^2*x^2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 3*sqrt(6)*(pi*b*d^2*x^
2 + 2*pi*b*c*d*x + pi*b*c^2)*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 2*(d*cos(b*x + a)^3 - 6*(b*d*x
 + b*c)*cos(b*x + a)^2*sin(b*x + a))*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x +
 c^2*d^2)
```

3.61.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)**3/(d*x+c)**(5/2),x)
```

output
```
Integral(cos(a + b*x)**3/(c + d*x)**(5/2), x)
```

3.61.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.87

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx =$$
$$-\frac{3\left(\sqrt{3}\left(\left((i-1)\sqrt{2}\Gamma\left(-\frac{3}{2},\frac{3i(dx+c)b}{d}\right)-(i+1)\sqrt{2}\Gamma\left(-\frac{3}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+\left((i+1)\sqrt{2}\Gamma\left(\right.\right.\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-3/16*(sqrt(3)*(((I - 1)*sqrt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) - (I + 1)*
sqrt(2)*gamma(-3/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I + 1)*
sqrt(2)*gamma(-3/2, 3*I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-3/2, -3*I*
(d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(3/2) - ((-(I - 1)*
sqrt(2)*gamma(-3/2, I*(d*x + c)*b/d) + (I + 1)*sqrt(2)*gamma(-3/2, -I*(d*x
 + c)*b/d))*cos(-(b*c - a*d)/d) + (-(I + 1)*sqrt(2)*gamma(-3/2, I*(d*x + c
)*b/d) + (I - 1)*sqrt(2)*gamma(-3/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)/d
))*((d*x + c)*b/d)^(3/2))/((d*x + c)^(3/2)*d)
```

3.61.8 Giac [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos(bx+a)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/(d*x + c)^(5/2), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cos(a+b\,x)^3}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(cos(a + b*x)^3/(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3/(c + d*x)^(5/2), x)
```

3.62 $\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx$

3.62.1 Optimal result

Integrand size = 18, antiderivative size = 356

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx =& -\frac{16b^2\cos(a+bx)}{5d^3\sqrt{c+dx}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}} \\ &+\frac{24b^2\cos^3(a+bx)}{5d^3\sqrt{c+dx}}+\frac{2b^{5/2}\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &+\frac{6b^{5/2}\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &+\frac{6b^{5/2}\sqrt{6\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{5d^{7/2}} \\ &+\frac{2b^{5/2}\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{5d^{7/2}}+\frac{4b\cos^2(a+bx)\sin(a+bx)}{5d^2(c+dx)^{3/2}}\end{aligned}$$

output
```
-2/5*cos(b*x+a)^3/d/(d*x+c)^(5/2)+4/5*b*cos(b*x+a)^2*sin(b*x+a)/d^2/(d*x+c
)^(3/2)+2/5*b^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)
^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)+2/5*b^(5/2)*FresnelC(b^(1/2)*2^(1
/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/d^(7/2)+
6/5*b^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*6^(1/2)*Pi^(1/2)/d^(7/2)+6/5*b^(5/2)*FresnelC(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/d^(7/2)-
16/5*b^2*cos(b*x+a)/d^3/(d*x+c)^(1/2)+24/5*b^2*cos(b*x+a)^3/d^3/(d*x+c)^(1
/2)
```

3.62.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{e^{-3ia}\left(2e^{4ia}\left(-3d^2e^{ibx}+2be^{-\frac{ibc}{d}}(c+dx)\left(e^{\frac{ib(c+dx)}{d}}(-id+2b(c+dx))-2id\left(-\frac{ib(c+dx)}{d}\right.\right.\right.\right.}{}$$

input
```
Integrate[Cos[a + b*x]^3/(c + d*x)^(7/2),x]
```

output
```
(2*E^((4*I)*a)*(-3*d^2*E^(I*b*x) + (2*b*(c + d*x)*(E^((I*b*(c + d*x))/d)*(
(-I)*d + 2*b*(c + d*x)) - (2*I)*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2,
((-I)*b*(c + d*x))/d]))/E^((I*b*c)/d)) + E^((2*I)*a - I*b*x)*(-6*d^2 + (4*
I)*b*d*(c + d*x) + 8*b^2*(c + d*x)^2 + 8*d^2*E^((I*b*(c + d*x))/d)*((I*b*(
c + d*x))/d)^(5/2)*Gamma[1/2, (I*b*(c + d*x))/d]) + 2*E^((6*I)*a)*(-(d^2*E
^((3*I)*b*x)) + (2*b*(c + d*x)*(E^(((3*I)*b*(c + d*x))/d)*((-I)*d + 6*b*(c
 + d*x)) - (6*I)*Sqrt[3]*d*(((-I)*b*(c + d*x))/d)^(3/2)*Gamma[1/2, ((-3*I)
*b*(c + d*x))/d]))/E^(((3*I)*b*c)/d)) + (2*(-d^2 - I*b*(c + d*x)*(-2*d + (
12*I)*b*(c + d*x) - 12*Sqrt[3]*d*E^(((3*I)*b*(c + d*x))/d)*((I*b*(c + d*x)
)/d)^(3/2)*Gamma[1/2, ((3*I)*b*(c + d*x))/d])))/E^((3*I)*b*x))/(40*d^3*E^(
(3*I)*a)*(c + d*x)^(5/2))
```

3.62.3 Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.42, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 3795, 3042, 3778, 25, 3042, 3787, 3042, 3785, 3786, 3794, 2009, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{12b^2\int \frac{\cos^3(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\int \frac{\cos(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx}{5d^2}-\frac{12b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{12b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(\frac{2b\int -\frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{12b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(-\frac{2b\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{12b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(-\frac{2b\int \frac{\sin(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{12b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(-\frac{2b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{12b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(-\frac{2b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{12b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int \cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{\sqrt{c+dx}}dx\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3786}$$

3.62. $\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx$

$$-\frac{12b^2\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}+\frac{8b^2\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 3794}$$

$$-\frac{12b^2\left(\frac{6b\int\left(-\frac{\sin(a+bx)}{4\sqrt{c+dx}}-\frac{\sin(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}-\frac{2\cos^3(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{8b^2\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{8b^2\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(a-\frac{bc}{d}\right)\int\sin\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{12b^2\left(\frac{6b\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\ c\cdots}{\cdots}\cdots\right)}{d}\cdots\right)}{5d^2}$$

$$\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 3832}$$

3.62. $\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx$

$$\frac{8b^2\left(-\frac{2b\left(\frac{2\sin\left(a-\frac{bc}{d}\right)\int\cos\left(\frac{b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-$$

$$\frac{12b^2\left(\frac{6b\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{6}}\,\cdots}{\cdots}\cdots\right)}{d}\cdots\right)}{5d^2}\cdots$$

$$\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

$\Big\downarrow$ 3833

$$-\frac{12b^2\left(\frac{6b\left(-\frac{\sqrt{\frac{\pi}{6}}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\cdots\right)}{d}\cdots\right)}{5d^2}\cdots$$

$$\frac{8b^2\left(-\frac{2b\left(\frac{\sqrt{2\pi}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{2\pi}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2\cos(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}+$$

$$\frac{4b\sin(a+bx)\cos^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cos^3(a+bx)}{5d(c+dx)^{5/2}}$$

input
```
Int[Cos[a + b*x]^3/(c + d*x)^(7/2),x]
```

3.62. $\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx$

output
```
(-2*Cos[a + b*x]^3)/(5*d*(c + d*x)^(5/2)) - (12*b^2*((-2*Cos[a + b*x]^3)/(
d*Sqrt[c + d*x]) + (6*b*(-1/2*(Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*Cos
[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(2
*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(2*Sqrt[b]*Sqrt[d]) - (Sqrt[Pi/2]*Fresnel
C[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(2*Sqrt[b]
*Sqrt[d])))/d))/(5*d^2) + (8*b^2*((-2*Cos[a + b*x])/(d*Sqrt[c + d*x]) - (2
*b*((Sqrt[2*Pi]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + (Sqrt[2*Pi]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(Sqrt[b]*Sqrt[d])))/d))/(5*d^2)
+ (4*b*Cos[a + b*x]^2*Sin[a + b*x])/(5*d^2*(c + d*x)^(3/2))
```

3.62.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.62.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{10(dx+c)^{\frac{5}{2}}}-\frac{3b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{d\sqrt{}}\right.}{3d}\right.}{5d}$
default	$-\frac{3\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{10(dx+c)^{\frac{5}{2}}}-\frac{3b\left(-\frac{\sin\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{3(dx+c)^{\frac{3}{2}}}+\frac{2b\left(-\frac{\cos\left(\frac{b(dx+c)}{d}+\frac{ad-bc}{d}\right)}{\sqrt{dx+c}}-\frac{b\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-bc}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{d\sqrt{}}\right.}{3d}\right.}{5d}$

input
```
int(cos(b*x+a)^3/(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/20/(d*x+c)^(5/2)*cos(b*(d*x+c)/d+(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+
c)^(3/2)*sin(b*(d*x+c)/d+(a*d-b*c)/d)+2/3*b/d*(-1/(d*x+c)^(1/2)*cos(b*(d*x
+c)/d+(a*d-b*c)/d)-b/d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*Fres
nelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/20/(d*x+c)^(5/2)
*cos(3*b*(d*x+c)/d+3*(a*d-b*c)/d)-3/10*b/d*(-1/3/(d*x+c)^(3/2)*sin(3*b*(d*
x+c)/d+3*(a*d-b*c)/d)+2*b/d*(-1/(d*x+c)^(1/2)*cos(3*b*(d*x+c)/d+3*(a*d-b*c
)/d)-b/d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS
(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d
)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{2\left(3\sqrt{6}(\pi b^2d^3x^3+3\,\pi b^2cd^2x^2+3\,\pi b^2c^2dx+\pi b^2c^3)\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
2/5*(3*sqrt(6)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + p
i*b^2*c^3)*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d
*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3
*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_s
in(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(2)*(pi*b^2*d^3*x^3 + 3*pi*
b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(pi*d))*fresnel_cos(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 3*sqrt(6)*(pi*
b^2*d^3*x^3 + 3*pi*b^2*c*d^2*x^2 + 3*pi*b^2*c^2*d*x + pi*b^2*c^3)*sqrt(b/(
pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + ((12*b^2*d^2*x^2 + 24*b^2*c*d*x + 12*b^2*c^2 - d^2)*cos(b*x + a)^3
+ 2*(b*d^2*x + b*c*d)*cos(b*x + a)^2*sin(b*x + a) - 8*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(b*x + a))*sqrt(d*x + c))/(d^6*x^3 + 3*c*d^5*x^2 + 3*
c^2*d^4*x + c^3*d^3)
```

3.62.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos^3(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)**3/(d*x+c)**(7/2),x)
```

output
```
Integral(cos(a + b*x)**3/(c + d*x)**(7/2), x)
```

3.62.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx =$$

$$-\frac{3\left(3\sqrt{3}\left(\left(-(i+1)\sqrt{2}\Gamma\left(-\frac{5}{2},\frac{3i(dx+c)b}{d}\right)+(i-1)\sqrt{2}\Gamma\left(-\frac{5}{2},-\frac{3i(dx+c)b}{d}\right)\right)\cos\left(-\frac{3(bc-ad)}{d}\right)+\left((i-1)\sqrt{2}\right.\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-3/16*(3*sqrt(3)*((-(I + 1)*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) + (I -
1)*sqrt(2)*gamma(-5/2, -3*I*(d*x + c)*b/d))*cos(-3*(b*c - a*d)/d) + ((I -
1)*sqrt(2)*gamma(-5/2, 3*I*(d*x + c)*b/d) - (I + 1)*sqrt(2)*gamma(-5/2, -3
*I*(d*x + c)*b/d))*sin(-3*(b*c - a*d)/d))*((d*x + c)*b/d)^(5/2) - (((I + 1
)*sqrt(2)*gamma(-5/2, I*(d*x + c)*b/d) - (I - 1)*sqrt(2)*gamma(-5/2, -I*(d
*x + c)*b/d))*cos(-(b*c - a*d)/d) + (-(I - 1)*sqrt(2)*gamma(-5/2, I*(d*x +
 c)*b/d) + (I + 1)*sqrt(2)*gamma(-5/2, -I*(d*x + c)*b/d))*sin(-(b*c - a*d)
/d))*((d*x + c)*b/d)^(5/2))/((d*x + c)^(5/2)*d)
```

3.62.8 Giac [F]

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos(bx+a)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cos(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^3/(d*x + c)^(7/2), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cos(a+b\,x)^3}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(cos(a + b*x)^3/(c + d*x)^(7/2),x)
```

output
```
int(cos(a + b*x)^3/(c + d*x)^(7/2), x)
```

3.63 $\int x^{3/2} \cos(x)\, dx$

3.63.1 Optimal result

Integrand size = 8, antiderivative size = 49

$$\int x^{3/2} \cos(x)\, dx = \frac{3}{2}\sqrt{x}\cos(x) - \frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right) + x^{3/2}\sin(x)$$

output
```
x^(3/2)*sin(x)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+3/2
*x^(1/2)*cos(x)
```

3.63.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int x^{3/2} \cos(x)\, dx = \frac{\sqrt{x}\Gamma\left(\frac{5}{2}, -ix\right)}{2\sqrt{-ix}} + \frac{\sqrt{x}\Gamma\left(\frac{5}{2}, ix\right)}{2\sqrt{ix}}$$

input
```
Integrate[x^(3/2)*Cos[x],x]
```

output
```
(Sqrt[x]*Gamma[5/2, (-I)*x])/(2*Sqrt[(-I)*x]) + (Sqrt[x]*Gamma[5/2, I*x])/
(2*Sqrt[I*x])
```

3.63.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \cos(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{3/2} \sin\left(x+\frac{\pi}{2}\right) dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{3}{2} \int -\sqrt{x} \sin(x) dx + x^{3/2} \sin(x)$$

$$\Big\downarrow \text{25}$$

$$x^{3/2} \sin(x) - \frac{3}{2} \int \sqrt{x} \sin(x) dx$$

$$\Big\downarrow \text{3042}$$

$$x^{3/2} \sin(x) - \frac{3}{2} \int \sqrt{x} \sin(x) dx$$

$$\Big\downarrow \text{3777}$$

$$x^{3/2} \sin(x) - \frac{3}{2} \left(\frac{1}{2} \int \frac{\cos(x)}{\sqrt{x}} dx - \sqrt{x} \cos(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$x^{3/2} \sin(x) - \frac{3}{2} \left(\frac{1}{2} \int \frac{\sin\left(x+\frac{\pi}{2}\right)}{\sqrt{x}} dx - \sqrt{x} \cos(x)\right)$$

$$\Big\downarrow \text{3785}$$

$$x^{3/2} \sin(x) - \frac{3}{2} \left(\int \cos(x) d\sqrt{x} - \sqrt{x} \cos(x)\right)$$

$$\Big\downarrow \text{3833}$$

$$x^{3/2} \sin(x) - \frac{3}{2} \left(\sqrt{\frac{\pi}{2}} \text{FresnelC}\left(\sqrt{\frac{2}{\pi}} \sqrt{x}\right) - \sqrt{x} \cos(x)\right)$$

input
```
Int[x^(3/2)*Cos[x],x]
```

output
```
(-3*(-(Sqrt[x]*Cos[x]) + Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[x]]))/2 + x^(
3/2)*Sin[x]
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.63.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$x^{\frac{3}{2}} \sin(x) - \frac{3\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{4} + \frac{3\sqrt{x}\,\cos(x)}{2}$	34
default	$x^{\frac{3}{2}} \sin(x) - \frac{3\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{4} + \frac{3\sqrt{x}\,\cos(x)}{2}$	34
meijerg	$2\sqrt{2}\,\sqrt{\pi}\left(\frac{3\sqrt{x}\,\sqrt{2}\,\cos(x)}{8\sqrt{\pi}} + \frac{x^{\frac{3}{2}}\sqrt{2}\,\sin(x)}{4\sqrt{\pi}} - \frac{3\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)}{8}\right)$	49

input
```
int(x^(3/2)*cos(x),x,method=_RETURNVERBOSE)
```

output
```
x^(3/2)*sin(x)-3/4*FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+3/2
*x^(1/2)*cos(x)
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int x^{3/2}\cos(x)\,dx = -\frac{3}{4}\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)+\frac{1}{2}\left(2\,x\sin\left(x\right)+3\,\cos\left(x\right)\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*cos(x),x, algorithm="fricas")
```

output
```
-3/4*sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x)/sqrt(pi)) + 1/2*(2*x*sin
(x) + 3*cos(x))*sqrt(x)
```

3.63.6 Sympy [A] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int x^{3/2}\cos(x)\,dx = \frac{5x^{\frac{3}{2}}\sin\left(x\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{x}\cos\left(x\right)\Gamma\left(\frac{5}{4}\right)}{8\Gamma\left(\frac{9}{4}\right)}-\frac{15\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{16\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**(3/2)*cos(x),x)
```

output
```
5*x**(3/2)*sin(x)*gamma(5/4)/(4*gamma(9/4)) + 15*sqrt(x)*cos(x)*gamma(5/4)
/(8*gamma(9/4)) - 15*sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*sqrt(x)/sqrt(pi))*g
amma(5/4)/(16*gamma(9/4))
```

3.63.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int x^{3/2}\cos(x)\,dx = x^{\frac{3}{2}}\sin(x) - \frac{3}{32}\sqrt{\pi}\left(-(i-1)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - (i+1)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) + (i+1)\sqrt{2}\,\mathrm{erf}\left(\sqrt{}\right.\right. + \frac{3}{2}\sqrt{x}\cos(x)$$

input
```
integrate(x^(3/2)*cos(x),x, algorithm="maxima")
```

output
```
x^(3/2)*sin(x) - 3/32*sqrt(pi)*(-(I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)
*sqrt(x)) - (I + 1)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (I + 1)*s
qrt(2)*erf(sqrt(-I)*sqrt(x)) - (I - 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x))) +
3/2*sqrt(x)*cos(x)
```

3.63.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int x^{3/2}\cos(x)\,dx = \left(\frac{3}{16}i+\frac{3}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - \left(\frac{3}{16}i-\frac{3}{16}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - \frac{1}{4}\left(2i\,x^{\frac{3}{2}}-3\sqrt{x}\right)e^{(i\,x)} - \frac{1}{4}\left(-2i\,x^{\frac{3}{2}}-3\sqrt{x}\right)e^{(-i\,x)}$$

input
```
integrate(x^(3/2)*cos(x),x, algorithm="giac")
```

output
```
(3/16*I + 3/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (3/1
6*I - 3/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x)) - 1/4*(2*
I*x^(3/2) - 3*sqrt(x))*e^(I*x) - 1/4*(-2*I*x^(3/2) - 3*sqrt(x))*e^(-I*x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\cos(x)\,dx = \int x^{3/2}\,\cos(x)\,dx$$

input
```
int(x^(3/2)*cos(x),x)
```

output
```
int(x^(3/2)*cos(x), x)
```

3.64 $\int \sqrt{x}\cos(x)\,dx$

3.64.1 Optimal result

Integrand size = 8, antiderivative size = 36

$$\int \sqrt{x}\cos(x)\,dx = -\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)+\sqrt{x}\sin(x)$$

output
```
-1/2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+sin(x)*x^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \sqrt{x}\cos(x)\,dx = \frac{\sqrt{-ix}\,\Gamma\left(\frac{3}{2},-ix\right)+\sqrt{ix}\,\Gamma\left(\frac{3}{2},ix\right)}{2\sqrt{x}}$$

input
```
Integrate[Sqrt[x]*Cos[x],x]
```

output
```
(Sqrt[(-I)*x]*Gamma[3/2, (-I)*x] + Sqrt[I*x]*Gamma[3/2, I*x])/(2*Sqrt[x])
```

3.64.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3777, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{x}\sin\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\int -\frac{\sin(x)}{\sqrt{x}}dx+\sqrt{x}\sin(x)$$

$$\Big\downarrow \text{25}$$

$$\sqrt{x}\sin(x)-\frac{1}{2}\int \frac{\sin(x)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt{x}\sin(x)-\frac{1}{2}\int \frac{\sin(x)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{3786}$$

$$\sqrt{x}\sin(x)-\int \sin(x)d\sqrt{x}$$

$$\Big\downarrow \text{3832}$$

$$\sqrt{x}\sin(x)-\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

input
```
Int[Sqrt[x]*Cos[x],x]
```

output
```
-(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]) + Sqrt[x]*Sin[x]
```

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.64.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{2}+\sin\left(x\right)\sqrt{x}$	27
default	$-\frac{\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{2}+\sin\left(x\right)\sqrt{x}$	27
meijerg	$\sqrt{2}\,\sqrt{\pi}\left(\frac{\sqrt{x}\,\sqrt{2}\,\sin(x)}{2\sqrt{\pi}}-\frac{\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)}{2}\right)$	35

input
```
int(x^(1/2)*cos(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)+sin(x)*x^(1/2)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\cos(x)\,dx = -\frac{1}{2}\sqrt{2}\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right) + \sqrt{x}\sin\left(x\right)$$

input
```
integrate(x^(1/2)*cos(x),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*sqrt(pi)*fresnel_sin(sqrt(2)*sqrt(x)/sqrt(pi)) + sqrt(x)*sin(
x)
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \sqrt{x}\cos(x)\,dx = \frac{3\sqrt{x}\sin\left(x\right)\Gamma\left(\frac{3}{4}\right)}{4\Gamma\left(\frac{7}{4}\right)} - \frac{3\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{8\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**(1/2)*cos(x),x)
```

output
```
3*sqrt(x)*sin(x)*gamma(3/4)/(4*gamma(7/4)) - 3*sqrt(2)*sqrt(pi)*fresnels(s
qrt(2)*sqrt(x)/sqrt(pi))*gamma(3/4)/(8*gamma(7/4))
```

3.64.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

$$\int \sqrt{x}\cos(x)\,dx =$$

$$-\frac{1}{16}\sqrt{\pi}\left(\left(i+1\right)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) + \left(i-1\right)\sqrt{2}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - \left(i-1\right)\sqrt{2}\,\mathrm{erf}\left(\right.\right.$$
$$+\sqrt{x}\sin\left(x\right)$$

input
```
integrate(x^(1/2)*cos(x),x, algorithm="maxima")
```

output
```
-1/16*sqrt(pi)*((I + 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*sqrt(x)) + (I -
1)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (I - 1)*sqrt(2)*erf(sqrt(-
I)*sqrt(x)) + (I + 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x))) + sqrt(x)*sin(x)
```

3.64.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int \sqrt{x}\cos(x)\,dx = -\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) + \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) - \frac{1}{2}i\sqrt{x}e^{(ix)} + \frac{1}{2}i\sqrt{x}e^{(-ix)}$$

input
```
integrate(x^(1/2)*cos(x),x, algorithm="giac")
```

output
```
-(1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (1/8*
I + 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x)) - 1/2*I*sqrt
(x)*e^(I*x) + 1/2*I*sqrt(x)*e^(-I*x)
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\cos(x)\,dx = \sqrt{x}\,\sin(x) - \frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)}{2}$$

input
```
int(x^(1/2)*cos(x),x)
```

output
```
x^(1/2)*sin(x) - (2^(1/2)*pi^(1/2)*fresnels((2^(1/2)*x^(1/2))/pi^(1/2)))/2
```

3.65 $\int \frac{\cos(x)}{\sqrt{x}}\,dx$

3.65.1 Optimal result

Integrand size = 8, antiderivative size = 24

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

output
```
FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = -\frac{i\left(\sqrt{-ix}\,\Gamma\left(\frac{1}{2},-ix\right)-\sqrt{ix}\,\Gamma\left(\frac{1}{2},ix\right)\right)}{2\sqrt{x}}$$

input
```
Integrate[Cos[x]/Sqrt[x],x]
```

output
```
((-1/2*I)*(Sqrt[(-I)*x]*Gamma[1/2, (-I)*x] - Sqrt[I*x]*Gamma[1/2, I*x]))/S
qrt[x]
```

3.65.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{x}} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \; 3785$$

$$2\int \cos(x)d\sqrt{x}$$

$$\Big\downarrow \; 3833$$

$$\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

input
```
Int[Cos[x]/Sqrt[x],x]
```

output
```
Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[x]]
```

3.65.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.65.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}$	19
default	$\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}$	19
meijerg	$\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}$	19

input
```
int(cos(x)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
FresnelC(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)$$

input
```
integrate(cos(x)/x^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x)/sqrt(pi))
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \frac{\sqrt{2}\sqrt{\pi}C\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(cos(x)/x**(1/2),x)
```

output
```
sqrt(2)*sqrt(pi)*fresnelc(sqrt(2)*sqrt(x)/sqrt(pi))*gamma(1/4)/(4*gamma(5/
4))
```

3.65.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.50

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = -\frac{1}{8}\sqrt{\pi}\left((i-1)\sqrt{2}\,\text{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)+(i+1)\sqrt{2}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)-(i+1)\sqrt{2}\,\text{erf}\left(\sqrt{\right.\right.$$

input
```
integrate(cos(x)/x^(1/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(pi)*((I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*sqrt(x)) + (I + 1
)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) - (I + 1)*sqrt(2)*erf(sqrt(-I
)*sqrt(x)) + (I - 1)*sqrt(2)*erf((-1)^(1/4)*sqrt(x)))
```

3.65.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = -\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right) + \left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x}\right)$$

input
```
integrate(cos(x)/x^(1/2),x, algorithm="giac")
```

output
```
-(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x)) + (1/4*
I - 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x))
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)}{\sqrt{x}}\,dx = \sqrt{2}\,\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)$$

input
```
int(cos(x)/x^(1/2),x)
```

output
```
2^(1/2)*pi^(1/2)*fresnelc((2^(1/2)*x^(1/2))/pi^(1/2))
```

3.66 $\int \frac{\cos(x)}{x^{3/2}}\,dx$

3.66.1 Optimal result

Integrand size = 8, antiderivative size = 35

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = -\frac{2\cos(x)}{\sqrt{x}} - 2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)$$

output
```
-2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)-2*cos(x)/x^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = \frac{-e^{-ix}(1+e^{2ix}) + \sqrt{-ix}\,\Gamma\left(\frac{1}{2}, -ix\right) + \sqrt{ix}\,\Gamma\left(\frac{1}{2}, ix\right)}{\sqrt{x}}$$

input
```
Integrate[Cos[x]/x^(3/2),x]
```

output
```
(-((1 + E^((2*I)*x))/E^(I*x)) + Sqrt[(-I)*x]*Gamma[1/2, (-I)*x] + Sqrt[I*x
]*Gamma[1/2, I*x])/Sqrt[x]
```

3.66.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3778, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 3778}$$

$$2\int -\frac{\sin(x)}{\sqrt{x}}dx-\frac{2\cos(x)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{\sin(x)}{\sqrt{x}}dx-\frac{2\cos(x)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 3042}$$

$$-2\int \frac{\sin(x)}{\sqrt{x}}dx-\frac{2\cos(x)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 3786}$$

$$-4\int \sin(x)d\sqrt{x}-\frac{2\cos(x)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 3832}$$

$$-2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{x}\right)-\frac{2\cos(x)}{\sqrt{x}}$$

input
```
Int[Cos[x]/x^(3/2),x]
```

output
```
(-2*Cos[x])/Sqrt[x] - 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[x]]
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

3.66.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-2\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}-\frac{2\cos(x)}{\sqrt{x}}$	28
default	$-2\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}-\frac{2\cos(x)}{\sqrt{x}}$	28
meijerg	$\frac{\sqrt{2}\,\sqrt{\pi}\left(-\frac{4\sqrt{2}\,\cos(x)}{\sqrt{\pi}\,\sqrt{x}}-8\,\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{x}}{\sqrt{\pi}}\right)\right)}{4}$	36

input
```
int(cos(x)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*FresnelS(2^(1/2)/Pi^(1/2)*x^(1/2))*2^(1/2)*Pi^(1/2)-2*cos(x)/x^(1/2)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = -\frac{2\left(\sqrt{2}\sqrt{\pi}x\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)+\sqrt{x}\cos\left(x\right)\right)}{x}$$

input
```
integrate(cos(x)/x^(3/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*sqrt(pi)*x*fresnel_sin(sqrt(2)*sqrt(x)/sqrt(pi)) + sqrt(x)*cos
(x))/x
```

3.66.6 Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{\pi}S\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{2\Gamma\left(\frac{3}{4}\right)}+\frac{\cos\left(x\right)\Gamma\left(-\frac{1}{4}\right)}{2\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(cos(x)/x**(3/2),x)
```

output
```
sqrt(2)*sqrt(pi)*fresnels(sqrt(2)*sqrt(x)/sqrt(pi))*gamma(-1/4)/(2*gamma(3
/4)) + cos(x)*gamma(-1/4)/(2*sqrt(x)*gamma(3/4))
```

3.66.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = -\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\Gamma\left(-\frac{1}{2},i\,x\right)+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\Gamma\left(-\frac{1}{2},-i\,x\right)$$

input
```
integrate(cos(x)/x^(3/2),x, algorithm="maxima")
```

output
```
-(1/4*I + 1/4)*sqrt(2)*gamma(-1/2, I*x) + (1/4*I - 1/4)*sqrt(2)*gamma(-1/2
, -I*x)
```

3.66.8 Giac [F]

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = \int \frac{\cos(x)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(x)/x^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(x)/x^(3/2), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{x^{3/2}}\,dx = \int \frac{\cos(x)}{x^{3/2}}\,dx$$

input
```
int(cos(x)/x^(3/2),x)
```

output
```
int(cos(x)/x^(3/2), x)
```

3.67 $\int (c+dx)^{4/3}\cos(a+bx)\,dx$

3.67.1 Optimal result

Integrand size = 16, antiderivative size = 183

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx=\frac{4d\sqrt[3]{c+dx}\cos(a+bx)}{3b^2}+\frac{2id^2e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}-\frac{2id^2e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{9b^3(c+dx)^{2/3}}+\frac{(c+dx)^{4/3}\sin(a+bx)}{b}$$

output
```
4/3*d*(d*x+c)^(1/3)*cos(b*x+a)/b^2+2/9*I*d^2*exp(I*(a-b*c/d))*(-I*b*(d*x+c
)/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b^3/(d*x+c)^(2/3)-2/9*I*d^2*(I*b*(d*x
+c)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b^3/exp(I*(a-b*c/d))/(d*x+c)^(2/3)+(
d*x+c)^(4/3)*sin(b*x+a)/b
```

3.67.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx=\frac{de^{-\frac{i(bc+ad)}{d}}\sqrt[3]{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{7}{3},-\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{-\frac{ib(c+dx)}{d}}}+\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{7}{3},\frac{ib(c+dx)}{d}\right)}{\sqrt[3]{\frac{ib(c+dx)}{d}}}\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^(4/3)*Cos[a + b*x],x]
```

output
```
(d*(c + d*x)^(1/3)*((E^((2*I)*a)*Gamma[7/3, ((-I)*b*(c + d*x))/d])/(((-I)*
b*(c + d*x))/d)^(1/3) + (E^(((2*I)*b*c)/d)*Gamma[7/3, (I*b*(c + d*x))/d])/
((I*b*(c + d*x))/d)^(1/3)))/(2*b^2*E^((I*(b*c + a*d))/d))
```

3.67.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3777, 25, 3042, 3777, 3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{4/3}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{4d\int -\sqrt[3]{c+dx}\sin(a+bx)dx}{3b}+\frac{(c+dx)^{4/3}\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{4d\int \sqrt[3]{c+dx}\sin(a+bx)dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{4d\int \sqrt[3]{c+dx}\sin(a+bx)dx}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{4d\left(\frac{d\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}dx}{3b}-\frac{\sqrt[3]{c+dx}\cos(a+bx)}{b}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{4d\left(\frac{d\int\frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{2/3}}dx}{3b}-\frac{\sqrt[3]{c+dx}\cos(a+bx)}{b}\right)}{3b}$$

$$\Big\downarrow \text{3788}$$

$$\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{4d\left(-\frac{\sqrt[3]{c+dx}\cos(a+bx)}{b}+\frac{d\left(\frac{1}{2}i\int-\frac{ie^{-i(a+bx)}}{(c+dx)^{2/3}}dx-\frac{1}{2}i\int\frac{ie^{i(a+bx)}}{(c+dx)^{2/3}}dx\right)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{4d\left(-\frac{\sqrt[3]{c+dx}\cos(a+bx)}{b}+\frac{d\left(\frac{1}{2}\int\frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}dx+\frac{1}{2}\int\frac{e^{i(a+bx)}}{(c+dx)^{2/3}}dx\right)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{2612}$$

$$\frac{(c+dx)^{4/3}\sin(a+bx)}{b}-\frac{4d\left(-\frac{\sqrt[3]{c+dx}\cos(a+bx)}{b}+\frac{d\left(\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}\right)}{3b}\right)}{3b}$$

input
```
Int[(c + d*x)^(4/3)*Cos[a + b*x],x]
```

output
```
(-4*d*(-(((c + d*x)^(1/3)*Cos[a + b*x])/b) + (d*(((-1/2*I)*E^(I*(a - (b*c)
/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(b*(c
+ d*x)^(2/3)) + ((I/2)*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c + d*x)
)/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3))))/(3*b)))/(3*b) + ((c + d*x)
^(4/3)*Sin[a + b*x])/b
```

3.67. $\int(c+dx)^{4/3}\cos(a+bx)\,dx$

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.67.4 Maple [F]

$$\int (dx+c)^{\frac{4}{3}} \cos(bx+a)\,dx$$

input
```
int((d*x+c)^(4/3)*cos(b*x+a),x)
```

output
```
int((d*x+c)^(4/3)*cos(b*x+a),x)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.95

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx =$$

$$-\frac{2\left(i\,d^2\cos\left(-\frac{bc-ad}{d}\right)+d^2\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{i\,b}{d}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\frac{i\,bdx+i\,bc}{d}\right)+2\left(-i\,d^2\cos\left(-\frac{bc-ad}{d}\right)+d^2\sin\left(-\frac{bc-ad}{d}\right)\right)\left(-\frac{i}{d}\right.}{9\,b^3}$$

input
```
integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="fricas")
```

output
```
-1/9*(2*(I*d^2*cos(-(b*c - a*d)/d) + d^2*sin(-(b*c - a*d)/d))*(I*b/d)^(2/3
)*gamma(1/3, (I*b*d*x + I*b*c)/d) + 2*(-I*d^2*cos(-(b*c - a*d)/d) + d^2*si
n(-(b*c - a*d)/d))*(-I*b/d)^(2/3)*gamma(1/3, (-I*b*d*x - I*b*c)/d) - 3*(4*
b*d*cos(b*x + a) + 3*(b^2*d*x + b^2*c)*sin(b*x + a))*(d*x + c)^(1/3))/b^3
```

3.67.6 Sympy [F]

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \int (c+dx)^{\frac{4}{3}}\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(4/3)*cos(b*x+a),x)
```

output
```
Integral((c + d*x)**(4/3)*cos(a + b*x), x)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.28

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \frac{9\,(dx+c)^{\frac{4}{3}}b\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)+12\,(dx+c)^{\frac{1}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left(\left(\right.\right.}{}$$

input
```
integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="maxima")
```

output
```
1/9*(9*(d*x + c)^(4/3)*b*((d*x + c)*b/d)^(1/3)*d*sin(((d*x + c)*b - b*c +
a*d)/d) + 12*(d*x + c)^(1/3)*((d*x + c)*b/d)^(1/3)*d^2*cos(((d*x + c)*b -
b*c + a*d)/d) + (((sqrt(3) - I)*gamma(1/3, I*(d*x + c)*b/d) + (sqrt(3) + I
)*gamma(1/3, -I*(d*x + c)*b/d))*d^2*cos(-(b*c - a*d)/d) + ((-I*sqrt(3) - 1
)*gamma(1/3, I*(d*x + c)*b/d) + (I*sqrt(3) - 1)*gamma(1/3, -I*(d*x + c)*b/
d))*d^2*sin(-(b*c - a*d)/d))*(d*x + c)^(1/3))/(b^2*((d*x + c)*b/d)^(1/3)*d
)
```

3.67.8 Giac [F]

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \int (dx+c)^{\frac{4}{3}}\cos(bx+a)\,dx$$

input
```
integrate((d*x+c)^(4/3)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(4/3)*cos(b*x + a), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{4/3}\cos(a+bx)\,dx = \int \cos(a+b\,x)\,(c+d\,x)^{4/3}\,dx$$

input
```
int(cos(a + b*x)*(c + d*x)^(4/3),x)
```

output
```
int(cos(a + b*x)*(c + d*x)^(4/3), x)
```

3.68 $\int (c+dx)^{2/3}\cos(a+bx)\,dx$

3.68.1 Optimal result

Integrand size = 16, antiderivative size = 152

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{de^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{3b^2\sqrt[3]{c+dx}} + \frac{(c+dx)^{2/3}\sin(a+bx)}{b}$$

output
```
1/3*d*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/b^
2/(d*x+c)^(1/3)+1/3*d*(I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,I*b*(d*x+c)/d)/b^2/e
xp(I*(a-b*c/d))/(d*x+c)^(1/3)+(d*x+c)^(2/3)*sin(b*x+a)/b
```

3.68.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{5}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{5}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b^2\sqrt[3]{c+dx}}$$

input
```
Integrate[(c + d*x)^(2/3)*Cos[a + b*x],x]
```

output
```
(d*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[5/3, ((-I)*b*(c + d*x))
/d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[5/3, (I*b*(c + d*x
))/d]))/(2*b^2*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

3.68.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3777, 25, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^{2/3}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{2d\int -\frac{\sin(a+bx)}{\sqrt[3]{c+dx}}dx}{3b}+\frac{(c+dx)^{2/3}\sin(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(c+dx)^{2/3}\sin(a+bx)}{b}-\frac{2d\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}dx}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^{2/3}\sin(a+bx)}{b}-\frac{2d\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}dx}{3b}$$

$$\Big\downarrow \text{ 3789}$$

$$\frac{(c+dx)^{2/3}\sin(a+bx)}{b}-\frac{2d\left(\frac{1}{2}i\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}dx-\frac{1}{2}i\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}dx\right)}{3b}$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{(c+dx)^{2/3}\sin(a+bx)}{b}-\frac{2d\left(-\frac{e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}\right)}{3b}$$

input
```
Int[(c + d*x)^(2/3)*Cos[a + b*x],x]
```

output
```
(-2*d*(-1/2*(E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (
(-I)*b*(c + d*x))/d])/(b*(c + d*x)^(1/3)) - (((I*b*(c + d*x))/d)^(1/3)*Gam
ma[2/3, (I*b*(c + d*x))/d])/(2*b*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3))))/(3
*b) + ((c + d*x)^(2/3)*Sin[a + b*x])/b
```

3.68.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.68.4 Maple [F]

$$\int (dx+c)^{\frac{2}{3}} \cos(bx+a)\, dx$$

input
```
int((d*x+c)^(2/3)*cos(b*x+a),x)
```

output
```
int((d*x+c)^(2/3)*cos(b*x+a),x)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int (c+dx)^{2/3}\cos(a+bx)\, dx = \frac{3\,(dx+c)^{\frac{2}{3}} b \sin(bx+a) + \left(d\cos\left(-\frac{bc-ad}{d}\right) - i\,d\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{i\,b}{d}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}, \frac{i\,bdx+i\,bc}{d}\right) + \left(d\cos\left(-\frac{bc-}{d}\right.\right.}{3\,b^2}$$

input
```
integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/3*(3*(d*x + c)^(2/3)*b*sin(b*x + a) + (d*cos(-(b*c - a*d)/d) - I*d*sin(-
(b*c - a*d)/d))*(I*b/d)^(1/3)*gamma(2/3, (I*b*d*x + I*b*c)/d) + (d*cos(-(b
*c - a*d)/d) + I*d*sin(-(b*c - a*d)/d))*(-I*b/d)^(1/3)*gamma(2/3, (-I*b*d*
x - I*b*c)/d))/b^2
```

3.68.6 Sympy [F]

$$\int (c+dx)^{2/3}\cos(a+bx)\, dx = \int (c+dx)^{\frac{2}{3}} \cos(a+bx)\, dx$$

input
```
integrate((d*x+c)**(2/3)*cos(b*x+a),x)
```

output
```
Integral((c + d*x)**(2/3)*cos(a + b*x), x)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \frac{6\,(dx+c)^{\frac{2}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{2}{3}} d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right) + \left(\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3}, \frac{i\,(dx+c)b}{d}\right) + \left(\sqrt{3}-i\right)\Gamma\left(\frac{2}{3}, -\frac{i\,(dx+c)}{d}\right.\right.\right.}{6\,b}$$

input
```
integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(6*(d*x + c)^(2/3)*((d*x + c)*b/d)^(2/3)*d*sin(((d*x + c)*b - b*c + a*
d)/d) + (((sqrt(3) + I)*gamma(2/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(
2/3, -I*(d*x + c)*b/d))*d*cos(-(b*c - a*d)/d) + ((-I*sqrt(3) + 1)*gamma(2/
3, I*(d*x + c)*b/d) + (I*sqrt(3) + 1)*gamma(2/3, -I*(d*x + c)*b/d))*d*sin(
-(b*c - a*d)/d))*(d*x + c)^(2/3))/(b*((d*x + c)*b/d)^(2/3)*d)
```

3.68.8 Giac [F]

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \int (dx+c)^{\frac{2}{3}}\cos(bx+a)\,dx$$

input
```
integrate((d*x+c)^(2/3)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(2/3)*cos(b*x + a), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{2/3}\cos(a+bx)\,dx = \int \cos(a+b\,x)\,\left(c+d\,x\right)^{2/3}\,dx$$

input
```
int(cos(a + b*x)*(c + d*x)^(2/3),x)
```

output
```
int(cos(a + b*x)*(c + d*x)^(2/3), x)
```

3.69 $\int \sqrt[3]{c+dx}\cos(a+bx)\,dx$

3.69.1 Optimal result

Integrand size = 16, antiderivative size = 152

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \frac{de^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{de^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{6b^2(c+dx)^{2/3}} + \frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}$$

output
```
1/6*d*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b^
2/(d*x+c)^(2/3)+1/6*d*(I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b^2/e
xp(I*(a-b*c/d))/(d*x+c)^(2/3)+(d*x+c)^(1/3)*sin(b*x+a)/b
```

3.69.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{4}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{4}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b^2(c+dx)^{2/3}}$$

input
```
Integrate[(c + d*x)^(1/3)*Cos[a + b*x],x]
```

output
```
(d*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[4/3, ((-I)*b*(c + d*x))
/d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[4/3, (I*b*(c + d*x
))/d]))/(2*b^2*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

3.69.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3777, 25, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt[3]{c+dx}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{d\int -\frac{\sin(a+bx)}{(c+dx)^{2/3}}dx}{3b}+\frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}-\frac{d\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}dx}{3b}$$

$$\Big\downarrow \text{3789}$$

$$\frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}-\frac{d\left(\frac{1}{2}i\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}dx-\frac{1}{2}i\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}dx\right)}{3b}$$

$$\Big\downarrow \text{2612}$$

$$\frac{\sqrt[3]{c+dx}\sin(a+bx)}{b}-\frac{d\left(-\frac{e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}\right)}{3b}$$

input
```
Int[(c + d*x)^(1/3)*Cos[a + b*x],x]
```

output
```
-1/3*(d*(-1/2*(E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3,
 ((-I)*b*(c + d*x))/d])/(b*(c + d*x)^(2/3)) - (((I*b*(c + d*x))/d)^(2/3)*G
amma[1/3, (I*b*(c + d*x))/d])/(2*b*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3))))/
b + ((c + d*x)^(1/3)*Sin[a + b*x])/b
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.69.4 Maple [F]

$$\int (dx+c)^{\frac{1}{3}} \cos (bx+a)\, dx$$

input
```
int((d*x+c)^(1/3)*cos(b*x+a),x)
```

output
```
int((d*x+c)^(1/3)*cos(b*x+a),x)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int \sqrt[3]{c+dx} \cos(a+bx)\, dx$$
$$= \frac{\left(d \cos\left(-\frac{bc-ad}{d}\right) - i\, d \sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{i\, b}{d}\right)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, \frac{i\, bdx+i\, bc}{d}\right) + \left(d \cos\left(-\frac{bc-ad}{d}\right) + i\, d \sin\left(-\frac{bc-ad}{d}\right)\right)\left(-\frac{i\, b}{d}\right)^{\frac{2}{3}} \Gamma\left(\frac{1}{3},\right.}{6\, b^2}$$

input
```
integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/6*((d*cos(-(b*c - a*d)/d) - I*d*sin(-(b*c - a*d)/d))*(I*b/d)^(2/3)*gamma
(1/3, (I*b*d*x + I*b*c)/d) + (d*cos(-(b*c - a*d)/d) + I*d*sin(-(b*c - a*d)
/d))*(-I*b/d)^(2/3)*gamma(1/3, (-I*b*d*x - I*b*c)/d) + 6*(d*x + c)^(1/3)*b
*sin(b*x + a))/b^2
```

3.69.6 Sympy [F]

$$\int \sqrt[3]{c+dx} \cos(a+bx)\, dx = \int \sqrt[3]{c+dx} \cos(a+bx)\, dx$$

input
```
integrate((d*x+c)**(1/3)*cos(b*x+a),x)
```

output
```
Integral((c + d*x)**(1/3)*cos(a + b*x), x)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.22

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx$$
$$= \frac{12\,(dx+c)^{\frac{1}{3}}\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}} d\sin\left(\frac{(dx+c)b-bc+ad}{d}\right) + \left(\left(\left(i\sqrt{3}+1\right)\Gamma\left(\frac{1}{3}, \frac{i\,(dx+c)b}{d}\right) + \left(-i\sqrt{3}+1\right)\Gamma\left(\frac{1}{3}, -\frac{i\,(dx+c)b}{d}\right.\right.\right.}{12\,b\left(\frac{(dx+c)b}{d}\right.}$$

input
```
integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(12*(d*x + c)^(1/3)*((d*x + c)*b/d)^(1/3)*d*sin(((d*x + c)*b - b*c +
a*d)/d) + (((I*sqrt(3) + 1)*gamma(1/3, I*(d*x + c)*b/d) + (-I*sqrt(3) + 1)
*gamma(1/3, -I*(d*x + c)*b/d))*d*cos(-(b*c - a*d)/d) + ((sqrt(3) - I)*gamm
a(1/3, I*(d*x + c)*b/d) + (sqrt(3) + I)*gamma(1/3, -I*(d*x + c)*b/d))*d*si
n(-(b*c - a*d)/d))*(d*x + c)^(1/3))/(b*((d*x + c)*b/d)^(1/3)*d)
```

3.69.8 Giac [F]

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \int (dx+c)^{\frac{1}{3}}\cos\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^(1/3)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(1/3)*cos(b*x + a), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{c+dx}\cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\left(c+d\,x\right)^{1/3}\,dx$$

input
```
int(cos(a + b*x)*(c + d*x)^(1/3),x)
```

output
```
int(cos(a + b*x)*(c + d*x)^(1/3), x)
```

3.70 $\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx$

3.70.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

output

```
-1/2*I*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/b
/(d*x+c)^(1/3)+1/2*I*(I*b*(d*x+c)/d)^(1/3)*GAMMA(2/3,I*b*(d*x+c)/d)/b/exp(
I*(a-b*c/d))/(d*x+c)^(1/3)
```

3.70.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b\sqrt[3]{c+dx}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^(1/3),x]
```

output
```
((I/2)*(-(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c +
 d*x))/d]) + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(
c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

3.70.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}dx$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{1}{2}i\int -\frac{ie^{-i(a+bx)}}{\sqrt[3]{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{i(a+bx)}}{\sqrt[3]{c+dx}}dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}dx+\frac{1}{2}\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}dx$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^(1/3),x]
```

output
```
((-1/2*I)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I
)*b*(c + d*x))/d])/(b*(c + d*x)^(1/3)) + ((I/2)*((I*b*(c + d*x))/d)^(1/3)*
Gamma[2/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3))
```

3.70.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.70.4 Maple [F]

$$\int \frac{\cos\left(bx+a\right)}{\left(dx+c\right)^{\frac{1}{3}}}dx$$

input
```
int(cos(b*x+a)/(d*x+c)^(1/3),x)
```

output
```
int(cos(b*x+a)/(d*x+c)^(1/3),x)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \frac{\left(\frac{i\,b}{d}\right)^{\frac{1}{3}}\left(i\,\cos\left(-\frac{bc-ad}{d}\right)+\sin\left(-\frac{bc-ad}{d}\right)\right)\Gamma\left(\frac{2}{3},\frac{i\,bdx+i\,bc}{d}\right)+\left(-\frac{i\,b}{d}\right)^{\frac{1}{3}}\left(-i\,\cos\left(-\frac{bc-ad}{d}\right)+\sin\left(-\frac{bc-ad}{d}\right)\right)\Gamma\left(\frac{2}{3},\frac{-i\,bd\cdots}{\cdots}\right.}{2\,b}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
```

output
```
1/2*((I*b/d)^(1/3)*(I*cos(-(b*c - a*d)/d) + sin(-(b*c - a*d)/d))*gamma(2/3
, (I*b*d*x + I*b*c)/d) + (-I*b/d)^(1/3)*(-I*cos(-(b*c - a*d)/d) + sin(-(b*
c - a*d)/d))*gamma(2/3, (-I*b*d*x - I*b*c)/d))/b
```

3.70.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \int \frac{\cos{(a+bx)}}{\sqrt[3]{c+dx}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(1/3),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**(1/3), x)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \frac{(dx+c)^{\frac{2}{3}}\left(\left(\left(i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},\frac{i(dx+c)b}{d}\right)+\left(-i\sqrt{3}-1\right)\Gamma\left(\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left(\left(\sqrt{3}+i\right)\Gamma\left(\frac{2}{3},\cdots\right.\right.\right.}{4\left(\frac{(dx+c)b}{d}\right)^{\frac{2}{3}}d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

output
```
1/4*(d*x + c)^(2/3)*(((I*sqrt(3) - 1)*gamma(2/3, I*(d*x + c)*b/d) + (-I*sq
rt(3) - 1)*gamma(2/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((sqrt(3) +
 I)*gamma(2/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(2/3, -I*(d*x + c)*b/
d))*sin(-(b*c - a*d)/d))/(((d*x + c)*b/d)^(2/3)*d)
```

3.70.8 Giac [F]

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(1/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x + c)^(1/3), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{1/3}}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^(1/3),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(1/3), x)
```

3.71 $\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx$

3.71.1 Optimal result

Integrand size = 16, antiderivative size = 135

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = -\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

output
```
-1/2*I*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,-I*b*(d*x+c)/d)/b
/(d*x+c)^(2/3)+1/2*I*(I*b*(d*x+c)/d)^(2/3)*GAMMA(1/3,I*b*(d*x+c)/d)/b/exp(
I*(a-b*c/d))/(d*x+c)^(2/3)
```

3.71.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \frac{ie^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)\right)}{2b(c+dx)^{2/3}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^(2/3),x]
```

output
```
((I/2)*(-(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c +
 d*x))/d]) + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(
c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

3.71.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{2/3}}dx$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{1}{2}i\int -\frac{ie^{-i(a+bx)}}{(c+dx)^{2/3}}dx-\frac{1}{2}i\int \frac{ie^{i(a+bx)}}{(c+dx)^{2/3}}dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}dx+\frac{1}{2}\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}dx$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^(2/3),x]
```

output
```
((-1/2*I)*E^(I*(a - (b*c)/d))*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I
)*b*(c + d*x))/d])/(b*(c + d*x)^(2/3)) + ((I/2)*((I*b*(c + d*x))/d)^(2/3)*
Gamma[1/3, (I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(2/3))
```

3.71.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.71.4 Maple [F]

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{2}{3}}}dx$$

input
```
int(cos(b*x+a)/(d*x+c)^(2/3),x)
```

output
```
int(cos(b*x+a)/(d*x+c)^(2/3),x)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \frac{\left(\frac{ib}{d}\right)^{\frac{2}{3}}\left(i\cos\left(-\frac{bc-ad}{d}\right)+\sin\left(-\frac{bc-ad}{d}\right)\right)\Gamma\left(\frac{1}{3},\frac{ibdx+ibc}{d}\right)+\left(-\frac{ib}{d}\right)^{\frac{2}{3}}\left(-i\cos\left(-\frac{bc-ad}{d}\right)+\sin\left(\right.\right.}{2b}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

output
```
1/2*((I*b/d)^(2/3)*(I*cos(-(b*c - a*d)/d) + sin(-(b*c - a*d)/d))*gamma(1/3
, (I*b*d*x + I*b*c)/d) + (-I*b/d)^(2/3)*(-I*cos(-(b*c - a*d)/d) + sin(-(b*
c - a*d)/d))*gamma(1/3, (-I*b*d*x - I*b*c)/d))/b
```

3.71.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(2/3),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**(2/3), x)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = -\frac{(dx+c)^{\frac{1}{3}}\left(\left(\left(\sqrt{3}-i\right)\Gamma\left(\frac{1}{3},\frac{i(dx+c)b}{d}\right)+\left(\sqrt{3}+i\right)\Gamma\left(\frac{1}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\left(i\sqrt{3}+1\right)\Gamma\left(\frac{1}{3},\frac{i(dx+c)b}{d}\right.\right.}{4\left(\frac{(dx+c)b}{d}\right)^{\frac{1}{3}}d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

output
```
-1/4*(d*x + c)^(1/3)*(((sqrt(3) - I)*gamma(1/3, I*(d*x + c)*b/d) + (sqrt(3
) + I)*gamma(1/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((I*sqrt(3) + 1
)*gamma(1/3, I*(d*x + c)*b/d) + (-I*sqrt(3) + 1)*gamma(1/3, -I*(d*x + c)*b
/d))*sin(-(b*c - a*d)/d))/(((d*x + c)*b/d)^(1/3)*d)
```

3.71.8 Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{2}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x + c)^(2/3), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{2/3}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{2/3}}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^(2/3),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(2/3), x)
```

3.72 $\int \frac{\cos(a+bx)}{(c+dx)^{4/3}} dx$

3.72.1 Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = -\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2d\sqrt[3]{c+dx}}$$

output
```
-3*cos(b*x+a)/d/(d*x+c)^(1/3)+3/2*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(1/3)*
GAMMA(2/3,-I*b*(d*x+c)/d)/d/(d*x+c)^(1/3)+3/2*(I*b*(d*x+c)/d)^(1/3)*GAMMA(
2/3,I*b*(d*x+c)/d)/d/exp(I*(a-b*c/d))/(d*x+c)^(1/3)
```

3.72.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = -\frac{e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(-\frac{1}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(-\frac{1}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d\sqrt[3]{c+dx}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^(4/3),x]
```

output
```
-1/2*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[-1/3, ((-I)*b*(c + d*
x))/d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[-1/3, (I*b*(c +
 d*x))/d])/(d*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

3.72.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3778, 25, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{4/3}}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{3b\int -\frac{\sin(a+bx)}{\sqrt[3]{c+dx}}dx}{d}-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3b\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}dx}{d}-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3b\int \frac{\sin(a+bx)}{\sqrt[3]{c+dx}}dx}{d}-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}-\frac{3b\left(\frac{1}{2}i\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}dx-\frac{1}{2}i\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}dx\right)}{d}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{3\cos(a+bx)}{d\sqrt[3]{c+dx}}-\frac{3b\left(-\frac{e^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}\right)}{d}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^(4/3),x]
```

output
```
(-3*Cos[a + b*x])/(d*(c + d*x)^(1/3)) - (3*b*(-1/2*(E^(I*(a - (b*c)/d))*((
(-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(b*(c + d*x)^
(1/3)) - (((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c + d*x))/d])/(2*b*E^
(I*(a - (b*c)/d))*(c + d*x)^(1/3))))/d
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.72.4 Maple [F]

$$\int \frac{\cos (bx + a)}{(dx + c)^{\frac{4}{3}}} dx$$

input
```
int(cos(b*x+a)/(d*x+c)^(4/3),x)
```

output
```
int(cos(b*x+a)/(d*x+c)^(4/3),x)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12

$$\int \frac{\cos(a + bx)}{(c + dx)^{4/3}} dx = \frac{3 \left(\left((dx + c) \cos \left(-\frac{bc-ad}{d} \right) - (i\,dx + i\,c) \sin \left(-\frac{bc-ad}{d} \right) \right) \left(\frac{i\,b}{d} \right)^{\frac{1}{3}} \Gamma\left(\frac{2}{3}, \frac{i\,bdx+i\,bc}{d} \right) + \left((dx + c) \, c\right.\right.}{2\,(d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="fricas")
```

output
```
3/2*(((d*x + c)*cos(-(b*c - a*d)/d) - (I*d*x + I*c)*sin(-(b*c - a*d)/d))*(
I*b/d)^(1/3)*gamma(2/3, (I*b*d*x + I*b*c)/d) + ((d*x + c)*cos(-(b*c - a*d)
/d) - (-I*d*x - I*c)*sin(-(b*c - a*d)/d))*(-I*b/d)^(1/3)*gamma(2/3, (-I*b*
d*x - I*b*c)/d) - 2*(d*x + c)^(2/3)*cos(b*x + a))/(d^2*x + c*d)
```

3.72.6 Sympy [F]

$$\int \frac{\cos(a + bx)}{(c + dx)^{4/3}} dx = \int \frac{\cos (a + bx)}{(c + dx)^{\frac{4}{3}}} dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(4/3),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**(4/3), x)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx =$$

$$-\frac{\left(\left(\left(\sqrt{3}+i\right)\Gamma\left(-\frac{1}{3},\frac{i\,(dx+c)b}{d}\right)+\left(\sqrt{3}-i\right)\Gamma\left(-\frac{1}{3},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{1}{3},\frac{i\,(dx+c)b}{d}\right)+\right.\right.}{4\,(dx+c)^{\frac{1}{3}}d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="maxima")
```

output
```
-1/4*(((sqrt(3) + I)*gamma(-1/3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(-1
/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((I*sqrt(3) - 1)*gamma(-1/3,
I*(d*x + c)*b/d) + (-I*sqrt(3) - 1)*gamma(-1/3, -I*(d*x + c)*b/d))*sin(-(b
*c - a*d)/d))*((d*x + c)*b/d)^(1/3)/((d*x + c)^(1/3)*d)
```

3.72.8 Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{4}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(4/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x + c)^(4/3), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{4/3}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{4/3}}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^(4/3),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(4/3), x)
```

3.73 $\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx$

3.73.1 Optimal result

Integrand size = 16, antiderivative size = 153

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = -\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}} + \frac{3e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}} + \frac{3e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{4d(c+dx)^{2/3}}$$

output
```
-3/2*cos(b*x+a)/d/(d*x+c)^(2/3)+3/4*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^(2/3
)*GAMMA(1/3,-I*b*(d*x+c)/d)/d/(d*x+c)^(2/3)+3/4*(I*b*(d*x+c)/d)^(2/3)*GAMM
A(1/3,I*b*(d*x+c)/d)/d/exp(I*(a-b*c/d))/(d*x+c)^(2/3)
```

3.73.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = -\frac{e^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(-\frac{2}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d(c+dx)^{2/3}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^(5/3),x]
```

output
```
-1/2*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[-2/3, ((-I)*b*(c + d*
x))/d] + E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(2/3)*Gamma[-2/3, (I*b*(c +
 d*x))/d])/(d*E^((I*(b*c + a*d))/d)*(c + d*x)^(2/3))
```

3.73.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3778, 25, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{5/3}}dx$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{3b\int -\frac{\sin(a+bx)}{(c+dx)^{2/3}}dx}{2d}-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{3b\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}dx}{2d}-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3b\int \frac{\sin(a+bx)}{(c+dx)^{2/3}}dx}{2d}-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}$$

$$\Big\downarrow \text{ 3789}$$

$$-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}-\frac{3b\left(\frac{1}{2}i\int \frac{e^{-i(a+bx)}}{(c+dx)^{2/3}}dx-\frac{1}{2}i\int \frac{e^{i(a+bx)}}{(c+dx)^{2/3}}dx\right)}{2d}$$

$$\Big\downarrow \text{ 2612}$$

$$-\frac{3\cos(a+bx)}{2d(c+dx)^{2/3}}-\frac{3b\left(-\frac{e^{i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},-\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{2/3}\Gamma\left(\frac{1}{3},\frac{ib(c+dx)}{d}\right)}{2b(c+dx)^{2/3}}\right)}{2d}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^(5/3),x]
```

output
```
(-3*Cos[a + b*x])/(2*d*(c + d*x)^(2/3)) - (3*b*(-1/2*(E^(I*(a - (b*c)/d))*
(((-I)*b*(c + d*x))/d)^(2/3)*Gamma[1/3, ((-I)*b*(c + d*x))/d])/(b*(c + d*x
)^(2/3)) - (((I*b*(c + d*x))/d)^(2/3)*Gamma[1/3, (I*b*(c + d*x))/d])/(2*b*
E^(I*(a - (b*c)/d))*(c + d*x)^(2/3))))/(2*d)
```

3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

3.73.4 Maple [F]

$$\int \frac{\cos (bx+a)}{(dx+c)^{\frac{5}{3}}}dx$$

input
```
int(cos(b*x+a)/(d*x+c)^(5/3),x)
```

output
```
int(cos(b*x+a)/(d*x+c)^(5/3),x)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \frac{3\left(\left(\left((dx+c)\cos\left(-\frac{bc-ad}{d}\right)-(i\,dx+i\,c)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{i\,b}{d}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},\frac{i\,bdx+i\,bc}{d}\right)+\left((dx+c)\,c\right.\right.\right.}{4\,(d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="fricas")
```

output
```
3/4*(((d*x + c)*cos(-(b*c - a*d)/d) - (I*d*x + I*c)*sin(-(b*c - a*d)/d))*(
I*b/d)^(2/3)*gamma(1/3, (I*b*d*x + I*b*c)/d) + ((d*x + c)*cos(-(b*c - a*d)
/d) - (-I*d*x - I*c)*sin(-(b*c - a*d)/d))*(-I*b/d)^(2/3)*gamma(1/3, (-I*b*
d*x - I*b*c)/d) - 2*(d*x + c)^(1/3)*cos(b*x + a))/(d^2*x + c*d)
```

3.73.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(5/3),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**(5/3), x)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \frac{\left(\left(\left(-i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},\frac{i(dx+c)b}{d}\right)+\left(i\sqrt{3}-1\right)\Gamma\left(-\frac{2}{3},-\frac{i(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-\left(\left(\sqrt{3}\right.\right.\right.}{4\,(dx+c)^{\frac{2}{3}}\,d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="maxima")
```

output
```
1/4*(((-I*sqrt(3) - 1)*gamma(-2/3, I*(d*x + c)*b/d) + (I*sqrt(3) - 1)*gamm
a(-2/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) - ((sqrt(3) - I)*gamma(-2/3
, I*(d*x + c)*b/d) + (sqrt(3) + I)*gamma(-2/3, -I*(d*x + c)*b/d))*sin(-(b*
c - a*d)/d))*((d*x + c)*b/d)^(2/3)/((d*x + c)^(2/3)*d)
```

3.73.8 Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{5}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(5/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x + c)^(5/3), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{5/3}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{5/3}}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^(5/3),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(5/3), x)
```

3.74 $\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx$

3.74.1 Optimal result

Integrand size = 16, antiderivative size = 182

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = -\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}} + \frac{9ibe^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} - \frac{9ibe^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{8d^2\sqrt[3]{c+dx}} + \frac{9b\sin(a+bx)}{4d^2\sqrt[3]{c+dx}}$$

output
```
-3/4*cos(b*x+a)/d/(d*x+c)^(4/3)+9/8*I*b*exp(I*(a-b*c/d))*(-I*b*(d*x+c)/d)^
(1/3)*GAMMA(2/3,-I*b*(d*x+c)/d)/d^2/(d*x+c)^(1/3)-9/8*I*b*(I*b*(d*x+c)/d)^
(1/3)*GAMMA(2/3,I*b*(d*x+c)/d)/d^2/exp(I*(a-b*c/d))/(d*x+c)^(1/3)+9/4*b*si
n(b*x+a)/d^2/(d*x+c)^(1/3)
```

3.74.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.69

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \frac{ibe^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt[3]{-\frac{ib(c+dx)}{d}}\,\Gamma\left(-\frac{4}{3},-\frac{ib(c+dx)}{d}\right) - e^{\frac{2ibc}{d}}\sqrt[3]{\frac{ib(c+dx)}{d}}\,\Gamma\left(-\frac{4}{3},\frac{ib(c+dx)}{d}\right)\right)}{2d^2\sqrt[3]{c+dx}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x)^(7/3),x]
```

output
```
((I/2)*b*(E^((2*I)*a)*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[-4/3, ((-I)*b*(c
+ d*x))/d] - E^(((2*I)*b*c)/d)*((I*b*(c + d*x))/d)^(1/3)*Gamma[-4/3, (I*b*
(c + d*x))/d]))/(d^2*E^((I*(b*c + a*d))/d)*(c + d*x)^(1/3))
```

3.74.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 3778, 25, 3042, 3778, 3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^{7/3}}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{3b\int -\frac{\sin(a+bx)}{(c+dx)^{4/3}}dx}{4d}-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3b\int \frac{\sin(a+bx)}{(c+dx)^{4/3}}dx}{4d}-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3b\int \frac{\sin(a+bx)}{(c+dx)^{4/3}}dx}{4d}-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{3b\left(\frac{3b\int \frac{\cos(a+bx)}{\sqrt[3]{c+dx}}dx}{d}-\frac{3\sin(a+bx)}{d\sqrt[3]{c+dx}}\right)}{4d}-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3b\left(\frac{3b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{\sqrt[3]{c+dx}}dx}{d}-\frac{3\sin(a+bx)}{d\sqrt[3]{c+dx}}\right)}{4d}-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}$$

$$\Big\downarrow \text{3788}$$

$$-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}-\frac{3b\left(-\frac{3\sin(a+bx)}{d\sqrt[3]{c+dx}}+\frac{3b\left(\frac{1}{2}i\int -\frac{ie^{-i(a+bx)}}{\sqrt[3]{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{i(a+bx)}}{\sqrt[3]{c+dx}}dx\right)}{d}\right)}{4d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}-\frac{3b\left(-\frac{3\sin(a+bx)}{d\sqrt[3]{c+dx}}+\frac{3b\left(\frac{1}{2}\int \frac{e^{-i(a+bx)}}{\sqrt[3]{c+dx}}dx+\frac{1}{2}\int \frac{e^{i(a+bx)}}{\sqrt[3]{c+dx}}dx\right)}{d}\right)}{4d}$$

$$\Big\downarrow \text{2612}$$

$$-\frac{3\cos(a+bx)}{4d(c+dx)^{4/3}}-\frac{3b\left(-\frac{3\sin(a+bx)}{d\sqrt[3]{c+dx}}+\frac{3b\left(\frac{ie^{-i\left(a-\frac{bc}{d}\right)}\sqrt[3]{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}\sqrt[3]{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{2}{3},-\frac{ib(c+dx)}{d}\right)}{2b\sqrt[3]{c+dx}}\right)}{d}\right)}{4d}$$

input
```
Int[Cos[a + b*x]/(c + d*x)^(7/3),x]
```

output
```
(-3*Cos[a + b*x])/(4*d*(c + d*x)^(4/3)) - (3*b*((3*b*(((-1/2*I)*E^(I*(a -
(b*c)/d))*(((-I)*b*(c + d*x))/d)^(1/3)*Gamma[2/3, ((-I)*b*(c + d*x))/d])/(
b*(c + d*x)^(1/3)) + ((I/2)*((I*b*(c + d*x))/d)^(1/3)*Gamma[2/3, (I*b*(c +
 d*x))/d])/(b*E^(I*(a - (b*c)/d))*(c + d*x)^(1/3))))/d - (3*Sin[a + b*x])/
(d*(c + d*x)^(1/3))))/(4*d)
```

3.74. $\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.74.4 Maple [F]

$$\int \frac{\cos(bx+a)}{(dx+c)^{\frac{7}{3}}}dx$$

input
```
int(cos(b*x+a)/(d*x+c)^(7/3),x)
```

output
```
int(cos(b*x+a)/(d*x+c)^(7/3),x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.42

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx =$$
$$-\frac{3\left(3\left((i\,bd^2x^2+2i\,bcdx+i\,bc^2)\cos\left(-\frac{bc-ad}{d}\right)+(bd^2x^2+2\,bcdx+bc^2)\sin\left(-\frac{bc-ad}{d}\right)\right)\left(\frac{i\,b}{d}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},\frac{i\,bdx+i\,bc}{d}\right)+\cdots\right.}{}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="fricas")
```

output
```
-3/8*(3*((I*b*d^2*x^2 + 2*I*b*c*d*x + I*b*c^2)*cos(-(b*c - a*d)/d) + (b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sin(-(b*c - a*d)/d))*(I*b/d)^(1/3)*gamma(2/3, (
I*b*d*x + I*b*c)/d) + 3*((-I*b*d^2*x^2 - 2*I*b*c*d*x - I*b*c^2)*cos(-(b*c
- a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(-(b*c - a*d)/d))*(-I*b/d)^
(1/3)*gamma(2/3, (-I*b*d*x - I*b*c)/d) + 2*(d*x + c)^(2/3)*(d*cos(b*x + a)
 - 3*(b*d*x + b*c)*sin(b*x + a)))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

3.74.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \int \frac{\cos(a+bx)}{(c+dx)^{\frac{7}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)**(7/3),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x)**(7/3), x)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.75

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx =$$
$$-\frac{\left(\left((i\sqrt{3}-1)\Gamma\left(-\frac{4}{3},\frac{i\,(dx+c)b}{d}\right)+(-i\sqrt{3}-1)\Gamma\left(-\frac{4}{3},-\frac{i\,(dx+c)b}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+\left((\sqrt{3}+i)\Gamma\left(-\frac{4}{3},\frac{i\,(dx+c)b}{d}\right.\right.\cdots}{4\,(dx+c)^{\frac{4}{3}}d}$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="maxima")
```

output
```
-1/4*(((I*sqrt(3) - 1)*gamma(-4/3, I*(d*x + c)*b/d) + (-I*sqrt(3) - 1)*gam
ma(-4/3, -I*(d*x + c)*b/d))*cos(-(b*c - a*d)/d) + ((sqrt(3) + I)*gamma(-4/
3, I*(d*x + c)*b/d) + (sqrt(3) - I)*gamma(-4/3, -I*(d*x + c)*b/d))*sin(-(b
*c - a*d)/d))*((d*x + c)*b/d)^(4/3)/((d*x + c)^(4/3)*d)
```

3.74.8 Giac [F]

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \int \frac{\cos(bx+a)}{(dx+c)^{\frac{7}{3}}}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x+c)^(7/3),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x + c)^(7/3), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{(c+dx)^{7/3}}\,dx = \int \frac{\cos(a+b\,x)}{(c+d\,x)^{7/3}}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x)^(7/3),x)
```

output
```
int(cos(a + b*x)/(c + d*x)^(7/3), x)
```

3.75 $\int x\sqrt{\cos(a+bx)}\,dx$

3.75.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x\sqrt{\cos(a+bx)}\,dx = \text{Int}\left(x\sqrt{\cos(a+bx)},x\right)$$

output
```
Unintegrable(x*cos(b*x+a)^(1/2),x)
```

3.75.2 Mathematica [N/A]

Not integrable

Time = 26.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos(a+bx)}\,dx$$

input
```
Integrate[x*Sqrt[Cos[a + b*x]],x]
```

output
```
Integrate[x*Sqrt[Cos[a + b*x]], x]
```

3.75.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3807}$$

$$\int x\sqrt{\cos(a+bx)}dx$$

input
```
Int[x*Sqrt[Cos[a + b*x]],x]
```

output
```
$Aborted
```

3.75.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.75.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x\left(\sqrt{\cos}(bx+a)\right)dx$$

input
```
int(x*cos(b*x+a)^(1/2),x)
```

output
```
int(x*cos(b*x+a)^(1/2),x)
```

3.75.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\cos(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.75.6 Sympy [N/A]

Not integrable

Time = 6.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos\left(a+bx\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)**(1/2),x)
```

output
```
Integral(x*sqrt(cos(a + b*x)), x)
```

3.75.7 Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos(bx+a)}\,dx$$

input
```
integrate(x*cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cos(b*x + a)), x)
```

3.75.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\sqrt{\cos(bx+a)}\,dx$$

input
```
integrate(x*cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cos(b*x + a)), x)
```

3.75.9 Mupad [N/A]

Not integrable

Time = 13.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\sqrt{\cos(a+bx)}\,dx = \int x\,\sqrt{\cos(a+b\,x)}\,dx$$

input
```
int(x*cos(a + b*x)^(1/2),x)
```

output
```
int(x*cos(a + b*x)^(1/2), x)
```

3.76 $\int \sqrt{\cos(a+bx)}\,dx$

3.76.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

output
```
2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*
b*x),2^(1/2))/b
```

3.76.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

input
```
Integrate[Sqrt[Cos[a + b*x]],x]
```

output
```
(2*EllipticE[(a + b*x)/2, 2])/b
```

3.76.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

input
```
Int[Sqrt[Cos[a + b*x]],x]
```

output
```
(2*EllipticE[(a + b*x)/2, 2])/b
```

3.76.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(42) = 84.

Time = 1.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 8.31

method	result
default	$\frac{2\sqrt{\left(-1+2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,E\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{-2\left(\sin^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{\left(\mathrm{e}^{2i(bx+a)}+1\right)\mathrm{e}^{-i(bx+a)}}}{b}-\frac{i\left(-\frac{2\left(\mathrm{e}^{2i(bx+a)}+1\right)}{\sqrt{\left(\mathrm{e}^{2i(bx+a)}+1\right)\mathrm{e}^{i(bx+a)}}}+\frac{i\sqrt{-i\left(\mathrm{e}^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{i(bx+a)}-i\right)}\,\sqrt{i\mathrm{e}^{i(bx+a)}}\,\left(-2iE\left(\sqrt{-i(\mathrm{e}\cdots\right.\right.}{\sqrt{\mathrm{e}^{3i(bx+a)}+\mathrm{e}^{i(bx+\cdots}}}\right.}{b(\mathrm{e}\cdots}$

input
```
int(cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/
2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*
a),2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b
*x+1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.76.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int \sqrt{\cos(a+bx)}\,dx$$
$$=\frac{i\sqrt{2}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\text{weierstrassZe}\cdots}{b}$$

input
```
integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a)
+ I*sin(b*x + a))) - I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

3.76.6 Sympy [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos{(a+bx)}}\,dx$$

input
```
integrate(cos(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(cos(a + b*x)), x)
```

3.76.7 Maxima [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos{(bx+a)}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(b*x + a)), x)
```

3.76.8 Giac [F]

$$\int \sqrt{\cos(a+bx)}\,dx = \int \sqrt{\cos{(bx+a)}}\,dx$$

input
```
integrate(cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(b*x + a)), x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2\,\mathrm{E}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

input
```
int(cos(a + b*x)^(1/2),x)
```

output
```
(2*ellipticE(a/2 + (b*x)/2, 2))/b
```

3.77 $\int \frac{\sqrt{\cos(a+bx)}}{x} dx$

3.77.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\sqrt{\cos(a+bx)}}{x} dx = \text{Int}\left(\frac{\sqrt{\cos(a+bx)}}{x}, x\right)$$

output
```
Unintegrable(cos(b*x+a)^(1/2)/x,x)
```

3.77.2 Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\cos(a+bx)}}{x} dx = \int \frac{\sqrt{\cos(a+bx)}}{x} dx$$

input
```
Integrate[Sqrt[Cos[a + b*x]]/x,x]
```

output
```
Integrate[Sqrt[Cos[a + b*x]]/x, x]
```

3.77.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cos(a+bx)}}{x} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}{x} dx$$
$$\Big\downarrow \; 3807$$
$$\int \frac{\sqrt{\cos(a+bx)}}{x} dx$$

input
```
Int[Sqrt[Cos[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.77.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.77.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\cos}(bx + a)}{x} dx$$

input
```
int(cos(b*x+a)^(1/2)/x,x)
```

output
```
int(cos(b*x+a)^(1/2)/x,x)
```

3.77.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\cos(a + bx)}}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.77.6 Sympy [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\cos(a + bx)}}{x} dx = \int \frac{\sqrt{\cos (a + bx)}}{x} dx$$

input
```
integrate(cos(b*x+a)**(1/2)/x,x)
```

output
```
Integral(sqrt(cos(a + b*x))/x, x)
```

3.77.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos(bx+a)}}{x}\,dx$$

input
```
integrate(cos(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(cos(b*x + a))/x, x)
```

3.77.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos(bx+a)}}{x}\,dx$$

input
```
integrate(cos(b*x+a)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(cos(b*x + a))/x, x)
```

3.77.9 Mupad [N/A]

Not integrable

Time = 13.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cos(a+bx)}}{x}\,dx = \int \frac{\sqrt{\cos(a+b\,x)}}{x}\,dx$$

input
```
int(cos(a + b*x)^(1/2)/x,x)
```

output
```
int(cos(a + b*x)^(1/2)/x, x)
```

3.78 $\int x \cos^{\frac{3}{2}}(a+bx)\,dx$

3.78.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2} + \frac{2x\sqrt{\cos(a+bx)}\sin(a+bx)}{3b} + \frac{1}{3}\text{Int}\left(\frac{x}{\sqrt{\cos(a+bx)}},x\right)$$

output
```
4/9*cos(b*x+a)^(3/2)/b^2+2/3*x*sin(b*x+a)*cos(b*x+a)^(1/2)/b+1/3*Unintegra
ble(x/cos(b*x+a)^(1/2),x)
```

3.78.2 Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \int x \cos^{\frac{3}{2}}(a+bx)\,dx$$

input
```
Integrate[x*Cos[a + b*x]^(3/2),x]
```

output
```
Integrate[x*Cos[a + b*x]^(3/2), x]
```

3.78.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3791, 3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{1}{3}\int\frac{x}{\sqrt{\cos(a+bx)}}dx+\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int\frac{x}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3807}$$

$$\frac{1}{3}\int\frac{x}{\sqrt{\cos(a+bx)}}dx+\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

input
```
Int[x*Cos[a + b*x]^(3/2),x]
```

output
```
$Aborted
```

3.78.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.78.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int x\left(\cos^{\frac{3}{2}}(bx+a)\right)dx$$

input
```
int(x*cos(b*x+a)^(3/2),x)
```

output
```
int(x*cos(b*x+a)^(3/2),x)
```

3.78.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.78.6 Sympy [N/A]

Not integrable

Time = 87.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx=\int x\cos^{\frac{3}{2}}\left(a+bx\right)dx$$

input
```
integrate(x*cos(b*x+a)**(3/2),x)
```

output
```
Integral(x*cos(a + b*x)**(3/2), x)
```

3.78.7 Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx=\int x\cos\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)^(3/2), x)
```

3.78.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x\cos^{\frac{3}{2}}(a+bx)\,dx=\int x\cos\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)^(3/2), x)
```

3.78.9 Mupad [N/A]

Not integrable

Time = 13.73 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x \cos^{\frac{3}{2}}(a+bx)\,dx = \int x \cos\left(a+b\,x\right)^{3/2}\,dx$$

input
```
int(x*cos(a + b*x)^(3/2),x)
```

output
```
int(x*cos(a + b*x)^(3/2), x)
```

3.79 $\int \cos^{\frac{3}{2}}(a+bx)\,dx$

3.79.1 Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}$$

output
```
2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/
2*b*x),2^(1/2))/b+2/3*sin(b*x+a)*cos(b*x+a)^(1/2)/b
```

3.79.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\left(\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sqrt{\cos(a+bx)}\sin(a+bx)\right)}{3b}$$

input
```
Integrate[Cos[a + b*x]^(3/2),x]
```

output
```
(2*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[a + b*x]))/(3*b)
```

3.79.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

input
```
Int[Cos[a + b*x]^(3/2),x]
```

output
```
(2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/
(3*b)
```

3.79.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.79.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(62) = 124$.

Time = 2.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 4.26

method	result
default	$-\frac{2\sqrt{\left(-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\left(4\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$

input
```
int(cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*((-1+2*cos(1/2*b*x+1/2*a)^2)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b
*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(
sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos
(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(
1/2)/sin(1/2*b*x+1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.79.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{2\sqrt{\cos(bx+a)}\sin(bx+a) - i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)) + i\sqrt{2}\text{weiers}}{3b}$$

input
```
integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(cos(b*x + a))*sin(b*x + a) - I*sqrt(2)*weierstrassPInverse(-4,
 0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0,
cos(b*x + a) - I*sin(b*x + a)))/b
```

3.79.6 Sympy [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**(3/2),x)
```

output
```
Integral(cos(a + b*x)**(3/2), x)
```

3.79.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(3/2), x)
```

3.79.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \int \cos(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(3/2), x)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \cos^{\frac{3}{2}}(a+bx)\,dx = \frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{3\,b} + \frac{2\,\sqrt{\cos(a+b\,x)}\,\sin(a+b\,x)}{3\,b}$$

input
```
int(cos(a + b*x)^(3/2),x)
```

output
```
(2*ellipticF(a/2 + (b*x)/2, 2))/(3*b) + (2*cos(a + b*x)^(1/2)*sin(a + b*x)
)/(3*b)
```

3.80 $\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} \, dx$

3.80.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} \, dx = \text{Int}\left(\frac{\cos^{\frac{3}{2}}(a+bx)}{x}, x\right)$$

output
```
Unintegrable(cos(b*x+a)^(3/2)/x,x)
```

3.80.2 Mathematica [N/A]

Not integrable

Time = 7.42 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} \, dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} \, dx$$

input
```
Integrate[Cos[a + b*x]^(3/2)/x,x]
```

output
```
Integrate[Cos[a + b*x]^(3/2)/x, x]
```

3.80.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \; 3807$$

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}dx$$

input
```
Int[Cos[a + b*x]^(3/2)/x,x]
```

output
```
$Aborted
```

3.80.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.80.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cos^{\frac{3}{2}}(bx+a)}{x}dx$$

input
```
int(cos(b*x+a)^(3/2)/x,x)
```

output
```
int(cos(b*x+a)^(3/2)/x,x)
```

3.80.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(b*x+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.80.6 Sympy [N/A]

Not integrable

Time = 37.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx$$

input
```
integrate(cos(b*x+a)**(3/2)/x,x)
```

output
```
Integral(cos(a + b*x)**(3/2)/x, x)
```

3.80.7 Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos(bx+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cos(b*x+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(3/2)/x, x)
```

3.80.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos(bx+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cos(b*x+a)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(3/2)/x, x)
```

3.80.9 Mupad [N/A]

Not integrable

Time = 13.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(a+bx)}{x}\,dx = \int \frac{\cos(a+b\,x)^{3/2}}{x}\,dx$$

input
```
int(cos(a + b*x)^(3/2)/x,x)
```

output
```
int(cos(a + b*x)^(3/2)/x, x)
```

3.81 $\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx$

3.81.1 Optimal result

Integrand size = 28, antiderivative size = 42

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sqrt{\cos(a+bx)}\sin(a+bx)}{3b}$$

output
```
4/9*cos(b*x+a)^(3/2)/b^2+2/3*x*sin(b*x+a)*cos(b*x+a)^(1/2)/b
```

3.81.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\frac{\sqrt{\cos(a+bx)}\left(\frac{8\cos(a+bx)}{3b}+4x\sin(a+bx)\right)}{6b}$$

input
```
Integrate[-1/3*x/Sqrt[Cos[a + b*x]] + x*Cos[a + b*x]^(3/2),x]
```

output
```
(Sqrt[Cos[a + b*x]]*((8*Cos[a + b*x])/(3*b) + 4*x*Sin[a + b*x]))/(6*b)
```

3.81.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\cos^{\frac{3}{2}}(a+bx)-\frac{x}{3\sqrt{\cos(a+bx)}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{4\cos^{\frac{3}{2}}(a+bx)}{9b^2}+\frac{2x\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

input
```
Int[-1/3*x/Sqrt[Cos[a + b*x]] + x*Cos[a + b*x]^(3/2),x]
```

output
```
(4*Cos[a + b*x]^(3/2))/(9*b^2) + (2*x*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*
b)
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.81.4 Maple [F]

$$\int \left(x\left(\cos^{\frac{3}{2}}(bx+a)\right)-\frac{x}{3\sqrt{\cos(bx+a)}}\right)dx$$

input
```
int(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x)
```

output
```
int(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x)
```

3.81.5 Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.81.6 Sympy [F]

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\frac{\int \left(-\frac{x}{\sqrt{\cos{(a+bx)}}}\right)\,dx+\int 3x\cos^{\frac{3}{2}}{(a+bx)}\,dx}{3}$$

input
```
integrate(x*cos(b*x+a)**(3/2)-1/3*x/cos(b*x+a)**(1/2),x)
```

output
```
(Integral(-x/sqrt(cos(a + b*x)), x) + Integral(3*x*cos(a + b*x)**(3/2), x)
)/3
```

3.81.7 Maxima [F]

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\int x\cos{(bx+a)}^{\frac{3}{2}}-\frac{x}{3\sqrt{\cos{(bx+a)}}}\,dx$$

input
```
integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)^(3/2) - 1/3*x/sqrt(cos(b*x + a)), x)
```

3.81.8 Giac [F]

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\int x\cos\left(bx+a\right)^{\frac{3}{2}}-\frac{x}{3\sqrt{\cos\left(bx+a\right)}}\,dx$$

input
```
integrate(x*cos(b*x+a)^(3/2)-1/3*x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)^(3/2) - 1/3*x/sqrt(cos(b*x + a)), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{x}{3\sqrt{\cos(a+bx)}}+x\cos^{\frac{3}{2}}(a+bx)\right)\,dx=\int x\cos\left(a+b\,x\right)^{3/2}-\frac{x}{3\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

input
```
int(x*cos(a + b*x)^(3/2) - x/(3*cos(a + b*x)^(1/2)),x)
```

output
```
int(x*cos(a + b*x)^(3/2) - x/(3*cos(a + b*x)^(1/2)), x)
```

3.82 $\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} \, dx$

3.82.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} \, dx = -\frac{\cos^{\frac{3}{2}}(x)}{2x^2} + \frac{3\sqrt{\cos(x)}\sin(x)}{4x} + \frac{3}{8}\text{Int}\left(\frac{1}{x\sqrt{\cos(x)}}, x\right) - \frac{9}{8}\text{Int}\left(\frac{\cos^{\frac{3}{2}}(x)}{x}, x\right)$$

output
```
-1/2*cos(x)^(3/2)/x^2+3/4*sin(x)*cos(x)^(1/2)/x-9/8*Unintegrable(cos(x)^(3
/2)/x,x)+3/8*Unintegrable(1/x/cos(x)^(1/2),x)
```

3.82.2 Mathematica [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} \, dx = \int \frac{\cos^{\frac{3}{2}}(x)}{x^3} \, dx$$

input
```
Integrate[Cos[x]^(3/2)/x^3,x]
```

output
```
Integrate[Cos[x]^(3/2)/x^3, x]
```

3.82.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3795, 3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$-\frac{9}{8}\int \frac{\cos^{\frac{3}{2}}(x)}{x}dx+\frac{3}{8}\int \frac{1}{x\sqrt{\cos(x)}}dx-\frac{\cos^{\frac{3}{2}}(x)}{2x^2}+\frac{3\sin(x)\sqrt{\cos(x)}}{4x}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{8}\int \frac{1}{x\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx-\frac{9}{8}\int \frac{\sin\left(x+\frac{\pi}{2}\right)^{3/2}}{x}dx-\frac{\cos^{\frac{3}{2}}(x)}{2x^2}+\frac{3\sin(x)\sqrt{\cos(x)}}{4x}$$

$$\Big\downarrow \text{3807}$$

$$-\frac{9}{8}\int \frac{\cos^{\frac{3}{2}}(x)}{x}dx+\frac{3}{8}\int \frac{1}{x\sqrt{\cos(x)}}dx-\frac{\cos^{\frac{3}{2}}(x)}{2x^2}+\frac{3\sin(x)\sqrt{\cos(x)}}{4x}$$

input
```
Int[Cos[x]^(3/2)/x^3,x]
```

output
```
$Aborted
```

3.82.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.82.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} dx$$

input
```
int(cos(x)^(3/2)/x^3,x)
```

output
```
int(cos(x)^(3/2)/x^3,x)
```

3.82.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.82.6 Sympy [N/A]

Not integrable

Time = 92.97 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx$$

input
```
integrate(cos(x)**(3/2)/x**3,x)
```

output
```
Integral(cos(x)**(3/2)/x**3, x)
```

3.82.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos(x)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(cos(x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(cos(x)^(3/2)/x^3, x)
```

3.82.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cos(x)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(cos(x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate(cos(x)^(3/2)/x^3, x)
```

3.82.9 Mupad [N/A]

Not integrable

Time = 13.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos^{\frac{3}{2}}(x)}{x^3} \, dx = \int \frac{\cos(x)^{3/2}}{x^3} \, dx$$

input
```
int(cos(x)^(3/2)/x^3,x)
```

output
```
int(cos(x)^(3/2)/x^3, x)
```

3.83 $\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$

3.83.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \text{Int}\left(\frac{x}{\sqrt{\cos(a+bx)}},x\right)$$

output
```
Unintegrable(x/cos(b*x+a)^(1/2),x)
```

3.83.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$

input
```
Integrate[x/Sqrt[Cos[a + b*x]],x]
```

output
```
Integrate[x/Sqrt[Cos[a + b*x]], x]
```

3.83.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{x}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \; 3807$$
$$\int \frac{x}{\sqrt{\cos(a+bx)}}dx$$

input
```
Int[x/Sqrt[Cos[a + b*x]],x]
```

output
```
$Aborted
```

3.83.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.83.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{\cos\left(bx+a\right)}}dx$$

input
```
int(x/cos(b*x+a)^(1/2),x)
```

output
```
int(x/cos(b*x+a)^(1/2),x)
```

3.83.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.83.6 Sympy [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos\left(a+bx\right)}}\,dx$$

input
```
integrate(x/cos(b*x+a)**(1/2),x)
```

output
```
Integral(x/sqrt(cos(a + b*x)), x)
```

3.83.7 Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos\left(bx+a\right)}}\,dx$$

input
```
integrate(x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(cos(b*x + a)), x)
```

3.83.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos\left(bx+a\right)}}\,dx$$

input
```
integrate(x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(cos(b*x + a)), x)
```

3.83.9 Mupad [N/A]

Not integrable

Time = 13.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x}{\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

input
```
int(x/cos(a + b*x)^(1/2),x)
```

output
```
int(x/cos(a + b*x)^(1/2), x)
```

3.84 $\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$

3.84.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

output
```
2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*
b*x),2^(1/2))/b
```

3.84.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

input
```
Integrate[1/Sqrt[Cos[a + b*x]],x]
```

output
```
(2*EllipticF[(a + b*x)/2, 2])/b
```

3.84.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b}$$

input
```
Int[1/Sqrt[Cos[a + b*x]],x]
```

output
```
(2*EllipticF[(a + b*x)/2, 2])/b
```

3.84.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.84.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{2\ \mathrm{am}^{-1}\left(\frac{bx}{2}+\frac{a}{2} \middle\vert \sqrt{2}\right)}{b}$	18

input
```
int(1/cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*InverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

3.84.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$$
$$= \frac{-i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos{(bx+a)}+i\sin{(bx+a)})+i\sqrt{2}\text{weierstrassPInverse}(-4,0,\cos{(bx-}}{b}$$

input
```
integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*
sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

3.84.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos{(a+bx)}}}\,dx$$

input
```
integrate(1/cos(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(cos(a + b*x)), x)
```

3.84.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos{(bx+a)}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(cos(b*x + a)), x)
```

3.84.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cos{(bx+a)}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(cos(b*x + a)), x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 13.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

input
```
int(1/cos(a + b*x)^(1/2),x)
```

output
```
(2*ellipticF(a/2 + (b*x)/2, 2))/b
```

3.85 $\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$

3.85.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{\cos(a+bx)}},x\right)$$

output
```
Unintegrable(1/x/cos(b*x+a)^(1/2),x)
```

3.85.2 Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[Cos[a + b*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[Cos[a + b*x]]), x]
```

3.85.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{x\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}dx$$

input
```
Int[1/(x*Sqrt[Cos[a + b*x]]),x]
```

output
```
$Aborted
```

3.85.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.85.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\sqrt{\cos\left(bx+a\right)}}dx$$

input
```
int(1/x/cos(b*x+a)^(1/2),x)
```

output
```
int(1/x/cos(b*x+a)^(1/2),x)
```

3.85.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.85.6 Sympy [N/A]

Not integrable

Time = 10.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos\left(a+bx\right)}}\,dx$$

input
```
integrate(1/x/cos(b*x+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(cos(a + b*x))), x)
```

3.85.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos(bx+a)}}\,dx$$

input
```
integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(cos(b*x + a))), x)
```

3.85.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\sqrt{\cos(bx+a)}}\,dx$$

input
```
integrate(1/x/cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(cos(b*x + a))), x)
```

3.85.9 Mupad [N/A]

Not integrable

Time = 13.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cos(a+bx)}}\,dx = \int \frac{1}{x\,\sqrt{\cos(a+b\,x)}}\,dx$$

input
```
int(1/(x*cos(a + b*x)^(1/2)),x)
```

output
```
int(1/(x*cos(a + b*x)^(1/2)), x)
```

3.86 $\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)} dx$

3.86.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)} dx = \frac{4\sqrt{\cos(a+bx)}}{b^2} + \frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}} - \text{Int}\left(x\sqrt{\cos(a+bx)}, x\right)$$

output
```
2*x*sin(b*x+a)/b/cos(b*x+a)^(1/2)+4*cos(b*x+a)^(1/2)/b^2-Unintegrable(x*co
s(b*x+a)^(1/2),x)
```

3.86.2 Mathematica [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)} dx = \int \frac{x}{\cos^{\frac{3}{2}}(a+bx)} dx$$

input
```
Integrate[x/Cos[a + b*x]^(3/2),x]
```

output
```
Integrate[x/Cos[a + b*x]^(3/2), x]
```

3.86.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3796, 3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x}{\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3796}$$

$$-\int x\sqrt{\cos(a+bx)}dx+\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\int x\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

$$\Big\downarrow \text{ 3807}$$

$$-\int x\sqrt{\cos(a+bx)}dx+\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

input
```
Int[x/Cos[a + b*x]^(3/2),x]
```

output
```
$Aborted
```

3.86.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3796
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (-Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x] + Simp[(
n + 2)/(b^2*(n + 1))   Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x]) /; F
reeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.86.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x/cos(b*x+a)^(3/2),x)
```

output
```
int(x/cos(b*x+a)^(3/2),x)
```

3.86.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.86.6 Sympy [N/A]

Not integrable

Time = 14.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(x/cos(b*x+a)**(3/2),x)
```

output
```
Integral(x/cos(a + b*x)**(3/2), x)
```

3.86.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/cos(b*x + a)^(3/2), x)
```

3.86.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/cos(b*x + a)^(3/2), x)
```

3.86.9 Mupad [N/A]

Not integrable

Time = 13.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{\cos^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{x}{\cos\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int(x/cos(a + b*x)^(3/2),x)
```

output
```
int(x/cos(a + b*x)^(3/2), x)
```

3.87 $\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

3.87.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b} + \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

output
```
-2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2
*b*x),2^(1/2))/b+2*sin(b*x+a)/b/cos(b*x+a)^(1/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b} + \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

input
```
Integrate[Cos[a + b*x]^(-3/2),x]
```

output
```
(-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

3.87.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

input
```
Int[Cos[a + b*x]^(-3/2),x]
```

output
```
(-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

3.87.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(62) = 124$.

Time = 1.51 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.79

method	result
default	$-\frac{2\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\right.\right.}}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,b}$

input
```
int(1/cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-2*cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(
1/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*
a)^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)*Ellipti
cE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a
)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(-1+2*cos(1/2*b*x+1/2*a)^2)^(1/2)/b
```

3.87.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.45

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$
$$= \frac{-i\sqrt{2}\cos(bx+a)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a)))+i\sqrt{}}{}$$

input
```
integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(2)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*cos(b*x + a)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*sqrt
(cos(b*x + a))*sin(b*x + a))/(b*cos(b*x + a))
```

3.87.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/cos(b*x+a)**(3/2),x)
```

output
```
Integral(cos(a + b*x)**(-3/2), x)
```

3.87.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{1}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)^(-3/2), x)
```

3.87.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx=\int \frac{1}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^(-3/2), x)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(a+b\,x\right)^2\right)}{b\,\sqrt{\cos\left(a+b\,x\right)}\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

input
```
int(1/cos(a + b*x)^(3/2),x)
```

output
```
(2*sin(a + b*x)*hypergeom([-1/4, 1/2], 3/4, cos(a + b*x)^2))/(b*cos(a + b*
x)^(1/2)*(sin(a + b*x)^2)^(1/2))
```

3.88 $\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)} dx$

3.88.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)} dx = \text{Int}\left(\frac{1}{x \cos^{\frac{3}{2}}(a+bx)}, x\right)$$

output
```
Unintegrable(1/x/cos(b*x+a)^(3/2),x)
```

3.88.2 Mathematica [N/A]

Not integrable

Time = 10.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)} dx = \int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)} dx$$

input
```
Integrate[1/(x*Cos[a + b*x]^(3/2)),x]
```

output
```
Integrate[1/(x*Cos[a + b*x]^(3/2)), x]
```

3.88.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}dx$$

input
```
Int[1/(x*Cos[a + b*x]^(3/2)),x]
```

output
```
$Aborted
```

3.88.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.88.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x \cos \left(bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/cos(b*x+a)^(3/2),x)
```

output
```
int(1/x/cos(b*x+a)^(3/2),x)
```

3.88.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.88.6 Sympy [N/A]

Not integrable

Time = 36.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x \cos^{\frac{3}{2}}{\left(a+bx\right)}}\,dx$$

input
```
integrate(1/x/cos(b*x+a)**(3/2),x)
```

output
```
Integral(1/(x*cos(a + b*x)**(3/2)), x)
```

3.88.7 Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x\cos{(bx+a)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cos(b*x + a)^(3/2)), x)
```

3.88.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x\cos{(bx+a)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*cos(b*x + a)^(3/2)), x)
```

3.88.9 Mupad [N/A]

Not integrable

Time = 13.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{x\,{\cos\left(a+b\,x\right)}^{3/2}}\,dx$$

input
```
int(1/(x*cos(a + b*x)^(3/2)),x)
```

output
```
int(1/(x*cos(a + b*x)^(3/2)), x)
```

3.89 $\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right) dx$

3.89.1 Optimal result

Integrand size = 25, antiderivative size = 38

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right) dx = \frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

output
```
2*x*sin(b*x+a)/b/cos(b*x+a)^(1/2)+4*cos(b*x+a)^(1/2)/b^2
```

3.89.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right) dx = \frac{2(2\cos(a+bx)+bx\sin(a+bx))}{b^2\sqrt{\cos(a+bx)}}$$

input
```
Integrate[x/Cos[a + b*x]^(3/2) + x*Sqrt[Cos[a + b*x]],x]
```

output
```
(2*(2*Cos[a + b*x] + b*x*Sin[a + b*x]))/(b^2*Sqrt[Cos[a + b*x]])
```

3.89.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right) dx$$
$$\Big\downarrow \ 2009$$
$$\frac{4\sqrt{\cos(a+bx)}}{b^2}+\frac{2x\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

input
```
Int[x/Cos[a + b*x]^(3/2) + x*Sqrt[Cos[a + b*x]],x]
```

output
```
(4*Sqrt[Cos[a + b*x]])/b^2 + (2*x*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.89.4 Maple [F]

$$\int \left(\frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}}+x\left(\sqrt{\cos}(bx+a)\right)\right) dx$$

input
```
int(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x)
```

output
```
int(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x)
```

3.89.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.89.6 Sympy [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right)\,dx=\int \frac{x(\cos^2{(a+bx)}+1)}{\cos^{\frac{3}{2}}{(a+bx)}}\,dx$$

input
```
integrate(x/cos(b*x+a)**(3/2)+x*cos(b*x+a)**(1/2),x)
```

output
```
Integral(x*(cos(a + b*x)**2 + 1)/cos(a + b*x)**(3/2), x)
```

3.89.7 Maxima [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right)\,dx=\int x\sqrt{\cos{(bx+a)}}+\frac{x}{\cos{(bx+a)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cos(b*x + a)) + x/cos(b*x + a)^(3/2), x)
```

3.89.8 Giac [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right)\,dx=\int x\sqrt{\cos\left(bx+a\right)}+\frac{x}{\cos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/cos(b*x+a)^(3/2)+x*cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cos(b*x + a)) + x/cos(b*x + a)^(3/2), x)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 13.83 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(a+bx)}+x\sqrt{\cos(a+bx)}\right)\,dx = \frac{2\,\sqrt{\cos\left(a+b\,x\right)}\,\left(2\,\cos\left(2\,a+2\,b\,x\right)+b\,x\,\sin\left(2\,a+2\,b\,x\right)+2\right)}{b^2\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

input
```
int(x*cos(a + b*x)^(1/2) + x/cos(a + b*x)^(3/2),x)
```

output
```
(2*cos(a + b*x)^(1/2)*(2*cos(2*a + 2*b*x) + b*x*sin(2*a + 2*b*x) + 2))/(b^
2*(cos(2*a + 2*b*x) + 1))
```

3.90 $\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)}+x\sqrt{\cos(x)}\right) dx$

3.90.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)}+x\sqrt{\cos(x)}\right) dx = 4\sqrt{\cos(x)}+\frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

output
```
2*x*sin(x)/cos(x)^(1/2)+4*cos(x)^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)}+x\sqrt{\cos(x)}\right) dx = \frac{2(2\cos(x)+x\sin(x))}{\sqrt{\cos(x)}}$$

input
```
Integrate[x/Cos[x]^(3/2) + x*Sqrt[Cos[x]],x]
```

output
```
(2*(2*Cos[x] + x*Sin[x]))/Sqrt[Cos[x]]
```

3.90.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)}+x\sqrt{\cos(x)}\right)\,dx$$
$$\Big\downarrow \text{2009}$$
$$4\sqrt{\cos(x)}+\frac{2x\sin(x)}{\sqrt{\cos(x)}}$$

input
```
Int[x/Cos[x]^(3/2) + x*Sqrt[Cos[x]],x]
```

output
```
4*Sqrt[Cos[x]] + (2*x*Sin[x])/Sqrt[Cos[x]]
```

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.90.4 Maple [F]

$$\int \left(\frac{x}{\cos\left(x\right)^{\frac{3}{2}}}+x\left(\sqrt{\cos}\left(x\right)\right)\right)dx$$

input
```
int(x/cos(x)^(3/2)+x*cos(x)^(1/2),x)
```

output
```
int(x/cos(x)^(3/2)+x*cos(x)^(1/2),x)
```

3.90.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.90.6 Sympy [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)}\right) dx = \int \frac{x(\cos^2(x) + 1)}{\cos^{\frac{3}{2}}(x)}\, dx$$

input
```
integrate(x/cos(x)**(3/2)+x*cos(x)**(1/2),x)
```

output
```
Integral(x*(cos(x)**2 + 1)/cos(x)**(3/2), x)
```

3.90.7 Maxima [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)}\right) dx = \int x\sqrt{\cos(x)} + \frac{x}{\cos(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cos(x)) + x/cos(x)^(3/2), x)
```

3.90.8 Giac [F]

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)}\right) dx = \int x\sqrt{\cos(x)} + \frac{x}{\cos(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x/cos(x)^(3/2)+x*cos(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cos(x)) + x/cos(x)^(3/2), x)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(\frac{x}{\cos^{\frac{3}{2}}(x)} + x\sqrt{\cos(x)}\right) dx = \frac{4\cos(x) + 2x\sin(x)}{\sqrt{\cos(x)}}$$

input
```
int(x*cos(x)^(1/2) + x/cos(x)^(3/2),x)
```

output
```
(4*cos(x) + 2*x*sin(x))/cos(x)^(1/2)
```

3.91 $\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right)\,dx$

3.91.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right)\,dx=-\frac{4}{3\sqrt{\cos(x)}}+\frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}$$

output
```
2/3*x*sin(x)/cos(x)^(3/2)-4/3/cos(x)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right)\,dx=-\frac{8-4x\tan(x)}{6\sqrt{\cos(x)}}$$

input
```
Integrate[x/Cos[x]^(5/2) - x/(3*Sqrt[Cos[x]]),x]
```

output
```
-1/6*(8 - 4*x*Tan[x])/Sqrt[Cos[x]]
```

3.91.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{2x\sin(x)}{3\cos^{\frac{3}{2}}(x)}-\frac{4}{3\sqrt{\cos(x)}}$$

input
```
Int[x/Cos[x]^(5/2) - x/(3*Sqrt[Cos[x]]),x]
```

output
```
-4/(3*Sqrt[Cos[x]]) + (2*x*Sin[x])/(3*Cos[x]^(3/2))
```

3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.91.4 Maple [F]

$$\int \left(\frac{x}{\cos\left(x\right)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\cos\left(x\right)}}\right)dx$$

input
```
int(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x)
```

output
```
int(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}}\right) dx = \frac{2\left(x\sin\left(x\right) - 2\,\cos\left(x\right)\right)}{3\,\cos\left(x\right)^{\frac{3}{2}}}$$

input
```
integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x*sin(x) - 2*cos(x))/cos(x)^(3/2)
```

3.91.6 Sympy [F]

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}}\right) dx = -\frac{\int \left(-\frac{3x}{\cos^{\frac{5}{2}}\left(x\right)}\right) dx + \int \frac{x}{\sqrt{\cos\left(x\right)}}\, dx}{3}$$

input
```
integrate(x/cos(x)**(5/2)-1/3*x/cos(x)**(1/2),x)
```

output
```
-(Integral(-3*x/cos(x)**(5/2), x) + Integral(x/sqrt(cos(x)), x))/3
```

3.91.7 Maxima [F]

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cos(x)}}\right) dx = \int -\frac{x}{3\,\sqrt{\cos\left(x\right)}} + \frac{x}{\cos\left(x\right)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x/sqrt(cos(x)) + x/cos(x)^(5/2), x)
```

3.91.8 Giac [F]

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right)\,dx=\int -\frac{x}{3\,\sqrt{\cos\left(x\right)}}+\frac{x}{\cos\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/cos(x)^(5/2)-1/3*x/cos(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x/sqrt(cos(x)) + x/cos(x)^(5/2), x)
```

3.91.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(\frac{x}{\cos^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cos(x)}}\right)\,dx=-\frac{4\,\cos\left(x\right)-2\,x\,\sin\left(x\right)}{3\cos\left(x\right)^{3/2}}$$

input
```
int(x/cos(x)^(5/2) - x/(3*cos(x)^(1/2)),x)
```

output
```
-(4*cos(x) - 2*x*sin(x))/(3*cos(x)^(3/2))
```

3.92 $\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right)\,dx$

3.92.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right)\,dx=-\frac{4}{15\cos^{\frac{3}{2}}(x)}+\frac{12\sqrt{\cos(x)}}{5}+\frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)}+\frac{6x\sin(x)}{5\sqrt{\cos(x)}}$$

output
```
-4/15/cos(x)^(3/2)+2/5*x*sin(x)/cos(x)^(5/2)+6/5*x*sin(x)/cos(x)^(1/2)+12/
5*cos(x)^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right)\,dx=\frac{46\cos(x)+18\cos(3x)+21x\sin(x)+9x\sin(3x)}{30\cos^{\frac{5}{2}}(x)}$$

input
```
Integrate[x/Cos[x]^(7/2) + (3*x*Sqrt[Cos[x]])/5,x]
```

output
```
(46*Cos[x] + 18*Cos[3*x] + 21*x*Sin[x] + 9*x*Sin[3*x])/(30*Cos[x]^(5/2))
```

3.92.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{15\cos^{\frac{3}{2}}(x)}+\frac{12\sqrt{\cos(x)}}{5}+\frac{2x\sin(x)}{5\cos^{\frac{5}{2}}(x)}+\frac{6x\sin(x)}{5\sqrt{\cos(x)}}$$

input
```
Int[x/Cos[x]^(7/2) + (3*x*Sqrt[Cos[x]])/5,x]
```

output
```
-4/(15*Cos[x]^(3/2)) + (12*Sqrt[Cos[x]])/5 + (2*x*Sin[x])/(5*Cos[x]^(5/2))
 + (6*x*Sin[x])/(5*Sqrt[Cos[x]])
```

3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.92.4 Maple [F]

$$\int \left(\frac{x}{\cos\left(x\right)^{\frac{7}{2}}}+\frac{3x\left(\sqrt{\cos}(x)\right)}{5}\right) dx$$

input
```
int(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x)
```

output
```
int(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x)
```

3.92.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right)\,dx=\text{Timed out}$$

input
```
integrate(x/cos(x)**(7/2)+3/5*x*cos(x)**(1/2),x)
```

output
```
Timed out
```

3.92.7 Maxima [F]

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\cos\left(x\right)}+\frac{x}{\cos\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x*sqrt(cos(x)) + x/cos(x)^(7/2), x)
```

3.92.8 Giac [F]

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\cos\left(x\right)}+\frac{x}{\cos\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/cos(x)^(7/2)+3/5*x*cos(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x*sqrt(cos(x)) + x/cos(x)^(7/2), x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 13.76 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \left(\frac{x}{\cos^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cos(x)}\right)\,dx=\frac{36\cos\left(x\right)^3+18\,x\,\sin\left(x\right)\,\cos\left(x\right)^2-4\,\cos\left(x\right)+6\,x\,\sin\left(x\right)}{15\cos\left(x\right)^{5/2}}$$

input
```
int((3*x*cos(x)^(1/2))/5 + x/cos(x)^(7/2),x)
```

output
```
(36*cos(x)^3 - 4*cos(x) + 6*x*sin(x) + 18*x*cos(x)^2*sin(x))/(15*cos(x)^(5
/2))
```

3.93 $\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)}+x^2\sqrt{\cos(x)}\right)\,dx$

3.93.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)}+x^2\sqrt{\cos(x)}\right)\,dx = 8x\sqrt{\cos(x)}-16E\left(\frac{x}{2}\middle|2\right)+\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}$$

output
```
-16*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/2))+2*x^2*si
n(x)/cos(x)^(1/2)+8*x*cos(x)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)}+x^2\sqrt{\cos(x)}\right)\,dx = 2\left(-8E\left(\frac{x}{2}\middle|2\right)+\frac{x(4\cos(x)+x\sin(x))}{\sqrt{\cos(x)}}\right)$$

input
```
Integrate[x^2/Cos[x]^(3/2) + x^2*Sqrt[Cos[x]],x]
```

output
```
2*(-8*EllipticE[x/2, 2] + (x*(4*Cos[x] + x*Sin[x]))/Sqrt[Cos[x]])
```

3.93.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)}+x^2\sqrt{\cos(x)}\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2x^2\sin(x)}{\sqrt{\cos(x)}}+8x\sqrt{\cos(x)}-16E\left(\frac{x}{2}\middle|2\right)$$

input
```
Int[x^2/Cos[x]^(3/2) + x^2*Sqrt[Cos[x]],x]
```

output
```
8*x*Sqrt[Cos[x]] - 16*EllipticE[x/2, 2] + (2*x^2*Sin[x])/Sqrt[Cos[x]]
```

3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.93.4 Maple [F]

$$\int \left(\frac{x^2}{\cos\left(x\right)^{\frac{3}{2}}}+x^2\left(\sqrt{\cos}\left(x\right)\right)\right) dx$$

input
```
int(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x)
```

output
```
int(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x)
```

3.93.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2\sqrt{\cos(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.93.6 Sympy [F]

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2\sqrt{\cos(x)}\right) dx = \int \frac{x^2(\cos^2(x) + 1)}{\cos^{\frac{3}{2}}(x)}\, dx$$

input
```
integrate(x**2/cos(x)**(3/2)+x**2*cos(x)**(1/2),x)
```

output
```
Integral(x**2*(cos(x)**2 + 1)/cos(x)**(3/2), x)
```

3.93.7 Maxima [F]

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2\sqrt{\cos(x)}\right) dx = \int x^2\sqrt{\cos(x)} + \frac{x^2}{\cos(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(cos(x)) + x^2/cos(x)^(3/2), x)
```

3.93.8 Giac [F]

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2\sqrt{\cos(x)}\right) dx = \int x^2\sqrt{\cos(x)} + \frac{x^2}{\cos(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2/cos(x)^(3/2)+x^2*cos(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(cos(x)) + x^2/cos(x)^(3/2), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\cos^{\frac{3}{2}}(x)} + x^2\sqrt{\cos(x)}\right) dx = \int x^2\,\sqrt{\cos(x)} + \frac{x^2}{\cos(x)^{3/2}}\, dx$$

input
```
int(x^2*cos(x)^(1/2) + x^2/cos(x)^(3/2),x)
```

output
```
int(x^2*cos(x)^(1/2) + x^2/cos(x)^(3/2), x)
```

3.94 $\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)\,dx$

3.94.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)\,dx=\frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}$$

output
```
4/9/sec(x)^(3/2)+2/3*x*sin(x)/sec(x)^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)\,dx=\frac{2(2+3x\tan(x))}{9\sec^{\frac{3}{2}}(x)}$$

input
```
Integrate[x/Sec[x]^(3/2) - (x*Sqrt[Sec[x]])/3,x]
```

output
```
(2*(2 + 3*x*Tan[x]))/(9*Sec[x]^(3/2))
```

3.94.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{9\sec^{\frac{3}{2}}(x)}+\frac{2x\sin(x)}{3\sqrt{\sec(x)}}$$

input
```
Int[x/Sec[x]^(3/2) - (x*Sqrt[Sec[x]])/3,x]
```

output
```
4/(9*Sec[x]^(3/2)) + (2*x*Sin[x])/(3*Sqrt[Sec[x]])
```

3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.94.4 Maple [F]

$$\int \left(\frac{x}{\sec\left(x\right)^{\frac{3}{2}}}-\frac{x\left(\sqrt{\sec}(x)\right)}{3}\right)dx$$

input
```
int(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x)
```

output
```
int(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x)
```

3.94.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.94.6 Sympy [F]

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)\,dx=-\frac{\int\left(-\frac{3x}{\sec^{\frac{3}{2}}(x)}\right)\,dx+\int x\sqrt{\sec(x)}\,dx}{3}$$

input
```
integrate(x/sec(x)**(3/2)-1/3*x*sec(x)**(1/2),x)
```

output
```
-(Integral(-3*x/sec(x)**(3/2), x) + Integral(x*sqrt(sec(x)), x))/3
```

3.94.7 Maxima [F]

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\sec(x)}\right)\,dx=\int -\frac{1}{3}\,x\sqrt{\sec(x)}+\frac{x}{\sec(x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x*sqrt(sec(x)) + x/sec(x)^(3/2), x)
```

3.94.8 Giac [F]

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x\sqrt{\sec(x)}\right) dx = \int -\frac{1}{3}x\sqrt{\sec(x)} + \frac{x}{\sec(x)^{\frac{3}{2}}} dx$$

input
```
integrate(x/sec(x)^(3/2)-1/3*x*sec(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x*sqrt(sec(x)) + x/sec(x)^(3/2), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x\sqrt{\sec(x)}\right) dx = -\int \frac{x\sqrt{\frac{1}{\cos(x)}}}{3} - \frac{x}{\left(\frac{1}{\cos(x)}\right)^{3/2}} dx$$

input
```
int(x/(1/cos(x))^(3/2) - (x*(1/cos(x))^(1/2))/3,x)
```

output
```
-int((x*(1/cos(x))^(1/2))/3 - x/(1/cos(x))^(3/2), x)
```

3.95 $\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)\,dx$

3.95.1 Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)\,dx=\frac{4}{25\sec^{\frac{5}{2}}(x)}+\frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}$$

output
```
4/25/sec(x)^(5/2)+2/5*x*sin(x)/sec(x)^(3/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)\,dx=\frac{2(2+5x\tan(x))}{25\sec^{\frac{5}{2}}(x)}$$

input
```
Integrate[x/Sec[x]^(5/2) - (3*x)/(5*Sqrt[Sec[x]]),x]
```

output
```
(2*(2 + 5*x*Tan[x]))/(25*Sec[x]^(5/2))
```

3.95.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{4}{25\sec^{\frac{5}{2}}(x)}+\frac{2x\sin(x)}{5\sec^{\frac{3}{2}}(x)}$$

input
```
Int[x/Sec[x]^(5/2) - (3*x)/(5*Sqrt[Sec[x]]),x]
```

output
```
4/(25*Sec[x]^(5/2)) + (2*x*Sin[x])/(5*Sec[x]^(3/2))
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.95.4 Maple [F]

$$\int \left(\frac{x}{\sec(x)^{\frac{5}{2}}}-\frac{3x}{5\sqrt{\sec(x)}}\right) dx$$

input
```
int(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x)
```

output
```
int(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x)
```

3.95.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.95.6 Sympy [F]

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}}\right) dx = -\frac{\int \left(-\frac{5x}{\sec^{\frac{5}{2}}(x)}\right) dx + \int \frac{3x}{\sqrt{\sec(x)}}\, dx}{5}$$

input
```
integrate(x/sec(x)**(5/2)-3/5*x/sec(x)**(1/2),x)
```

output
```
-(Integral(-5*x/sec(x)**(5/2), x) + Integral(3*x/sqrt(sec(x)), x))/5
```

3.95.7 Maxima [F]

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)} - \frac{3x}{5\sqrt{\sec(x)}}\right) dx = \int -\frac{3\,x}{5\,\sqrt{\sec(x)}} + \frac{x}{\sec(x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-3/5*x/sqrt(sec(x)) + x/sec(x)^(5/2), x)
```

3.95.8 Giac [F]

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)\,dx=\int -\frac{3\,x}{5\,\sqrt{\sec\left(x\right)}}+\frac{x}{\sec\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/sec(x)^(5/2)-3/5*x/sec(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-3/5*x/sqrt(sec(x)) + x/sec(x)^(5/2), x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\sec^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\sec(x)}}\right)\,dx=-\int \frac{3\,x}{5\,\sqrt{\frac{1}{\cos(x)}}}-\frac{x}{\left(\frac{1}{\cos(x)}\right)^{5/2}}\,dx$$

input
```
int(x/(1/cos(x))^(5/2) - (3*x)/(5*(1/cos(x))^(1/2)),x)
```

output
```
-int((3*x)/(5*(1/cos(x))^(1/2)) - x/(1/cos(x))^(5/2), x)
```

3.96 $\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx$

3.96.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = \frac{4}{49 \sec^{\frac{7}{2}}(x)} + \frac{20}{63 \sec^{\frac{3}{2}}(x)} + \frac{2x \sin(x)}{7 \sec^{\frac{5}{2}}(x)} + \frac{10x \sin(x)}{21 \sqrt{\sec(x)}}$$

output
```
4/49/sec(x)^(7/2)+20/63/sec(x)^(3/2)+2/7*x*sin(x)/sec(x)^(5/2)+10/21*x*sin
(x)/sec(x)^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\sec(x)} \right) dx = \sqrt{\sec(x)} \left(\frac{167}{882} + \frac{88}{441} \cos(2x) + \frac{1}{98} \cos(4x) + \frac{13}{42} x \sin(2x) + \frac{1}{28} x \sin(4x) \right)$$

input
```
Integrate[x/Sec[x]^(7/2) - (5*x*Sqrt[Sec[x]])/21,x]
```

output
```
Sqrt[Sec[x]]*(167/882 + (88*Cos[2*x])/441 + Cos[4*x]/98 + (13*x*Sin[2*x])/
42 + (x*Sin[4*x])/28)
```

3.96.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\sec(x)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{20}{63\sec^{\frac{3}{2}}(x)}+\frac{4}{49\sec^{\frac{7}{2}}(x)}+\frac{2x\sin(x)}{7\sec^{\frac{5}{2}}(x)}+\frac{10x\sin(x)}{21\sqrt{\sec(x)}}$$

input
```
Int[x/Sec[x]^(7/2) - (5*x*Sqrt[Sec[x]])/21,x]
```

output
```
4/(49*Sec[x]^(7/2)) + 20/(63*Sec[x]^(3/2)) + (2*x*Sin[x])/(7*Sec[x]^(5/2))
 + (10*x*Sin[x])/(21*Sqrt[Sec[x]])
```

3.96.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.96.4 Maple [F]

$$\int \left(\frac{x}{\sec\left(x\right)^{\frac{7}{2}}}-\frac{5x\left(\sqrt{\sec}(x)\right)}{21}\right)dx$$

input
```
int(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x)
```

output
```
int(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x)
```

3.96.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\sec(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.96.6 Sympy [F]

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\sec(x)}\right) dx = -\frac{\int \left(-\frac{21x}{\sec^{\frac{7}{2}}(x)}\right) dx + \int 5x\sqrt{\sec(x)}\, dx}{21}$$

input
```
integrate(x/sec(x)**(7/2)-5/21*x*sec(x)**(1/2),x)
```

output
```
-(Integral(-21*x/sec(x)**(7/2), x) + Integral(5*x*sqrt(sec(x)), x))/21
```

3.96.7 Maxima [F]

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\sec(x)}\right) dx = \int -\frac{5}{21}x\sqrt{\sec(x)} + \frac{x}{\sec(x)^{\frac{7}{2}}}\, dx$$

input
```
integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-5/21*x*sqrt(sec(x)) + x/sec(x)^(7/2), x)
```

3.96.8 Giac [F]

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\sec(x)}\right) dx = \int -\frac{5}{21}\,x\sqrt{\sec\left(x\right)} + \frac{x}{\sec\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/sec(x)^(7/2)-5/21*x*sec(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-5/21*x*sqrt(sec(x)) + x/sec(x)^(7/2), x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\sec^{\frac{7}{2}}(x)} - \frac{5}{21}x\sqrt{\sec(x)}\right) dx = -\int \frac{5\,x\,\sqrt{\frac{1}{\cos(x)}}}{21} - \frac{x}{\left(\frac{1}{\cos(x)}\right)^{7/2}}\,dx$$

input
```
int(x/(1/cos(x))^(7/2) - (5*x*(1/cos(x))^(1/2))/21,x)
```

output
```
-int((5*x*(1/cos(x))^(1/2))/21 - x/(1/cos(x))^(7/2), x)
```

3.97 $\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3} x^2 \sqrt{\sec (x)}\right) \, dx$

3.97.1 Optimal result

Integrand size = 24, antiderivative size = 62

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3} x^2 \sqrt{\sec (x)}\right) \, dx=\frac{8 x}{9 \sec^{\frac{3}{2}}(x)}-\frac{16}{27} \sqrt{\cos (x)} \text{EllipticF}\left(\frac{x}{2},2\right) \sqrt{\sec (x)}-\frac{16 \sin (x)}{27 \sqrt{\sec (x)}}+\frac{2 x^2 \sin (x)}{3 \sqrt{\sec (x)}}$$

output
```
8/9*x/sec(x)^(3/2)-16/27*sin(x)/sec(x)^(1/2)+2/3*x^2*sin(x)/sec(x)^(1/2)-1
6/27*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))*cos(x)^
(1/2)*sec(x)^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3} x^2 \sqrt{\sec (x)}\right) \, dx=\frac{1}{27} \sqrt{\sec (x)} \left(12 x+12 x \cos (2 x)-16 \sqrt{\cos (x)} \text{EllipticF}\left(\frac{x}{2},2\right)-8 \sin (2 x)+9 x^2 \sin (2 x)\right)$$

input
```
Integrate[x^2/Sec[x]^(3/2) - (x^2*Sqrt[Sec[x]])/3,x]
```

output
```
(Sqrt[Sec[x]]*(12*x + 12*x*Cos[2*x] - 16*Sqrt[Cos[x]]*EllipticF[x/2, 2] -
8*Sin[2*x] + 9*x^2*Sin[2*x]))/27
```

3.97.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\sec(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^2\sin(x)}{3\sqrt{\sec(x)}}+\frac{8x}{9\sec^{\frac{3}{2}}(x)}-\frac{16\sin(x)}{27\sqrt{\sec(x)}}-\frac{16}{27}\sqrt{\cos(x)}\sqrt{\sec(x)}\,\text{EllipticF}\left(\frac{x}{2},2\right)$$

input
```
Int[x^2/Sec[x]^(3/2) - (x^2*Sqrt[Sec[x]])/3,x]
```

output
```
(8*x)/(9*Sec[x]^(3/2)) - (16*Sqrt[Cos[x]]*EllipticF[x/2, 2]*Sqrt[Sec[x]])/
27 - (16*Sin[x])/(27*Sqrt[Sec[x]]) + (2*x^2*Sin[x])/(3*Sqrt[Sec[x]])
```

3.97.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.97.4 Maple [F]

$$\int \left(\frac{x^2}{\sec\left(x\right)^{\frac{3}{2}}}-\frac{x^2\left(\sqrt{\sec}(x)\right)}{3}\right)dx$$

input
```
int(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x)
```

output
```
int(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x)
```

3.97.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\sec(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.97.6 Sympy [F]

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\sec(x)}\right) dx = -\frac{\int \left(-\frac{3x^2}{\sec^{\frac{3}{2}}(x)}\right) dx + \int x^2\sqrt{\sec(x)}\, dx}{3}$$

input
```
integrate(x**2/sec(x)**(3/2)-1/3*x**2*sec(x)**(1/2),x)
```

output
```
-(Integral(-3*x**2/sec(x)**(3/2), x) + Integral(x**2*sqrt(sec(x)), x))/3
```

3.97.7 Maxima [F]

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\sec(x)}\right) dx = \int -\frac{1}{3}\, x^2\sqrt{\sec(x)} + \frac{x^2}{\sec(x)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x^2*sqrt(sec(x)) + x^2/sec(x)^(3/2), x)
```

3.97.8 Giac [F]

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\sec(x)}\right)\,dx=\int -\frac{1}{3}\,x^2\sqrt{\sec\left(x\right)}+\frac{x^2}{\sec\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/sec(x)^(3/2)-1/3*x^2*sec(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x^2*sqrt(sec(x)) + x^2/sec(x)^(3/2), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\sec^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\sec(x)}\right)\,dx=-\int \frac{x^2\,\sqrt{\frac{1}{\cos(x)}}}{3}-\frac{x^2}{{\left(\frac{1}{\cos(x)}\right)}^{3/2}}\,dx$$

input
```
int(x^2/(1/cos(x))^(3/2) - (x^2*(1/cos(x))^(1/2))/3,x)
```

output
```
-int((x^2*(1/cos(x))^(1/2))/3 - x^2/(1/cos(x))^(3/2), x)
```

3.98 $\int (c+dx)^m (b\cos(e+fx))^n\,dx$

3.98.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m (b\cos(e+fx))^n\,dx = \mathrm{Int}\left((c+dx)^m (b\cos(e+fx))^n, x\right)$$

output
```
Unintegrable((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

3.98.2 Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cos(e+fx))^n\,dx = \int (c+dx)^m (b\cos(e+fx))^n\,dx$$

input
```
Integrate[(c + d*x)^m*(b*Cos[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(b*Cos[e + f*x])^n, x]
```

3.98.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (b\cos(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m \left(b\sin\left(e+fx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3807}$$

$$\int (c+dx)^m (b\cos(e+fx))^n dx$$

input
```
Int[(c + d*x)^m*(b*Cos[e + f*x])^n,x]
```

output
```
$Aborted
```

3.98.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.98.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx+c)^m (b\cos(fx+e))^n \, dx$$

input
```
int((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(b*cos(f*x+e))^n,x)
```

3.98.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int (dx+c)^m (b\cos(fx+e))^n \, dx$$

input
```
integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

3.98.6 Sympy [N/A]

Not integrable

Time = 8.74 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (c+dx)^m (b\cos(e+fx))^n \, dx = \int (b\cos(e+fx))^n (c+dx)^m \, dx$$

input
```
integrate((d*x+c)**m*(b*cos(f*x+e))**n,x)
```

output
```
Integral((b*cos(e + f*x))**n*(c + d*x)**m, x)
```

3.98.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\cos(e+fx))^n\,dx = \int (dx+c)^m(b\cos(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

3.98.8 Giac [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\cos(e+fx))^n\,dx = \int (dx+c)^m(b\cos(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*cos(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*cos(f*x + e))^n, x)
```

3.98.9 Mupad [N/A]

Not integrable

Time = 14.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\cos(e+fx))^n\,dx = \int (b\,\cos(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((b*cos(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((b*cos(e + f*x))^n*(c + d*x)^m, x)
```

3.99 $\int (c+dx)^m \cos^3(a+bx)\,dx$

3.99.1 Optimal result

Integrand size = 16, antiderivative size = 275

$$\int (c+dx)^m \cos^3(a+bx)\,dx = -\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} + \frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{8b} - \frac{i3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{8b} + \frac{i3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{8b}$$

output
```
-3/8*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+
c)/d)^m)+3/8*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)-1/8*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*
b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/8*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I
*b*(d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

3.99.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.92

$$\int (c+dx)^m \cos^3(a+bx)\,dx$$
$$= \frac{i3^{-1-m}e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{2+m}e^{2i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)+3^{2+m}e^{2ia+\cdots}\right.}{}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^3,x]
```

output
```
((I/8)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((2*I)*(2*a + (b*c)/d))*((I*b
*(c + d*x))/d)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]) + 3^(2 + m)*E^((2*I)*
a + ((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, (I*b*(c + d*x))/d
] - E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]
 + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d
*x))/d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

3.99.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^3(c+dx)^m dx$$
$$\Big\downarrow \text{3793}$$
$$\int \left(\frac{3}{4}\cos(a+bx)(c+dx)^m+\frac{1}{4}\cos(3a+3bx)(c+dx)^m\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}+\frac{3ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^3,x]
```

output
```
(((-3*I)/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x)
)/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (((3*I)/8)*(c + d*x)^m*Gamma[1 + m, (
I*b*(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - ((I/8)*
3^(-1 - m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*(c + d*x)^m*Ga
mma[1 + m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d
*x))/d)^m)
```

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.99.4 Maple [F]

$$\int (dx+c)^m \left(\cos^3(bx+a)\right) dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^3,x)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \frac{9i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right) - i\,e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right) - 9i\,e^{\left(-\frac{dm\,\mathrm{l}\cdots}{\cdots}\right)}\cdots}{24\,b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/24*(9*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x +
I*b*c)/d) - I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1,
-3*(I*b*d*x + I*b*c)/d) - 9*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gam
ma(m + 1, (-I*b*d*x - I*b*c)/d) + I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*
a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

3.99.6 Sympy [F]

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \int (c+dx)^m \cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)**3, x)
```

3.99.7 Maxima [F]

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \int (dx+c)^m \cos{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3, x)
```

3.99.8 Giac [F]

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \int (dx+c)^m \cos{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3, x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^3(a+bx)\,dx = \int \cos{(a+b\,x)}^3\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^3*(c + d*x)^m, x)
```

3.100 $\int (c+dx)^m \cos^2(a+bx)\,dx$

3.100.1 Optimal result

Integrand size = 16, antiderivative size = 162

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \frac{(c+dx)^{1+m}}{2d(1+m)} - \frac{i2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{i2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}$$

output
```
1/2*(d*x+c)^(1+m)/d/(1+m)-I*2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+
m,-2*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+I*2^(-3-m)*(d*x+c)^m*GAMMA(1+m,
2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

3.100.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \frac{1}{8}(c+dx)^m \left(\frac{4c+4dx}{d+dm} - \frac{i2^{-m}e^{2i\left(a-\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{i2^{-m}e^{-2i\left(a-\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}\right)$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2,x]
```

output
```
((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) - (I*E^((2*I)*(a - (b*c)/d))*Gamma[1
 + m, ((-2*I)*b*(c + d*x))/d])/(2^m*b*(((-I)*b*(c + d*x))/d)^m) + (I*Gamma
[1 + m, ((2*I)*b*(c + d*x))/d])/(2^m*b*E^((2*I)*(a - (b*c)/d))*((I*b*(c +
d*x))/d)^m)))/8
```

3.100.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)^2 (c+dx)^m dx$$

$$\Big\downarrow \ 3793$$

$$\int \left(\frac{1}{2}\cos(2a+2bx)(c+dx)^m + \frac{1}{2}(c+dx)^m\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{i2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{2d(m+1)}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2,x]
```

output
```
(c + d*x)^(1 + m)/(2*d*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c
 + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^
m) + (I*2^(-3 - m)*(c + d*x)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^(
(2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.100.4 Maple [F]

$$\int (dx+c)^m \left(\cos^2(bx+a)\right) dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^2,x)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int (c+dx)^m \cos^2(a+bx)\,dx$$
$$= \frac{(-i\,dm - i\,d)e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1, -\frac{2\,(i\,bdx+i\,bc)}{d}\right) + (i\,dm + i\,d)e^{\left(-\frac{dm\log\left(\frac{2i\,b}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m+\right.}{8\,(bdm+bd)}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*((-I*d*m - I*d)*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m
 + 1, -2*(I*b*d*x + I*b*c)/d) + (I*d*m + I*d)*e^(-(d*m*log(2*I*b/d) - 2*I*
b*c + 2*I*a*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) + 4*(b*d*x + b*c)*
(d*x + c)^m)/(b*d*m + b*d)
```

3.100.6 Sympy [F]

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int (c+dx)^m \cos^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)**2, x)
```

3.100.7 Maxima [F]

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int (dx+c)^m \cos{(bx+a)}^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((d*m + d)*integrate((d*x + c)^m*cos(2*b*x + 2*a), x) + e^(m*log(d*x +
 c) + log(d*x + c)))/(d*m + d)
```

3.100.8 Giac [F]

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int (dx+c)^m \cos{(bx+a)}^2 \,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2, x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^2(a+bx)\,dx = \int \cos{(a+b\,x)}^2\,{(c+d\,x)}^m \,dx$$

input
```
int(cos(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*(c + d*x)^m, x)
```

3.101 $\int (c+dx)^m \cos(a+bx)\,dx$

3.101.1 Optimal result

Integrand size = 14, antiderivative size = 131

$$\int (c+dx)^m \cos(a+bx)\,dx=-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b}$$

output
```
-1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+
c)/d)^m)+1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)
```

3.101.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int (c+dx)^m \cos(a+bx)\,dx=-\frac{ie^{-\frac{i(bc+ad)}{d}}(c+dx)^m\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)\right)}{2b}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x],x]
```

output
```
((-1/2*I)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/((
(-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d
])/((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d))
```

3.101.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(a+bx+\frac{\pi}{2}\right)(c+dx)^m dx$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{1}{2}i\int -ie^{-i(a+bx)}(c+dx)^m dx-\frac{1}{2}i\int ie^{i(a+bx)}(c+dx)^m dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m dx+\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m dx$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x],x]
```

output
```
((-1/2*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/
d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c
 + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

3.101.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.101.4 Maple [F]

$$\int (dx+c)^m \cos(bx+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a),x)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

$$\int (c+dx)^m \cos(a+bx)\,dx = \frac{i\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)-i\,e^{\left(-\frac{dm\log\left(-\frac{i\,b}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-i\,bdx-i\,bc}{d}\right)}{2\,b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I*b
*c)/d) - I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x
 - I*b*c)/d))/b
```

3.101.6 Sympy [F]

$$\int (c+dx)^m \cos(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x), x)
```

3.101.7 Maxima [F]

$$\int (c+dx)^m \cos(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a), x)
```

3.101.8 Giac [F]

$$\int (c+dx)^m \cos(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\left(c+d\,x\right)^m\,dx$$

input
```
int(cos(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*(c + d*x)^m, x)
```

3.102 $\int (c+dx)^m \sec(a+bx) \, dx$

3.102.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \sec(a+bx) \, dx = \text{Int}((c+dx)^m \sec(a+bx), x)$$

output
```
Unintegrable((d*x+c)^m*sec(b*x+a),x)
```

3.102.2 Mathematica [N/A]

Not integrable

Time = 9.54 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \sec(a+bx) \, dx = \int (c+dx)^m \sec(a+bx) \, dx$$

input
```
Integrate[(c + d*x)^m*Sec[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sec[a + b*x], x]
```

3.102.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)(c+dx)^m \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)(c+dx)^m dx$$

$$\Big\downarrow \text{4680}$$

$$\int \sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Sec[a + b*x],x]
```

output
```
$Aborted
```

3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.102.4 Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec (bx+a)\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a),x)
```

output
```
int((d*x+c)^m*sec(b*x+a),x)
```

3.102.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (dx+c)^m \sec (bx+a)\, dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sec(b*x + a), x)
```

3.102.6 Sympy [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (c+dx)^m \sec (a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sec(a + b*x), x)
```

3.102.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a), x)
```

3.102.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \sec(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a), x)
```

3.102.9 Mupad [N/A]

Not integrable

Time = 14.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (c+dx)^m \sec(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/cos(a + b*x),x)
```

output
```
int((c + d*x)^m/cos(a + b*x), x)
```

3.103 $\int (c+dx)^m \sec^2(a+bx)\,dx$

3.103.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \text{Int}\big((c+dx)^m \sec^2(a+bx),x\big)$$

output
```
Unintegrable((d*x+c)^m*sec(b*x+a)^2,x)
```

3.103.2 Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sec[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Sec[a + b*x]^2, x]
```

3.103.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(a+bx+\frac{\pi}{2}\right)^2(c+dx)^m dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

3.103.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

3.103.4 Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \left(\sec^2(bx+a)\right) dx$$

input
```
int((d*x+c)^m*sec(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sec(b*x+a)^2,x)
```

3.103.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sec(b*x + a)^2, x)
```

3.103.6 Sympy [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sec(a + b*x)**2, x)
```

3.103.7 Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)^2, x)
```

3.103.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)^2, x)
```

3.103.9 Mupad [N/A]

Not integrable

Time = 13.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^m/cos(a + b*x)^2,x)
```

output
```
int((c + d*x)^m/cos(a + b*x)^2, x)
```

3.104 $\int x^{3+m} \cos(a+bx)\,dx$

3.104.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{3+m}\cos(a+bx)\,dx=-\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

output
```
-1/2*exp(I*a)*x^m*GAMMA(4+m,-I*b*x)/b^4/((-I*b*x)^m)-1/2*x^m*GAMMA(4+m,I*b
*x)/b^4/exp(I*a)/((I*b*x)^m)
```

3.104.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^{3+m}\cos(a+bx)\,dx=-\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(4+m,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(4+m,ibx)}{2b^4}$$

input
```
Integrate[x^(3 + m)*Cos[a + b*x],x]
```

output
```
-1/2*(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - (x^m*Gamma[
4 + m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

3.104.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+3}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{-i(a+bx)}x^{m+3}dx-\frac{1}{2}i\int ie^{i(a+bx)}x^{m+3}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-i(a+bx)}x^{m+3}dx+\frac{1}{2}\int e^{i(a+bx)}x^{m+3}dx$$

$$\Big\downarrow \text{2612}$$

$$-\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(m+4,-ibx)}{2b^4}-\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(m+4,ibx)}{2b^4}$$

input
```
Int[x^(3 + m)*Cos[a + b*x],x]
```

output
```
-1/2*(E^(I*a)*x^m*Gamma[4 + m, (-I)*b*x])/(b^4*((-I)*b*x)^m) - (x^m*Gamma[
4 + m, I*b*x])/(2*b^4*E^(I*a)*(I*b*x)^m)
```

3.104.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.104.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 455, normalized size of antiderivative = 6.07

method	result
meijerg	$\frac{2^{3+m}\left(b^{2}\right)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{3\,2^{-4-m}x^{3+m}b^{3}\left(b^{2}\right)^{\frac{m}{2}}\left(\frac{8}{3}+\frac{2m}{3}\right)\sin(bx)}{\sqrt{\pi}\,(4+m)}-\frac{2^{-3-m}x^{1+m}b\left(b^{2}\right)^{\frac{m}{2}}\left(-m^{2}-7m-12\right)(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,(4+m)}+\frac{2^{-3-m}x^{2+m}b^{}\cdots}{\cdots}\right.}{\cdots}$

input
```
int(x^(3+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(3+m)/b^4*(b^2)^(-1/2*m)*Pi^(1/2)*(3*2^(-4-m)/Pi^(1/2)/(4+m)*x^(3+m)*b^3
*(b^2)^(1/2*m)*(8/3+2/3*m)*sin(b*x)-2^(-3-m)/Pi^(1/2)/(4+m)*x^(1+m)*b*(b^2
)^(1/2*m)*(-m^2-7*m-12)*(cos(b*x)*x*b-sin(b*x))+2^(-3-m)/Pi^(1/2)/(4+m)*x^
(2+m)*b^2*(b^2)^(1/2*m)*(-m^3-8*m^2-19*m-12)*(b*x)^(-3/2-m)*LommelS1(3/2+m
,3/2,b*x)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2+m)*(1+m)
*(3+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(1/2+m,1/2,b*x))*cos
(a)-2^(3+m)*b^(-4-m)*Pi^(1/2)*(2^(-3-m)/Pi^(1/2)/(5+m)*x^(2+m)*b^(2+m)*(m^
2+7*m+10)*sin(b*x)-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(cos(b*x)*x*b-sin(b*x
))-2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*m*(3+m)*(2+m)*(b*x)^(-3/2-m)*LommelS1
(1/2+m,3/2,b*x)*sin(b*x)+2^(-3-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(3+m)*(2+m)*(b*
x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(3/2+m,1/2,b*x))*sin(a)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int x^{3+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m+3)\log(i\,b)-i\,a)}\Gamma(m+4,i\,bx)-i\,e^{(-(m+3)\log(-i\,b)+i\,a)}\Gamma(m+4,-i\,bx)}{2\,b}$$

input
```
integrate(x^(3+m)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(I*e^(-(m + 3)*log(I*b) - I*a)*gamma(m + 4, I*b*x) - I*e^(-(m + 3)*log
(-I*b) + I*a)*gamma(m + 4, -I*b*x))/b
```

3.104.6 Sympy [F]

$$\int x^{3+m}\cos(a+bx)\,dx = \int x^{m+3}\cos{(a+bx)}\,dx$$

input
```
integrate(x**(3+m)*cos(b*x+a),x)
```

output
```
Integral(x**(m + 3)*cos(a + b*x), x)
```

3.104.7 Maxima [F]

$$\int x^{3+m}\cos(a+bx)\,dx = \int x^{m+3}\cos(bx+a)\,dx$$

input
```
integrate(x^(3+m)*cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m + 3)*cos(b*x + a), x)
```

3.104.8 Giac [F]

$$\int x^{3+m}\cos(a+bx)\,dx = \int x^{m+3}\cos(bx+a)\,dx$$

input
```
integrate(x^(3+m)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*cos(b*x + a), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\cos(a+bx)\,dx = \int x^{m+3}\cos(a+b\,x)\,dx$$

input
```
int(x^(m + 3)*cos(a + b*x),x)
```

output
```
int(x^(m + 3)*cos(a + b*x), x)
```

3.105 $\int x^{2+m}\cos(a+bx)\,dx$

3.105.1 Optimal result

Integrand size = 12, antiderivative size = 79

$$\int x^{2+m}\cos(a+bx)\,dx=\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

output
```
1/2*I*exp(I*a)*x^m*GAMMA(3+m,-I*b*x)/b^3/((-I*b*x)^m)-1/2*I*x^m*GAMMA(3+m,
I*b*x)/b^3/exp(I*a)/((I*b*x)^m)
```

3.105.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^{2+m}\cos(a+bx)\,dx=\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(3+m,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(3+m,ibx)}{2b^3}$$

input
```
Integrate[x^(2 + m)*Cos[a + b*x],x]
```

output
```
((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m
*Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

3.105.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+2}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{-i(a+bx)}x^{m+2}dx-\frac{1}{2}i\int ie^{i(a+bx)}x^{m+2}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-i(a+bx)}x^{m+2}dx+\frac{1}{2}\int e^{i(a+bx)}x^{m+2}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(m+3,-ibx)}{2b^3}-\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(m+3,ibx)}{2b^3}$$

input
```
Int[x^(2 + m)*Cos[a + b*x],x]
```

output
```
((I/2)*E^(I*a)*x^m*Gamma[3 + m, (-I)*b*x])/(b^3*((-I)*b*x)^m) - ((I/2)*x^m
*Gamma[3 + m, I*b*x])/(b^3*E^(I*a)*(I*b*x)^m)
```

3.105.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.105.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.48

method	result
meijerg	$\frac{2^{2+m} \left(b^{2}\right)^{-\frac{1}{2}-\frac{m}{2}} \sqrt{\pi}\, \left(\frac{3\, 2^{-3-m} x^{2+m} \left(b^{2}\right)^{\frac{3}{2}+\frac{m}{2}} \left(2+\frac{2m}{3}\right) \sin(bx)}{\sqrt{\pi}\, (3+m) b}-\frac{2^{-2-m} x^{2+m} \left(b^{2}\right)^{\frac{3}{2}+\frac{m}{2}} (2+m) m (bx)^{-\frac{3}{2}-m} s^{(+)}_{\frac{1}{2}+m,\frac{3}{2}}(bx) \sin(bx)}{\sqrt{\pi}\, b}+ \frac{2^{-2}\cdots}{}\right.}{b^{2}}$

input
```
int(x^(2+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(2+m)/b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-3-m)/Pi^(1/2)/(3+m)*x^(2+m)
*(b^2)^(3/2+1/2*m)*(2+2/3*m)/b*sin(b*x)-2^(-2-m)/Pi^(1/2)*x^(2+m)*(b^2)^(3
/2+1/2*m)/b*(2+m)*m*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)*sin(b*x)+2^(-2-
m)/Pi^(1/2)*x^(2+m)*(b^2)^(3/2+1/2*m)/b*(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b
-sin(b*x))*LommelS1(3/2+m,1/2,b*x))*cos(a)-2^(2+m)*b^(-3-m)*Pi^(1/2)*(-2^(
-2-m)/Pi^(1/2)*x^(1+m)*b^(1+m)*(cos(b*x)*x*b-sin(b*x))+2^(-2-m)/Pi^(1/2)/(
4+m)*x^(2+m)*b^(2+m)*(m^2+5*m+4)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/2,b*x)*si
n(b*x)+2^(-2-m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(2+m)*(1+m)*(b*x)^(-5/2-m)*(cos(b
*x)*x*b-sin(b*x))*LommelS1(1/2+m,1/2,b*x))*sin(a)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int x^{2+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m+2)\log(i\,b)-i\,a)}\Gamma(m+3,i\,bx)-i\,e^{(-(m+2)\log(-i\,b)+i\,a)}\Gamma(m+3,-i\,bx)}{2\,b}$$

input
```
integrate(x^(2+m)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(I*e^(-(m + 2)*log(I*b) - I*a)*gamma(m + 3, I*b*x) - I*e^(-(m + 2)*log
(-I*b) + I*a)*gamma(m + 3, -I*b*x))/b
```

3.105.6 Sympy [F]

$$\int x^{2+m}\cos(a+bx)\,dx = \int x^{m+2}\cos{(a+bx)}\,dx$$

input
```
integrate(x**(2+m)*cos(b*x+a),x)
```

output
```
Integral(x**(m + 2)*cos(a + b*x), x)
```

3.105.7 Maxima [F]

$$\int x^{2+m}\cos(a+bx)\,dx = \int x^{m+2}\cos(bx+a)\;dx$$

input
```
integrate(x^(2+m)*cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m + 2)*cos(b*x + a), x)
```

3.105.8 Giac [F]

$$\int x^{2+m} \cos(a+bx)\,dx = \int x^{m+2} \cos(bx+a)\,dx$$

input
```
integrate(x^(2+m)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*cos(b*x + a), x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int x^{2+m} \cos(a+bx)\,dx = \int x^{m+2} \cos(a+b\,x)\,dx$$

input
```
int(x^(m + 2)*cos(a + b*x),x)
```

output
```
int(x^(m + 2)*cos(a + b*x), x)
```

3.106 $\int x^{1+m} \cos(a+bx)\,dx$

3.106.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{1+m}\cos(a+bx)\,dx=\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}$$

output
```
1/2*exp(I*a)*x^m*GAMMA(2+m,-I*b*x)/b^2/((-I*b*x)^m)+1/2*x^m*GAMMA(2+m,I*b*
x)/b^2/exp(I*a)/((I*b*x)^m)
```

3.106.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^{1+m}\cos(a+bx)\,dx=\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(2+m,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(2+m,ibx)}{2b^2}$$

input
```
Integrate[x^(1 + m)*Cos[a + b*x],x]
```

output
```
(E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
 m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

3.106.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+1}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{-i(a+bx)}x^{m+1}dx-\frac{1}{2}i\int ie^{i(a+bx)}x^{m+1}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-i(a+bx)}x^{m+1}dx+\frac{1}{2}\int e^{i(a+bx)}x^{m+1}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{e^{ia}x^m(-ibx)^{-m}\Gamma(m+2,-ibx)}{2b^2}+\frac{e^{-ia}x^m(ibx)^{-m}\Gamma(m+2,ibx)}{2b^2}$$

input
```
Int[x^(1 + m)*Cos[a + b*x],x]
```

output
```
(E^(I*a)*x^m*Gamma[2 + m, (-I)*b*x])/(2*b^2*((-I)*b*x)^m) + (x^m*Gamma[2 +
 m, I*b*x])/(2*b^2*E^(I*a)*(I*b*x)^m)
```

3.106.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.106.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.88

method	result
meijerg	$\frac{2^{1+m} \left(b^2\right)^{-\frac{m}{2}} \sqrt{\pi}\, \left(\frac{2^{-1-m} x^{1+m} b \left(b^2\right)^{\frac{m}{2}} \sin(bx)}{\sqrt{\pi}\,(2+m)}+\frac{3\, 2^{-2-m} x^{2+m} b^2 \left(b^2\right)^{\frac{m}{2}} \left(\frac{2}{3}+\frac{2m}{3}\right) (bx)^{-\frac{3}{2}-m} s^{(+)}_{\frac{3}{2}+m,\frac{3}{2}}(bx) \sin(bx)}{\sqrt{\pi}\,(2+m)}+\frac{2^{-1-m} x^{2+m} b^2 \left(b^2 \right.}{}$

input
```
int(x^(1+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(1+m)/b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2)/(2+m)*x^(1+m)*b*(b^
2)^(1/2*m)*sin(b*x)+3*2^(-2-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)*(2
/3+2/3*m)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/2,b*x)*sin(b*x)+2^(-1-m)/Pi^(1/2
)*x^(2+m)*b^2*(b^2)^(1/2*m)*(1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*L
ommelS1(1/2+m,1/2,b*x))*cos(a)-2^(1+m)*b^(-2-m)*Pi^(1/2)*(2^(-1-m)/Pi^(1/2
)*x^(2+m)*b^(2+m)*m*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)*sin(b*x)-2^(-1-
m)/Pi^(1/2)*x^(2+m)*b^(2+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS
1(3/2+m,1/2,b*x))*sin(a)
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int x^{1+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m+1)\log(i\,b)-i\,a)}\Gamma(m+2,i\,bx)-i\,e^{(-(m+1)\log(-i\,b)+i\,a)}\Gamma(m+2,-i\,bx)}{2\,b}$$

input
```
integrate(x^(1+m)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(I*e^(-(m + 1)*log(I*b) - I*a)*gamma(m + 2, I*b*x) - I*e^(-(m + 1)*log
(-I*b) + I*a)*gamma(m + 2, -I*b*x))/b
```

3.106.6 Sympy [F]

$$\int x^{1+m}\cos(a+bx)\,dx = \int x^{m+1}\cos\left(a+bx\right)dx$$

input
```
integrate(x**(1+m)*cos(b*x+a),x)
```

output
```
Integral(x**(m + 1)*cos(a + b*x), x)
```

3.106.7 Maxima [F]

$$\int x^{1+m}\cos(a+bx)\,dx = \int x^{m+1}\cos\left(bx+a\right)\,dx$$

input
```
integrate(x^(1+m)*cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m + 1)*cos(b*x + a), x)
```

3.106.8 Giac [F]

$$\int x^{1+m}\cos(a+bx)\,dx = \int x^{m+1}\cos\left(bx+a\right)\,dx$$

input
```
integrate(x^(1+m)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*cos(b*x + a), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\cos(a+bx)\,dx = \int x^{m+1}\cos\left(a+b\,x\right)\,dx$$

input
```
int(x^(m + 1)*cos(a + b*x),x)
```

output
```
int(x^(m + 1)*cos(a + b*x), x)
```

3.107 $\int x^m \cos(a+bx)\,dx$

3.107.1 Optimal result

Integrand size = 10, antiderivative size = 79

$$\int x^m \cos(a+bx)\,dx = -\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b} + \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

output
```
-1/2*I*exp(I*a)*x^m*GAMMA(1+m,-I*b*x)/b/((-I*b*x)^m)+1/2*I*x^m*GAMMA(1+m,I
*b*x)/b/exp(I*a)/((I*b*x)^m)
```

3.107.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^m \cos(a+bx)\,dx = -\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(1+m,-ibx)}{2b} + \frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(1+m,ibx)}{2b}$$

input
```
Integrate[x^m*Cos[a + b*x],x]
```

output
```
((-1/2*I)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^
m*Gamma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

3.107.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int x^m \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 3788$$

$$\frac{1}{2}i\int -ie^{-i(a+bx)}x^m dx-\frac{1}{2}i\int ie^{i(a+bx)}x^m dx$$

$$\Big\downarrow \; 26$$

$$\frac{1}{2}\int e^{-i(a+bx)}x^m dx+\frac{1}{2}\int e^{i(a+bx)}x^m dx$$

$$\Big\downarrow \; 2612$$

$$\frac{ie^{-ia}x^m(ibx)^{-m}\Gamma(m+1,ibx)}{2b}-\frac{ie^{ia}x^m(-ibx)^{-m}\Gamma(m+1,-ibx)}{2b}$$

input
```
Int[x^m*Cos[a + b*x],x]
```

output
```
((-1/2*I)*E^(I*a)*x^m*Gamma[1 + m, (-I)*b*x])/(b*((-I)*b*x)^m) + ((I/2)*x^
m*Gamma[1 + m, I*b*x])/(b*E^(I*a)*(I*b*x)^m)
```

3.107.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.107.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.47 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.80

method	result
meijerg	$2^{m} \left(b^{2}\right)^{-\frac{1}{2}-\frac{m}{2}} \sqrt{\pi}\, \left(\frac{3\, 2^{-1-m} \left(b^{2}\right)^{\frac{1}{2}+\frac{m}{2}} x^{m} (6+2m) \sin(bx)}{\sqrt{\pi}\, (1+m)(9+3m) b}+\frac{\left(b^{2}\right)^{\frac{1}{2}+\frac{m}{2}} x^{m} 2^{-m} (\cos(bx) x b-\sin(bx))}{\sqrt{\pi}\, (1+m) b}+\frac{2^{-m} x^{2+m} \left(b^{2}\right)^{\frac{1}{2}+\frac{m}{2}}}{}\right.$

input
```
int(x^m*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^m*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(-1-m)/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*
m)*x^m*(6+2*m)/(9+3*m)/b*sin(b*x)+1/Pi^(1/2)/(1+m)*(b^2)^(1/2+1/2*m)*x^m*2
^(-m)/b*(cos(b*x)*x*b-sin(b*x))+2^(-m)/Pi^(1/2)/(1+m)*x^(2+m)*(b^2)^(1/2+1
/2*m)*b*m*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)*sin(b*x)-2^(-m)/Pi^(1/2)/
(1+m)*x^(2+m)*(b^2)^(1/2+1/2*m)*b*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*L
ommelS1(3/2+m,1/2,b*x))*cos(a)-2^m*b^(-1-m)*Pi^(1/2)*(1/Pi^(1/2)/(2+m)*x^(
1+m)*b^(1+m)*2^(-m)*sin(b*x)-2^(-m)/Pi^(1/2)/(2+m)*x^(2+m)*b^(2+m)*(b*x)^(
-3/2-m)*LommelS1(3/2+m,3/2,b*x)*sin(b*x)-3*2^(-1-m)/Pi^(1/2)/(2+m)*x^(2+m)
*b^(2+m)*(4/3+2/3*m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(1/2+m
,1/2,b*x))*sin(a)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int x^m \cos(a+bx)\,dx = \frac{i\,e^{(-m\log(i\,b)-i\,a)}\Gamma(m+1,i\,bx) - i\,e^{(-m\log(-i\,b)+i\,a)}\Gamma(m+1,-i\,bx)}{2\,b}$$

input
```
integrate(x^m*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(I*e^(-m*log(I*b) - I*a)*gamma(m + 1, I*b*x) - I*e^(-m*log(-I*b) + I*a
)*gamma(m + 1, -I*b*x))/b
```

3.107.6 Sympy [F]

$$\int x^m \cos(a+bx)\,dx = \int x^m \cos{(a+bx)}\,dx$$

input
```
integrate(x**m*cos(b*x+a),x)
```

output
```
Integral(x**m*cos(a + b*x), x)
```

3.107.7 Maxima [F]

$$\int x^m \cos(a+bx)\,dx = \int x^m \cos{(bx+a)}\ dx$$

input
```
integrate(x^m*cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^m*cos(b*x + a), x)
```

3.107.8 Giac [F]

$$\int x^m \cos(a+bx)\,dx = \int x^m \cos(bx+a)\,dx$$

input
```
integrate(x^m*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*cos(b*x + a), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cos(a+bx)\,dx = \int x^m \cos(a+b\,x)\,dx$$

input
```
int(x^m*cos(a + b*x),x)
```

output
```
int(x^m*cos(a + b*x), x)
```

3.108 $\int x^{-1+m}\cos(a+bx)\,dx$

3.108.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int x^{-1+m}\cos(a+bx)\,dx=-\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx)-\frac{1}{2}e^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)$$

output
```
-1/2*exp(I*a)*x^m*GAMMA(m,-I*b*x)/((-I*b*x)^m)-1/2*x^m*GAMMA(m,I*b*x)/exp(
I*a)/((I*b*x)^m)
```

3.108.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int x^{-1+m}\cos(a+bx)\,dx=\frac{1}{2}e^{-ia}x^m\left(-e^{2ia}(-ibx)^{-m}\Gamma(m,-ibx)-(ibx)^{-m}\Gamma(m,ibx)\right)$$

input
```
Integrate[x^(-1 + m)*Cos[a + b*x],x]
```

output
```
(x^m*(-((E^((2*I)*a)*Gamma[m, (-I)*b*x])/((-I)*b*x)^m) - Gamma[m, I*b*x]/(
I*b*x)^m))/(2*E^(I*a))
```

3.108.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^{m-1}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{1}{2}i\int -ie^{-i(a+bx)}x^{m-1}dx-\frac{1}{2}i\int ie^{i(a+bx)}x^{m-1}dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}\int e^{-i(a+bx)}x^{m-1}dx+\frac{1}{2}\int e^{i(a+bx)}x^{m-1}dx$$

$$\Big\downarrow \text{ 2612}$$

$$-\frac{1}{2}e^{ia}x^m(-ibx)^{-m}\Gamma(m,-ibx)-\frac{1}{2}e^{-ia}x^m(ibx)^{-m}\Gamma(m,ibx)$$

input
```
Int[x^(-1 + m)*Cos[a + b*x],x]
```

output
```
-1/2*(E^(I*a)*x^m*Gamma[m, (-I)*b*x])/((-I)*b*x)^m - (x^m*Gamma[m, I*b*x])
/(2*E^(I*a)*(I*b*x)^m)
```

3.108.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.108.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.44 (sec) , antiderivative size = 427, normalized size of antiderivative = 6.57

method	result
meijerg	$2^{-1+m}(b^2)^{-\frac{m}{2}}\sqrt{\pi}\left(\frac{3x^{-1+m}2^{-m}(b^2)^{\frac{m}{2}}(2x^2b^2+2m+4)\sin(bx)}{\sqrt{\pi}\,m(6+3m)b}+\frac{2^{1-m}x^{-1+m}(b^2)^{\frac{m}{2}}(\cos(bx)xb-\sin(bx))}{\sqrt{\pi}\,mb}-\frac{3x^{2+m}2^{1-m}\cdots}{\cdots}\right.$

input
```
int(x^(-1+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(-1+m)*(b^2)^(-1/2*m)*Pi^(1/2)*(3/Pi^(1/2)/m*x^(-1+m)*2^(-m)*(b^2)^(1/2*
m)*(2*b^2*x^2+2*m+4)/(6+3*m)/b*sin(b*x)+2^(1-m)/Pi^(1/2)/m*x^(-1+m)*(b^2)^
(1/2*m)/b*(cos(b*x)*x*b-sin(b*x))-3/Pi^(1/2)/m*x^(2+m)*2^(1-m)*(b^2)^(1/2*
m)*b^2/(6+3*m)*(b*x)^(-3/2-m)*LommelS1(3/2+m,3/2,b*x)*sin(b*x)-1/Pi^(1/2)/
m*x^(2+m)*2^(1-m)*(b^2)^(1/2*m)*b^2*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))
*LommelS1(1/2+m,1/2,b*x))*cos(a)-2^(-1+m)*b^(-m)*Pi^(1/2)*(2^(1-m)/Pi^(1/2
)/(1+m)*x^m*b^m*sin(b*x)-2^(1-m)/Pi^(1/2)/(1+m)*x^m*b^m/m*(cos(b*x)*x*b-si
n(b*x))-1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)*(b*x)^(-3/2-m)*LommelS1(1
/2+m,3/2,b*x)*sin(b*x)+1/Pi^(1/2)/(1+m)*x^(2+m)*b^(2+m)*2^(1-m)/m*(b*x)^(-
5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(3/2+m,1/2,b*x))*sin(a)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int x^{-1+m}\cos(a+bx)\,dx=\frac{i\,e^{(-(m-1)\log(i\,b)-i\,a)}\Gamma(m,i\,bx)-i\,e^{(-(m-1)\log(-i\,b)+i\,a)}\Gamma(m,-i\,bx)}{2\,b}$$

input
```
integrate(x^(-1+m)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(I*e^(-(m - 1)*log(I*b) - I*a)*gamma(m, I*b*x) - I*e^(-(m - 1)*log(-I*
b) + I*a)*gamma(m, -I*b*x))/b
```

3.108.6 Sympy [F]

$$\int x^{-1+m}\cos(a+bx)\,dx=\int x^{m-1}\cos\left(a+bx\right)dx$$

input
```
integrate(x**(-1+m)*cos(b*x+a),x)
```

output
```
Integral(x**(m - 1)*cos(a + b*x), x)
```

3.108.7 Maxima [F]

$$\int x^{-1+m}\cos(a+bx)\,dx=\int x^{m-1}\cos\left(bx+a\right)\,dx$$

input
```
integrate(x^(-1+m)*cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m - 1)*cos(b*x + a), x)
```

3.108.8 Giac [F]

$$\int x^{-1+m}\cos(a+bx)\,dx=\int x^{m-1}\cos\left(bx+a\right)\,dx$$

input
```
integrate(x^(-1+m)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*cos(b*x + a), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\cos(a+bx)\,dx=\int x^{m-1}\,\cos\left(a+b\,x\right)\,dx$$

input
```
int(x^(m - 1)*cos(a + b*x),x)
```

output
```
int(x^(m - 1)*cos(a + b*x), x)
```

3.109 $\int x^{-2+m} \cos(a+bx)\,dx$

3.109.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{-2+m} \cos(a+bx)\,dx = \frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx) - \frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)$$

output
```
1/2*I*b*exp(I*a)*x^m*GAMMA(-1+m,-I*b*x)/((-I*b*x)^m)-1/2*I*b*x^m*GAMMA(-1+
m,I*b*x)/exp(I*a)/((I*b*x)^m)
```

3.109.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^{-2+m} \cos(a+bx)\,dx = \frac{1}{2}ibe^{ia}x^m(-ibx)^{-m}\Gamma(-1+m,-ibx) - \frac{1}{2}ibe^{-ia}x^m(ibx)^{-m}\Gamma(-1+m,ibx)$$

input
```
Integrate[x^(-2 + m)*Cos[a + b*x],x]
```

output
```
((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*
Gamma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

3.109.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-2}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{-i(a+bx)}x^{m-2}dx-\frac{1}{2}i\int ie^{i(a+bx)}x^{m-2}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-i(a+bx)}x^{m-2}dx+\frac{1}{2}\int e^{i(a+bx)}x^{m-2}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{1}{2}ie^{ia}bx^m(-ibx)^{-m}\Gamma(m-1,-ibx)-\frac{1}{2}ie^{-ia}bx^m(ibx)^{-m}\Gamma(m-1,ibx)$$

input
```
Int[x^(-2 + m)*Cos[a + b*x],x]
```

output
```
((I/2)*b*E^(I*a)*x^m*Gamma[-1 + m, (-I)*b*x])/((-I)*b*x)^m - ((I/2)*b*x^m*
Gamma[-1 + m, I*b*x])/(E^(I*a)*(I*b*x)^m)
```

3.109.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.109.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 530, normalized size of antiderivative = 7.07

method	result
meijerg	$2^{-2+m} b^2 (b^2)^{-\frac{1}{2}-\frac{m}{2}} \sqrt{\pi} \left(\frac{3\, 2^{1-m} x^{-2+m} (b^2)^{-\frac{1}{2}+\frac{m}{2}} (2x^2b^2+2m+2) \sin(bx)}{\sqrt{\pi}\,(-1+m)(3+3m)b} - \frac{2^{2-m} x^{-2+m} (b^2)^{-\frac{1}{2}+\frac{m}{2}} (x^2b^2-m^2-m)(\mathrm{co}}{\sqrt{\pi}\,(-1+m)b(1+m)m} \right.$

input
```
int(x^(-2+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(-2+m)*b^2*(b^2)^(-1/2-1/2*m)*Pi^(1/2)*(3*2^(1-m)/Pi^(1/2)/(-1+m)*x^(-2+
m)*(b^2)^(-1/2+1/2*m)*(2*b^2*x^2+2*m+2)/(3+3*m)/b*sin(b*x)-2^(2-m)/Pi^(1/2
)/(-1+m)*x^(-2+m)*(b^2)^(-1/2+1/2*m)/b*(b^2*x^2-m^2-m)/(1+m)/m*(cos(b*x)*x
*b-sin(b*x))-3*2^(2-m)/Pi^(1/2)/(-1+m)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/(3+3
*m)*(b*x)^(-3/2-m)*LommelS1(1/2+m,3/2,b*x)*sin(b*x)+2^(2-m)/Pi^(1/2)/(-1+m
)*x^(2+m)*(b^2)^(-1/2+1/2*m)*b^3/(1+m)/m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(
b*x))*LommelS1(3/2+m,1/2,b*x))*cos(a)-2^(-2+m)*b^(1-m)*Pi^(1/2)*(2^(1-m)/P
i^(1/2)/m*x^(-1+m)*b^(-1+m)*(-2*b^2*x^2+2*m^2+2*m-4)/(2+m)/(-1+m)*sin(b*x)
-3*2^(2-m)/Pi^(1/2)/m*x^(-1+m)*b^(-1+m)/(-3+3*m)*(cos(b*x)*x*b-sin(b*x))+2
^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(3/2
+m,3/2,b*x)*sin(b*x)+3*2^(2-m)/Pi^(1/2)/m*x^(2+m)*b^(2+m)/(-3+3*m)*(b*x)^(
-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(1/2+m,1/2,b*x))*sin(a)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int x^{-2+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m-2)\log(i\,b)-i\,a)}\Gamma(m-1,i\,bx)-i\,e^{(-(m-2)\log(-i\,b)+i\,a)}\Gamma(m-1,-i\,bx)}{2\,b}$$

input
```
integrate(x^(-2+m)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(I*e^(-(m - 2)*log(I*b) - I*a)*gamma(m - 1, I*b*x) - I*e^(-(m - 2)*log
(-I*b) + I*a)*gamma(m - 1, -I*b*x))/b
```

3.109.6 Sympy [F]

$$\int x^{-2+m}\cos(a+bx)\,dx = \int x^{m-2}\cos(a+bx)\,dx$$

input
```
integrate(x**(-2+m)*cos(b*x+a),x)
```

output
```
Integral(x**(m - 2)*cos(a + b*x), x)
```

3.109.7 Maxima [F]

$$\int x^{-2+m}\cos(a+bx)\,dx = \int x^{m-2}\cos(bx+a)\,dx$$

input
```
integrate(x^(-2+m)*cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m - 2)*cos(b*x + a), x)
```

3.109.8 Giac [F]

$$\int x^{-2+m}\cos(a+bx)\,dx = \int x^{m-2}\cos(bx+a)\,dx$$

input
```
integrate(x^(-2+m)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*cos(b*x + a), x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\cos(a+bx)\,dx = \int x^{m-2}\cos(a+b\,x)\,dx$$

input
```
int(x^(m - 2)*cos(a + b*x),x)
```

output
```
int(x^(m - 2)*cos(a + b*x), x)
```

3.110 $\int x^{-3+m}\cos(a+bx)\,dx$

3.110.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x^{-3+m}\cos(a+bx)\,dx=\frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx)+\frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)$$

output
```
1/2*b^2*exp(I*a)*x^m*GAMMA(-2+m,-I*b*x)/((-I*b*x)^m)+1/2*b^2*x^m*GAMMA(-2+
m,I*b*x)/exp(I*a)/((I*b*x)^m)
```

3.110.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^{-3+m}\cos(a+bx)\,dx=\frac{1}{2}b^2e^{ia}x^m(-ibx)^{-m}\Gamma(-2+m,-ibx)+\frac{1}{2}b^2e^{-ia}x^m(ibx)^{-m}\Gamma(-2+m,ibx)$$

input
```
Integrate[x^(-3 + m)*Cos[a + b*x],x]
```

output
```
(b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamm
a[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

3.110.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-3}\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{-i(a+bx)}x^{m-3}dx-\frac{1}{2}i\int ie^{i(a+bx)}x^{m-3}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-i(a+bx)}x^{m-3}dx+\frac{1}{2}\int e^{i(a+bx)}x^{m-3}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{1}{2}e^{ia}b^2x^m(-ibx)^{-m}\Gamma(m-2,-ibx)+\frac{1}{2}e^{-ia}b^2x^m(ibx)^{-m}\Gamma(m-2,ibx)$$

input
```
Int[x^(-3 + m)*Cos[a + b*x],x]
```

output
```
(b^2*E^(I*a)*x^m*Gamma[-2 + m, (-I)*b*x])/(2*((-I)*b*x)^m) + (b^2*x^m*Gamm
a[-2 + m, I*b*x])/(2*E^(I*a)*(I*b*x)^m)
```

3.110.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

3.110.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 600, normalized size of antiderivative = 8.00

method	result
meijerg	$2^{-3+m} b^2 (b^2)^{-\frac{m}{2}} \sqrt{\pi} \left(\frac{2^{2-m} x^{-3+m} (b^2)^{\frac{m}{2}} (-2x^4 b^4 + 2x^2 b^2 m^2 + 2x^2 b^2 m - 4x^2 b^2 + 2m^3 + 2m^2 - 4m) \sin(bx)}{\sqrt{\pi} (-2+m) b^3 m (2+m)(-1+m)} - \frac{2^{3-m} x^{-3+m} (\cdots}{\cdots} \right.$

input
```
int(x^(-3+m)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(-3+m)*b^2*(b^2)^(-1/2*m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-2+m)*x^(-3+m)/b^3
*(b^2)^(1/2*m)*(-2*b^4*x^4+2*b^2*m^2*x^2+2*b^2*m*x^2-4*b^2*x^2+2*m^3+2*m^2
-4*m)/m/(2+m)/(-1+m)*sin(b*x)-2^(3-m)/Pi^(1/2)/(-2+m)*x^(-3+m)/b^3*(b^2)^(
1/2*m)*(b^2*x^2-m^2+m)/m/(-1+m)*(cos(b*x)*x*b-sin(b*x))+2^(3-m)/Pi^(1/2)/(
-2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(2+m)/(-1+m)*(b*x)^(-3/2-m)*LommelS1(3/2
+m,3/2,b*x)*sin(b*x)+2^(3-m)/Pi^(1/2)/(-2+m)*x^(2+m)*b^2*(b^2)^(1/2*m)/m/(
-1+m)*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(1/2+m,1/2,b*x))*cos(
a)-2^(-3+m)*b^(2-m)*Pi^(1/2)*(2^(2-m)/Pi^(1/2)/(-1+m)*x^(-2+m)*b^(-2+m)*(-
2*b^2*x^2+2*m^2-2*m-4)/(1+m)/(-2+m)*sin(b*x)+2^(3-m)/Pi^(1/2)/(-1+m)*x^(-2
+m)*b^(-2+m)*(b^2*x^2-m^2-m)/(1+m)/(-2+m)/m*(cos(b*x)*x*b-sin(b*x))+2^(3-m
)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(-2+m)*(b*x)^(-3/2-m)*LommelS1(1/2
+m,3/2,b*x)*sin(b*x)-2^(3-m)/Pi^(1/2)/(-1+m)*x^(2+m)*b^(2+m)/(1+m)/(-2+m)/
m*(b*x)^(-5/2-m)*(cos(b*x)*x*b-sin(b*x))*LommelS1(3/2+m,1/2,b*x))*sin(a)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int x^{-3+m}\cos(a+bx)\,dx = \frac{i\,e^{(-(m-3)\log(i\,b)-i\,a)}\Gamma(m-2,i\,bx)-i\,e^{(-(m-3)\log(-i\,b)+i\,a)}\Gamma(m-2,-i\,bx)}{2\,b}$$

input
```
integrate(x^(-3+m)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(I*e^(-(m - 3)*log(I*b) - I*a)*gamma(m - 2, I*b*x) - I*e^(-(m - 3)*log
(-I*b) + I*a)*gamma(m - 2, -I*b*x))/b
```

3.110.6 Sympy [F]

$$\int x^{-3+m}\cos(a+bx)\,dx = \int x^{m-3}\cos\left(a+bx\right)\,dx$$

input
```
integrate(x**(-3+m)*cos(b*x+a),x)
```

output
```
Integral(x**(m - 3)*cos(a + b*x), x)
```

3.110.7 Maxima [F]

$$\int x^{-3+m}\cos(a+bx)\,dx = \int x^{m-3}\cos(bx+a)\,dx$$

input
```
integrate(x^(-3+m)*cos(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x^(m - 3)*cos(b*x + a), x)
```

3.110.8 Giac [F]

$$\int x^{-3+m}\cos(a+bx)\,dx = \int x^{m-3}\cos(bx+a)\,dx$$

input
```
integrate(x^(-3+m)*cos(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*cos(b*x + a), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\cos(a+bx)\,dx = \int x^{m-3}\cos(a+bx)\,dx$$

input
```
int(x^(m - 3)*cos(a + b*x),x)
```

output
```
int(x^(m - 3)*cos(a + b*x), x)
```

3.111 $\int x^{3+m}\cos^2(a+bx)\,dx$

3.111.1 Optimal result

Integrand size = 14, antiderivative size = 99

$$\int x^{3+m}\cos^2(a+bx)\,dx=\frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4}-\frac{2^{-6-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}$$

output
```
1/2*x^(4+m)/(4+m)-2^(-6-m)*exp(2*I*a)*x^m*GAMMA(4+m,-2*I*b*x)/b^4/((-I*b*x
)^m)-2^(-6-m)*x^m*GAMMA(4+m,2*I*b*x)/b^4/exp(2*I*a)/((I*b*x)^m)
```

3.111.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int x^{3+m}\cos^2(a+bx)\,dx=\frac{1}{64}x^m\left(\frac{32x^4}{4+m}-\frac{2^{-m}e^{2ia}(-ibx)^{-m}\Gamma(4+m,-2ibx)}{b^4}-\frac{2^{-m}e^{-2ia}(ibx)^{-m}\Gamma(4+m,2ibx)}{b^4}\right)$$

input
```
Integrate[x^(3 + m)*Cos[a + b*x]^2,x]
```

output
```
(x^m*((32*x^4)/(4 + m) - (E^((2*I)*a)*Gamma[4 + m, (-2*I)*b*x])/(2^m*b^4*(
(-I)*b*x)^m) - Gamma[4 + m, (2*I)*b*x]/(2^m*b^4*E^((2*I)*a)*(I*b*x)^m)))/6
4
```

3.111.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+3}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2}x^{m+3}\cos(2a+2bx)+\frac{x^{m+3}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2ia}2^{-m-6}x^m(-ibx)^{-m}\Gamma(m+4,-2ibx)}{b^4}-\frac{e^{-2ia}2^{-m-6}x^m(ibx)^{-m}\Gamma(m+4,2ibx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

input
```
Int[x^(3 + m)*Cos[a + b*x]^2,x]
```

output
```
x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^((2*I)*a)*x^m*Gamma[4 + m, (-2*I)*b*
x])/(b^4*((-I)*b*x)^m) - (2^(-6 - m)*x^m*Gamma[4 + m, (2*I)*b*x])/(b^4*E^(
(2*I)*a)*(I*b*x)^m)
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.111.4 Maple [F]

$$\int x^{3+m} \left(\cos^2 (bx + a)\right) dx$$

input
```
int(x^(3+m)*cos(b*x+a)^2,x)
```

output
```
int(x^(3+m)*cos(b*x+a)^2,x)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int x^{3+m} \cos^2(a + bx) \, dx = \frac{4\,bxx^{m+3} + (i\,m + 4i)e^{(-(m+3)\log(2i\,b)-2i\,a)}\Gamma(m+4, 2i\,bx) + (-i\,m - 4i)e^{(-(m+3)\log(-2i\,b)+2i\,a)}\Gamma(m+4, -2i}{8\,(bm + 4\,b)}$$

input
```
integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m + 3) + (I*m + 4*I)*e^(-(m + 3)*log(2*I*b) - 2*I*a)*gamma(m
 + 4, 2*I*b*x) + (-I*m - 4*I)*e^(-(m + 3)*log(-2*I*b) + 2*I*a)*gamma(m + 4
, -2*I*b*x))/(b*m + 4*b)
```

3.111.6 Sympy [F]

$$\int x^{3+m} \cos^2(a + bx) \, dx = \int x^{m+3} \cos^2{(a + bx)} \, dx$$

input
```
integrate(x**(3+m)*cos(b*x+a)**2,x)
```

output
```
Integral(x**(m + 3)*cos(a + b*x)**2, x)
```

3.111.7 Maxima [F]

$$\int x^{3+m}\cos^2(a+bx)\,dx = \int x^{m+3}\cos\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((m + 4)*integrate(x^3*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 4*log(
x)))/(m + 4)
```

3.111.8 Giac [F]

$$\int x^{3+m}\cos^2(a+bx)\,dx = \int x^{m+3}\cos\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(3+m)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*cos(b*x + a)^2, x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\cos^2(a+bx)\,dx = \int x^{m+3}\cos\left(a+b\,x\right)^2\,dx$$

input
```
int(x^(m + 3)*cos(a + b*x)^2,x)
```

output
```
int(x^(m + 3)*cos(a + b*x)^2, x)
```

3.112 $\int x^{2+m}\cos^2(a+bx)\,dx$

3.112.1 Optimal result

Integrand size = 14, antiderivative size = 103

$$\int x^{2+m}\cos^2(a+bx)\,dx=\frac{x^{3+m}}{2(3+m)}+\frac{i2^{-5-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3}-\frac{i2^{-5-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}$$

output
```
1/2*x^(3+m)/(3+m)+I*2^(-5-m)*exp(2*I*a)*x^m*GAMMA(3+m,-2*I*b*x)/b^3/((-I*b
*x)^m)-I*2^(-5-m)*x^m*GAMMA(3+m,2*I*b*x)/b^3/exp(2*I*a)/((I*b*x)^m)
```

3.112.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int x^{2+m}\cos^2(a+bx)\,dx=\frac{1}{32}x^m\left(\frac{16x^3}{3+m}+\frac{i2^{-m}e^{2ia}(-ibx)^{-m}\Gamma(3+m,-2ibx)}{b^3}-\frac{i2^{-m}e^{-2ia}(ibx)^{-m}\Gamma(3+m,2ibx)}{b^3}\right)$$

input
```
Integrate[x^(2 + m)*Cos[a + b*x]^2,x]
```

output
```
(x^m*((16*x^3)/(3 + m) + (I*E^((2*I)*a)*Gamma[3 + m, (-2*I)*b*x])/(2^m*b^3
*((-I)*b*x)^m) - (I*Gamma[3 + m, (2*I)*b*x])/(2^m*b^3*E^((2*I)*a)*(I*b*x)^
m)))/32
```

3.112.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+2}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2}x^{m+2}\cos(2a+2bx)+\frac{x^{m+2}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie^{2ia}2^{-m-5}x^m(-ibx)^{-m}\Gamma(m+3,-2ibx)}{b^3}-\frac{ie^{-2ia}2^{-m-5}x^m(ibx)^{-m}\Gamma(m+3,2ibx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

input
```
Int[x^(2 + m)*Cos[a + b*x]^2,x]
```

output
```
x^(3 + m)/(2*(3 + m)) + (I*2^(-5 - m)*E^((2*I)*a)*x^m*Gamma[3 + m, (-2*I)*
b*x])/(b^3*((-I)*b*x)^m) - (I*2^(-5 - m)*x^m*Gamma[3 + m, (2*I)*b*x])/(b^3
*E^((2*I)*a)*(I*b*x)^m)
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.112.4 Maple [F]

$$\int x^{2+m} \left(\cos^2(bx+a)\right) dx$$

input
```
int(x^(2+m)*cos(b*x+a)^2,x)
```

output
```
int(x^(2+m)*cos(b*x+a)^2,x)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int x^{2+m}\cos^2(a+bx)\,dx = \frac{4\,bxx^{m+2} + (i\,m+3i)e^{(-(m+2)\log(2i\,b)-2i\,a)}\Gamma(m+3, 2i\,bx) + (-i\,m-3i)e^{(-(m+2)\log(-2i\,b)+2i\,a)}\Gamma(m+3, -2i}{8\,(bm+3\,b)}$$

input
```
integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m + 2) + (I*m + 3*I)*e^(-(m + 2)*log(2*I*b) - 2*I*a)*gamma(m
 + 3, 2*I*b*x) + (-I*m - 3*I)*e^(-(m + 2)*log(-2*I*b) + 2*I*a)*gamma(m + 3
, -2*I*b*x))/(b*m + 3*b)
```

3.112.6 Sympy [F]

$$\int x^{2+m}\cos^2(a+bx)\,dx = \int x^{m+2}\cos^2(a+bx)\,dx$$

input
```
integrate(x**(2+m)*cos(b*x+a)**2,x)
```

output
```
Integral(x**(m + 2)*cos(a + b*x)**2, x)
```

3.112.7 Maxima [F]

$$\int x^{2+m}\cos^2(a+bx)\,dx = \int x^{m+2}\cos\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((m + 3)*integrate(x^2*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 3*log(
x)))/(m + 3)
```

3.112.8 Giac [F]

$$\int x^{2+m}\cos^2(a+bx)\,dx = \int x^{m+2}\cos\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(2+m)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*cos(b*x + a)^2, x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\cos^2(a+bx)\,dx = \int x^{m+2}\cos\left(a+b\,x\right)^2\,dx$$

input
```
int(x^(m + 2)*cos(a + b*x)^2,x)
```

output
```
int(x^(m + 2)*cos(a + b*x)^2, x)
```

3.113 $\int x^{1+m}\cos^2(a+bx)\,dx$

3.113.1 Optimal result

Integrand size = 14, antiderivative size = 97

$$\int x^{1+m}\cos^2(a+bx)\,dx=\frac{x^{2+m}}{2(2+m)}+\frac{2^{-4-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2}+\frac{2^{-4-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}$$

output
```
1/2*x^(2+m)/(2+m)+2^(-4-m)*exp(2*I*a)*x^m*GAMMA(2+m,-2*I*b*x)/b^2/((-I*b*x
)^m)+2^(-4-m)*x^m*GAMMA(2+m,2*I*b*x)/b^2/exp(2*I*a)/((I*b*x)^m)
```

3.113.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int x^{1+m}\cos^2(a+bx)\,dx=\frac{1}{16}x^m\left(\frac{8x^2}{2+m}+\frac{2^{-m}e^{2ia}(-ibx)^{-m}\Gamma(2+m,-2ibx)}{b^2}+\frac{2^{-m}e^{-2ia}(ibx)^{-m}\Gamma(2+m,2ibx)}{b^2}\right)$$

input
```
Integrate[x^(1 + m)*Cos[a + b*x]^2,x]
```

output
```
(x^m*((8*x^2)/(2 + m) + (E^((2*I)*a)*Gamma[2 + m, (-2*I)*b*x])/(2^m*b^2*((
-I)*b*x)^m) + Gamma[2 + m, (2*I)*b*x]/(2^m*b^2*E^((2*I)*a)*(I*b*x)^m)))/16
```

3.113.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^{m+1}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 3793}$$

$$\int\left(\frac{1}{2}x^{m+1}\cos(2a+2bx)+\frac{x^{m+1}}{2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^{2ia}2^{-m-4}x^m(-ibx)^{-m}\Gamma(m+2,-2ibx)}{b^2}+\frac{e^{-2ia}2^{-m-4}x^m(ibx)^{-m}\Gamma(m+2,2ibx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

input
```
Int[x^(1 + m)*Cos[a + b*x]^2,x]
```

output
```
x^(2 + m)/(2*(2 + m)) + (2^(-4 - m)*E^((2*I)*a)*x^m*Gamma[2 + m, (-2*I)*b*
x])/(b^2*((-I)*b*x)^m) + (2^(-4 - m)*x^m*Gamma[2 + m, (2*I)*b*x])/(b^2*E^(
(2*I)*a)*(I*b*x)^m)
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.113.4 Maple [F]

$$\int x^{1+m}\left(\cos^2(bx+a)\right)dx$$

input
```
int(x^(1+m)*cos(b*x+a)^2,x)
```

output
```
int(x^(1+m)*cos(b*x+a)^2,x)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int x^{1+m}\cos^2(a+bx)\,dx = \frac{4\,bxx^{m+1}+(i\,m+2i)e^{(-(m+1)\log(2i\,b)-2i\,a)}\Gamma(m+2,2i\,bx)+(-i\,m-2i)e^{(-(m+1)\log(-2i\,b)+2i\,a)}\Gamma(m+2,-2i}{8\,(bm+2\,b)}$$

input
```
integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m + 1) + (I*m + 2*I)*e^(-(m + 1)*log(2*I*b) - 2*I*a)*gamma(m
 + 2, 2*I*b*x) + (-I*m - 2*I)*e^(-(m + 1)*log(-2*I*b) + 2*I*a)*gamma(m + 2
, -2*I*b*x))/(b*m + 2*b)
```

3.113.6 Sympy [F]

$$\int x^{1+m}\cos^2(a+bx)\,dx = \int x^{m+1}\cos^2(a+bx)\,dx$$

input
```
integrate(x**(1+m)*cos(b*x+a)**2,x)
```

output
```
Integral(x**(m + 1)*cos(a + b*x)**2, x)
```

3.113.7 Maxima [F]

$$\int x^{1+m}\cos^2(a+bx)\,dx = \int x^{m+1}\cos(bx+a)^2\,dx$$

input
```
integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((m + 2)*integrate(x*x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + 2*log(x)
))/(m + 2)
```

3.113.8 Giac [F]

$$\int x^{1+m}\cos^2(a+bx)\,dx = \int x^{m+1}\cos(bx+a)^2\,dx$$

input
```
integrate(x^(1+m)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*cos(b*x + a)^2, x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\cos^2(a+bx)\,dx = \int x^{m+1}\cos(a+b\,x)^2\,dx$$

input
```
int(x^(m + 1)*cos(a + b*x)^2,x)
```

output
```
int(x^(m + 1)*cos(a + b*x)^2, x)
```

3.114 $\int x^m \cos^2(a+bx)\,dx$

3.114.1 Optimal result

Integrand size = 12, antiderivative size = 103

$$\int x^m \cos^2(a+bx)\,dx = \frac{x^{1+m}}{2(1+m)} - \frac{i2^{-3-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(1+m,-2ibx)}{b} + \frac{i2^{-3-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(1+m,2ibx)}{b}$$

output
```
1/2*x^(1+m)/(1+m)-I*2^(-3-m)*exp(2*I*a)*x^m*GAMMA(1+m,-2*I*b*x)/b/((-I*b*x
)^m)+I*2^(-3-m)*x^m*GAMMA(1+m,2*I*b*x)/b/exp(2*I*a)/((I*b*x)^m)
```

3.114.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int x^m \cos^2(a+bx)\,dx = \frac{1}{8}x^m\left(\frac{4x}{1+m} - 2^{-m}e^{2ia}x(-ibx)^{-1-m}\Gamma(1+m,-2ibx) - 2^{-m}e^{-2ia}x(ibx)^{-1-m}\Gamma(1+m,2ibx)\right)$$

input
```
Integrate[x^m*Cos[a + b*x]^2,x]
```

output
```
(x^m*((4*x)/(1 + m) - (E^((2*I)*a)*x*((-I)*b*x)^(-1 - m)*Gamma[1 + m, (-2*
I)*b*x])/2^m - (x*(I*b*x)^(-1 - m)*Gamma[1 + m, (2*I)*b*x])/(2^m*E^((2*I)*
a))))/8
```

3.114.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \sin\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2}x^m\cos(2a+2bx)+\frac{x^m}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^{2ia}2^{-m-3}x^m(-ibx)^{-m}\Gamma(m+1,-2ibx)}{b}+\frac{ie^{-2ia}2^{-m-3}x^m(ibx)^{-m}\Gamma(m+1,2ibx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

input
```
Int[x^m*Cos[a + b*x]^2,x]
```

output
```
x^(1 + m)/(2*(1 + m)) - (I*2^(-3 - m)*E^((2*I)*a)*x^m*Gamma[1 + m, (-2*I)*
b*x])/(b*((-I)*b*x)^m) + (I*2^(-3 - m)*x^m*Gamma[1 + m, (2*I)*b*x])/(b*E^(
(2*I)*a)*(I*b*x)^m)
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.114.4 Maple [F]

$$\int x^{m} \left(\cos^{2}\left(b x +a \right)\right)d x$$

input
```
int(x^m*cos(b*x+a)^2,x)
```

output
```
int(x^m*cos(b*x+a)^2,x)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.67

$$\int x^m \cos^2(a+bx) \, dx = \frac{4\,bxx^{m} + (i\,m + i)e^{(-m\log(2i\,b) - 2i\,a)}\Gamma(m+1, 2i\,bx) + (-i\,m - i)e^{(-m\log(-2i\,b) + 2i\,a)}\Gamma(m+1, -2i\,bx)}{8\,(bm+b)}$$

input
```
integrate(x^m*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^m + (I*m + I)*e^(-m*log(2*I*b) - 2*I*a)*gamma(m + 1, 2*I*b*x)
 + (-I*m - I)*e^(-m*log(-2*I*b) + 2*I*a)*gamma(m + 1, -2*I*b*x))/(b*m + b)
```

3.114.6 Sympy [F]

$$\int x^m \cos^2(a+bx) \, dx = \int x^{m} \cos^{2}\left(a + bx\right)\, dx$$

input
```
integrate(x**m*cos(b*x+a)**2,x)
```

output
```
Integral(x**m*cos(a + b*x)**2, x)
```

3.114.7 Maxima [F]

$$\int x^m \cos^2(a+bx)\,dx = \int x^m \cos{(bx+a)}^2\,dx$$

input
```
integrate(x^m*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((m + 1)*integrate(x^m*cos(2*b*x + 2*a), x) + e^(m*log(x) + log(x)))/(
m + 1)
```

3.114.8 Giac [F]

$$\int x^m \cos^2(a+bx)\,dx = \int x^m \cos{(bx+a)}^2\,dx$$

input
```
integrate(x^m*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cos(b*x + a)^2, x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int x^m \cos^2(a+bx)\,dx = \int x^m \cos{(a+b\,x)}^2\,dx$$

input
```
int(x^m*cos(a + b*x)^2,x)
```

output
```
int(x^m*cos(a + b*x)^2, x)
```

3.115 $\int x^{-1+m}\cos^2(a+bx)\,dx$

3.115.1 Optimal result

Integrand size = 14, antiderivative size = 85

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \frac{x^m}{2m} - 2^{-2-m}e^{2ia}x^m(-ibx)^{-m}\Gamma(m,-2ibx) - 2^{-2-m}e^{-2ia}x^m(ibx)^{-m}\Gamma(m,2ibx)$$

output
```
1/2*x^m/m-2^(-2-m)*exp(2*I*a)*x^m*GAMMA(m,-2*I*b*x)/((-I*b*x)^m)-2^(-2-m)*
x^m*GAMMA(m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

3.115.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{-1+m}\cos^2(a+bx)\,dx = \frac{1}{4}x^m\left(\frac{2}{m} - 2^{-m}e^{2ia}(-ibx)^{-m}\Gamma(m,-2ibx) - 2^{-m}e^{-2ia}(ibx)^{-m}\Gamma(m,2ibx)\right)$$

input
```
Integrate[x^(-1 + m)*Cos[a + b*x]^2,x]
```

output
```
(x^m*(2/m - (E^((2*I)*a)*Gamma[m, (-2*I)*b*x])/(2^m*((-I)*b*x)^m) - Gamma[
m, (2*I)*b*x]/(2^m*E^((2*I)*a)*(I*b*x)^m)))/4
```

3.115.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^{m-1}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 3793}$$

$$\int\left(\frac{1}{2}x^{m-1}\cos(2a+2bx)+\frac{x^{m-1}}{2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$e^{2ia}\left(-2^{-m-2}\right)x^m(-ibx)^{-m}\Gamma(m,-2ibx)-e^{-2ia}2^{-m-2}x^m(ibx)^{-m}\Gamma(m,2ibx)+\frac{x^m}{2m}$$

input
```
Int[x^(-1 + m)*Cos[a + b*x]^2,x]
```

output
```
x^m/(2*m) - (2^(-2 - m)*E^((2*I)*a)*x^m*Gamma[m, (-2*I)*b*x])/((-I)*b*x)^m
 - (2^(-2 - m)*x^m*Gamma[m, (2*I)*b*x])/(E^((2*I)*a)*(I*b*x)^m)
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.115.4 Maple [F]

$$\int x^{-1+m} \left(\cos^2 (bx + a)\right) dx$$

input
```
int(x^(-1+m)*cos(b*x+a)^2,x)
```

output
```
int(x^(-1+m)*cos(b*x+a)^2,x)
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.75

$$\int x^{-1+m} \cos^2(a + bx) \, dx = \frac{4 \, bxx^{m-1} + i \, me^{(-(m-1) \log(2i \, b) - 2i \, a)} \Gamma(m, 2i \, bx) - i \, me^{(-(m-1) \log(-2i \, b) + 2i \, a)} \Gamma(m, -2i \, bx)}{8 \, bm}$$

input
```
integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m - 1) + I*m*e^(-(m - 1)*log(2*I*b) - 2*I*a)*gamma(m, 2*I*b*
x) - I*m*e^(-(m - 1)*log(-2*I*b) + 2*I*a)*gamma(m, -2*I*b*x))/(b*m)
```

3.115.6 Sympy [F]

$$\int x^{-1+m} \cos^2(a + bx) \, dx = \int x^{m-1} \cos^2 (a + bx) \, dx$$

input
```
integrate(x**(-1+m)*cos(b*x+a)**2,x)
```

output
```
Integral(x**(m - 1)*cos(a + b*x)**2, x)
```

3.115.7 Maxima [F]

$$\int x^{-1+m}\cos^2(a+bx)\,dx=\int x^{m-1}\cos{(bx+a)}^2\,dx$$

input
```
integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(m*integrate(x^m*cos(2*b*x + 2*a)/x, x) + x^m)/m
```

3.115.8 Giac [F]

$$\int x^{-1+m}\cos^2(a+bx)\,dx=\int x^{m-1}\cos{(bx+a)}^2\,dx$$

input
```
integrate(x^(-1+m)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*cos(b*x + a)^2, x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\cos^2(a+bx)\,dx=\int x^{m-1}\cos{(a+b\,x)}^2\,dx$$

input
```
int(x^(m - 1)*cos(a + b*x)^2,x)
```

output
```
int(x^(m - 1)*cos(a + b*x)^2, x)
```

3.116 $\int x^{-2+m} \cos^2(a+bx) \, dx$

3.116.1 Optimal result

Integrand size = 14, antiderivative size = 101

$$\int x^{-2+m} \cos^2(a+bx) \, dx = -\frac{x^{-1+m}}{2(1-m)} + i2^{-1-m}be^{2ia}x^m(-ibx)^{-m}\Gamma(-1+m,-2ibx) - i2^{-1-m}be^{-2ia}x^m(ibx)^{-m}\Gamma(-1+m,2ibx)$$

output
```
-1/2*x^(-1+m)/(1-m)+I*2^(-1-m)*b*exp(2*I*a)*x^m*GAMMA(-1+m,-2*I*b*x)/((-I*
b*x)^m)-I*2^(-1-m)*b*x^m*GAMMA(-1+m,2*I*b*x)/exp(2*I*a)/((I*b*x)^m)
```

3.116.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int x^{-2+m} \cos^2(a+bx) \, dx = \frac{1}{2}x^m\left(\frac{1}{(-1+m)x} + i2^{-m}be^{2ia}(-ibx)^{-m}\Gamma(-1+m,-2ibx) - i2^{-m}be^{-2ia}(ibx)^{-m}\Gamma(-1+m,2ibx)\right)$$

input
```
Integrate[x^(-2 + m)*Cos[a + b*x]^2,x]
```

output
```
(x^m*(1/((-1 + m)*x) + (I*b*E^((2*I)*a)*Gamma[-1 + m, (-2*I)*b*x])/(2^m*((
-I)*b*x)^m) - (I*b*Gamma[-1 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(I*b*x)^m)))
/2
```

3.116.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-2}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int\left(\frac{1}{2}x^{m-2}\cos(2a+2bx)+\frac{x^{m-2}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ie^{2ia}b2^{-m-1}x^m(-ibx)^{-m}\Gamma(m-1,-2ibx)-ie^{-2ia}b2^{-m-1}x^m(ibx)^{-m}\Gamma(m-1,2ibx)-\frac{x^{m-1}}{2(1-m)}$$

input
```
Int[x^(-2 + m)*Cos[a + b*x]^2,x]
```

output
```
-1/2*x^(-1 + m)/(1 - m) + (I*2^(-1 - m)*b*E^((2*I)*a)*x^m*Gamma[-1 + m, (-
2*I)*b*x])/((-I)*b*x)^m - (I*2^(-1 - m)*b*x^m*Gamma[-1 + m, (2*I)*b*x])/(E
^((2*I)*a)*(I*b*x)^m)
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.116.4 Maple [F]

$$\int x^{-2+m} \left(\cos^2 (bx + a)\right) dx$$

input
```
int(x^(-2+m)*cos(b*x+a)^2,x)
```

output
```
int(x^(-2+m)*cos(b*x+a)^2,x)
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.76

$$\int x^{-2+m} \cos^2(a + bx) \, dx = \frac{4\, bxx^{m-2} + (i\, m - i)e^{(-(m-2)\log(2i\, b) - 2i\, a)}\Gamma(m - 1, 2i\, bx) + (-i\, m + i)e^{(-(m-2)\log(-2i\, b) + 2i\, a)}\Gamma(m - 1, -2i\, bx)}{8\,(bm - b)}$$

input
```
integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m - 2) + (I*m - I)*e^(-(m - 2)*log(2*I*b) - 2*I*a)*gamma(m -
 1, 2*I*b*x) + (-I*m + I)*e^(-(m - 2)*log(-2*I*b) + 2*I*a)*gamma(m - 1, -2
*I*b*x))/(b*m - b)
```

3.116.6 Sympy [F]

$$\int x^{-2+m} \cos^2(a + bx) \, dx = \int x^{m-2} \cos^2 (a + bx) \, dx$$

input
```
integrate(x**(-2+m)*cos(b*x+a)**2,x)
```

output
```
Integral(x**(m - 2)*cos(a + b*x)**2, x)
```

3.116.7 Maxima [F]

$$\int x^{-2+m}\cos^2(a+bx)\,dx = \int x^{m-2}\cos(bx+a)^2\,dx$$

input
```
integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((m - 1)*x*integrate(x^m*cos(2*b*x + 2*a)/x^2, x) + x^m)/((m - 1)*x)
```

3.116.8 Giac [F]

$$\int x^{-2+m}\cos^2(a+bx)\,dx = \int x^{m-2}\cos(bx+a)^2\,dx$$

input
```
integrate(x^(-2+m)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*cos(b*x + a)^2, x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\cos^2(a+bx)\,dx = \int x^{m-2}\cos(a+b\,x)^2\,dx$$

input
```
int(x^(m - 2)*cos(a + b*x)^2,x)
```

output
```
int(x^(m - 2)*cos(a + b*x)^2, x)
```

3.117 $\int x^{-3+m} \cos^2(a+bx)\,dx$

3.117.1 Optimal result

Integrand size = 14, antiderivative size = 95

$$\int x^{-3+m} \cos^2(a+bx)\,dx = -\frac{x^{-2+m}}{2(2-m)} + 2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx) + 2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)$$

output
```
-1/2*x^(-2+m)/(2-m)+b^2*exp(2*I*a)*x^m*GAMMA(-2+m,-2*I*b*x)/(2^m)/((-I*b*x
)^m)+b^2*x^m*GAMMA(-2+m,2*I*b*x)/(2^m)/exp(2*I*a)/((I*b*x)^m)
```

3.117.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int x^{-3+m} \cos^2(a+bx)\,dx = -\frac{x^{-2+m}}{2(2-m)} + 2^{-m}b^2e^{2ia}x^m(-ibx)^{-m}\Gamma(-2+m,-2ibx) + 2^{-m}b^2e^{-2ia}x^m(ibx)^{-m}\Gamma(-2+m,2ibx)$$

input
```
Integrate[x^(-3 + m)*Cos[a + b*x]^2,x]
```

output
```
-1/2*x^(-2 + m)/(2 - m) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/
(2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(
I*b*x)^m)
```

3.117.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-3}\sin\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int\left(\frac{1}{2}x^{m-3}\cos(2a+2bx)+\frac{x^{m-3}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{2ia}b^2 2^{-m}x^m(-ibx)^{-m}\Gamma(m-2,-2ibx)+e^{-2ia}b^2 2^{-m}x^m(ibx)^{-m}\Gamma(m-2,2ibx)-\frac{x^{m-2}}{2(2-m)}$$

input
```
Int[x^(-3 + m)*Cos[a + b*x]^2,x]
```

output
```
-1/2*x^(-2 + m)/(2 - m) + (b^2*E^((2*I)*a)*x^m*Gamma[-2 + m, (-2*I)*b*x])/
(2^m*((-I)*b*x)^m) + (b^2*x^m*Gamma[-2 + m, (2*I)*b*x])/(2^m*E^((2*I)*a)*(
I*b*x)^m)
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.117.4 Maple [F]

$$\int x^{-3+m}\left(\cos^2(bx+a)\right)dx$$

input
```
int(x^(-3+m)*cos(b*x+a)^2,x)
```

output
```
int(x^(-3+m)*cos(b*x+a)^2,x)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.81

$$\int x^{-3+m}\cos^2(a+bx)\,dx = \frac{4\,bxx^{m-3} + (i\,m-2i)e^{(-(m-3)\log(2i\,b)-2i\,a)}\Gamma(m-2,2i\,bx) + (-i\,m+2i)e^{(-(m-3)\log(-2i\,b)+2i\,a)}\Gamma(m-2,-2i}{8\,(bm-2\,b)}$$

input
```
integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*x^(m - 3) + (I*m - 2*I)*e^(-(m - 3)*log(2*I*b) - 2*I*a)*gamma(m
 - 2, 2*I*b*x) + (-I*m + 2*I)*e^(-(m - 3)*log(-2*I*b) + 2*I*a)*gamma(m - 2
, -2*I*b*x))/(b*m - 2*b)
```

3.117.6 Sympy [F]

$$\int x^{-3+m}\cos^2(a+bx)\,dx = \int x^{m-3}\cos^2(a+bx)\,dx$$

input
```
integrate(x**(-3+m)*cos(b*x+a)**2,x)
```

output
```
Integral(x**(m - 3)*cos(a + b*x)**2, x)
```

3.117.7 Maxima [F]

$$\int x^{-3+m} \cos^2(a+bx)\, dx = \int x^{m-3} \cos{(bx+a)}^2\, dx$$

input
```
integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*((m - 2)*x^2*integrate(x^m*cos(2*b*x + 2*a)/x^3, x) + x^m)/((m - 2)*x^
2)
```

3.117.8 Giac [F]

$$\int x^{-3+m} \cos^2(a+bx)\, dx = \int x^{m-3} \cos{(bx+a)}^2\, dx$$

input
```
integrate(x^(-3+m)*cos(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*cos(b*x + a)^2, x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int x^{-3+m} \cos^2(a+bx)\, dx = \int x^{m-3} \cos{(a+b\,x)}^2\, dx$$

input
```
int(x^(m - 3)*cos(a + b*x)^2,x)
```

output
```
int(x^(m - 3)*cos(a + b*x)^2, x)
```

3.118 $\int (c+dx)^3(a+a\cos(e+fx))\,dx$

3.118.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx = \frac{a(c+dx)^4}{4d} - \frac{6ad^3\cos(e+fx)}{f^4} + \frac{3ad(c+dx)^2\cos(e+fx)}{f^2} - \frac{6ad^2(c+dx)\sin(e+fx)}{f^3} + \frac{a(c+dx)^3\sin(e+fx)}{f}$$

output
```
1/4*a*(d*x+c)^4/d-6*a*d^3*cos(f*x+e)/f^4+3*a*d*(d*x+c)^2*cos(f*x+e)/f^2-6*
a*d^2*(d*x+c)*sin(f*x+e)/f^3+a*(d*x+c)^3*sin(f*x+e)/f
```

3.118.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.37

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx = a\left(\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) + \frac{3d(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)}{f^4} + \frac{(c+dx)\left(c^2f^2+2cdf^2x+d^2(-6+f^2x^2)\right)\sin(e+fx)}{f^3}\right)$$

input
```
Integrate[(c + d*x)^3*(a + a*Cos[e + f*x]),x]
```

output
```
a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (3*d*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x])/f^4 + ((c + d*x)*(c^2*f^2 + 2
*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Sin[e + f*x])/f^3)
```

3.118.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a\cos(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^3\cos(e+fx)+a(c+dx)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6ad^2(c+dx)\sin(e+fx)}{f^3}+\frac{3ad(c+dx)^2\cos(e+fx)}{f^2}+\frac{a(c+dx)^3\sin(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cos(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + a*Cos[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cos[e + f*x])/f^4 + (3*a*d*(c + d*x)^2*Co
s[e + f*x])/f^2 - (6*a*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (a*(c + d*x)^3*Si
n[e + f*x])/f
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.118.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

method	result
parallelrisch	$\frac{\left((dx+c)\left((dx+c)^2 f^2-6d^2\right) f \sin(fx+e)+3\left((dx+c)^2 f^2-2d^2\right) d \cos(fx+e)+\left(\frac{dx}{2}+c\right) x\left(\frac{1}{2} x^2 d^2+cdx+c^2\right) f^4-3c^2 d f^2+6\ldots\right.}{f^4}$
risch	$\frac{a d^3 x^4}{4}+ac d^2 x^3+\frac{3ad c^2 x^2}{2}+a c^3 x+\frac{a c^4}{4d}+\frac{3ad\left(d^2 x^2 f^2+2cd f^2 x+c^2 f^2-2d^2\right) \cos(fx+e)}{f^4}+\frac{a\left(d^3 f^2 x^3+\ldots\right.}{}$
norman	$\frac{\frac{6a c^2 d f^2-12a d^3}{f^4}+ac d^2 x^3+ac d^2 x^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{ac\left(c^2 f^2+6d^2\right) x}{f^2}+\frac{ac\left(c^2 f^2-6d^2\right) x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a d^3 x^4}{4}+\frac{a d^3 x^4 (\ldots}{}}{}$
parts	$\frac{a(dx+c)^4}{4d}+\frac{a\left(\frac{d^3\left((fx+e)^3 \sin(fx+e)+3(fx+e)^2 \cos(fx+e)-6 \cos(fx+e)-6(fx+e) \sin(fx+e)\right)}{f^3}+\frac{3c d^2\left((fx+e)^2 \sin(fx+e)-2 \sin\ldots\right.}{f\ldots}\right.}{}$
derivativedivides	$\frac{a c^3 \sin(fx+e)-\frac{3a c^2 de \sin(fx+e)}{f}+\frac{3a c^2 d(\cos(fx+e)+(fx+e) \sin(fx+e))}{f}+\frac{3ac d^2 e^2 \sin(fx+e)}{f^2}-\frac{6ac d^2 e(\cos(fx+e)+(fx+e) \sin\ldots}{f^2}}{}$
default	$\frac{a c^3 \sin(fx+e)-\frac{3a c^2 de \sin(fx+e)}{f}+\frac{3a c^2 d(\cos(fx+e)+(fx+e) \sin(fx+e))}{f}+\frac{3ac d^2 e^2 \sin(fx+e)}{f^2}-\frac{6ac d^2 e(\cos(fx+e)+(fx+e) \sin\ldots}{f^2}}{}$

input
```
int((d*x+c)^3*(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*((d*x+c)^2*f^2-6*d^2)*f*sin(f*x+e)+3*((d*x+c)^2*f^2-2*d^2)*d*cos(
f*x+e)+(1/2*d*x+c)*x*(1/2*x^2*d^2+c*d*x+c^2)*f^4-3*c^2*d*f^2+6*d^3)*a/f^4
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx = \frac{ad^3f^4x^4+4\,acd^2f^4x^3+6\,ac^2df^4x^2+4\,ac^3f^4x+12\,(ad^3f^2x^2+2\,acd^2f^2x+ac^2df^2-2\,ad^3)\cos(fx+e)}{4\,f^4}$$

input
```
integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
 + 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a*d^3)*cos(f*x +
e) + 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 - 6*a*c*d^2*f + 3*(a
*c^2*d*f^3 - 2*a*d^3*f)*x)*sin(f*x + e))/f^4
```

3.118.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(88) = 176.

Time = 0.30 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.97

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx = \begin{cases} ac^3x+\frac{ac^3\sin(e+fx)}{f}+\frac{3ac^2dx^2}{2}+\frac{3ac^2dx\sin(e+fx)}{f}+\frac{3ac^2d\cos(e+fx)}{f^2}+acd^2x^3+\frac{3acd^2x^2\sin(e+fx)}{f}+\frac{6acd^2x\cos(e+fx)}{f^2} \\ (a\cos(e)+a)\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+a*cos(f*x+e)),x)
```

output
```
Piecewise((a*c**3*x + a*c**3*sin(e + f*x)/f + 3*a*c**2*d*x**2/2 + 3*a*c**2
*d*x*sin(e + f*x)/f + 3*a*c**2*d*cos(e + f*x)/f**2 + a*c*d**2*x**3 + 3*a*c
*d**2*x**2*sin(e + f*x)/f + 6*a*c*d**2*x*cos(e + f*x)/f**2 - 6*a*c*d**2*si
n(e + f*x)/f**3 + a*d**3*x**4/4 + a*d**3*x**3*sin(e + f*x)/f + 3*a*d**3*x*
*2*cos(e + f*x)/f**2 - 6*a*d**3*x*sin(e + f*x)/f**3 - 6*a*d**3*cos(e + f*x
)/f**4, Ne(f, 0)), ((a*cos(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4), True))
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(87) = 174$.

Time = 0.31 (sec) , antiderivative size = 456, normalized size of antiderivative = 5.12

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx$$
$$= \frac{4\,(fx+e)ac^3 + \frac{(fx+e)^4ad^3}{f^3} - \frac{4\,(fx+e)^3ad^3e}{f^3} + \frac{6\,(fx+e)^2ad^3e^2}{f^3} - \frac{4\,(fx+e)ad^3e^3}{f^3} + \frac{4\,(fx+e)^3acd^2}{f^2} - \frac{12\,(fx+e)^2acd^2e}{f^2} + \frac{12\,(}{}}{}$$

input
```
integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*(4*(f*x + e)*a*c^3 + (f*x + e)^4*a*d^3/f^3 - 4*(f*x + e)^3*a*d^3*e/f^3
 + 6*(f*x + e)^2*a*d^3*e^2/f^3 - 4*(f*x + e)*a*d^3*e^3/f^3 + 4*(f*x + e)^3
*a*c*d^2/f^2 - 12*(f*x + e)^2*a*c*d^2*e/f^2 + 12*(f*x + e)*a*c*d^2*e^2/f^2
 + 6*(f*x + e)^2*a*c^2*d/f - 12*(f*x + e)*a*c^2*d*e/f + 4*a*c^3*sin(f*x +
e) - 4*a*d^3*e^3*sin(f*x + e)/f^3 + 12*a*c*d^2*e^2*sin(f*x + e)/f^2 - 12*a
*c^2*d*e*sin(f*x + e)/f + 12*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*d^3
*e^2/f^3 - 24*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*c*d^2*e/f^2 + 12*(
(f*x + e)*sin(f*x + e) + cos(f*x + e))*a*c^2*d/f - 12*(2*(f*x + e)*cos(f*x
 + e) + ((f*x + e)^2 - 2)*sin(f*x + e))*a*d^3*e/f^3 + 12*(2*(f*x + e)*cos(
f*x + e) + ((f*x + e)^2 - 2)*sin(f*x + e))*a*c*d^2/f^2 + 4*(3*((f*x + e)^2
 - 2)*cos(f*x + e) + ((f*x + e)^3 - 6*f*x - 6*e)*sin(f*x + e))*a*d^3/f^3)/
f
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.73

$$\begin{aligned}\int (c+dx)^3(a+a\cos(e+fx))\,dx &= \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x \\ &\quad + \frac{3\,(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 - 2\,ad^3)\cos(fx+e)}{f^4} \\ &\quad + \frac{(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x + ac^3f^3 - 6\,ad^3fx - 6\,acd^2f)\sin(fx+e)}{f^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+a*cos(f*x+e)),x, algorithm="giac")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3*(a*d^3*f^2*x^2
 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 - 2*a*d^3)*cos(f*x + e)/f^4 + (a*d^3*f^3*
x^3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + a*c^3*f^3 - 6*a*d^3*f*x - 6*a*
c*d^2*f)*sin(f*x + e)/f^4
```

3.118.9 Mupad [B] (verification not implemented)

Time = 14.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.12

$$\int (c+dx)^3(a+a\cos(e+fx))\,dx = \frac{\sin(e+f\,x)\,\left(a\,c^3\,f^2-6\,a\,c\,d^2\right)}{f^3} - \frac{3\,\cos(e+f\,x)\,\left(2\,a\,d^3-a\,c^2\,d\,f^2\right)}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x - \frac{3\,x\,\sin(e+f\,x)\,\left(2\,a\,d^3-a\,c^2\,d\,f^2\right)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 + \frac{3\,a\,d^3\,x^2\,\cos(e+f\,x)}{f^2} + \frac{a\,d^3\,x^3\,\sin(e+f\,x)}{f} + \frac{6\,a\,c\,d^2\,x\,\cos(e+f\,x)}{f^2} + \frac{3\,a\,c\,d^2\,x^2\,\sin(e+f\,x)}{f}$$

input
```
int((a + a*cos(e + f*x))*(c + d*x)^3,x)
```

output
```
(sin(e + f*x)*(a*c^3*f^2 - 6*a*c*d^2))/f^3 - (3*cos(e + f*x)*(2*a*d^3 - a*
c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x - (3*x*sin(e + f*x)*(2*a*d^3 - a
*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (3*a*d^3*x^2*cos(e +
f*x))/f^2 + (a*d^3*x^3*sin(e + f*x))/f + (6*a*c*d^2*x*cos(e + f*x))/f^2 +
(3*a*c*d^2*x^2*sin(e + f*x))/f
```

3.119 $\int (c+dx)^2(a+a\cos(e+fx))\,dx$

3.119.1 Optimal result

Integrand size = 18, antiderivative size = 67

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = \frac{a(c+dx)^3}{3d} + \frac{2ad(c+dx)\cos(e+fx)}{f^2} - \frac{2ad^2\sin(e+fx)}{f^3} + \frac{a(c+dx)^2\sin(e+fx)}{f}$$

output
```
1/3*a*(d*x+c)^3/d+2*a*d*(d*x+c)*cos(f*x+e)/f^2-2*a*d^2*sin(f*x+e)/f^3+a*(d
*x+c)^2*sin(f*x+e)/f
```

3.119.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = a\left(c^2x + cdx^2 + \frac{d^2x^3}{3} + \frac{2d(c+dx)\cos(e+fx)}{f^2} + \frac{(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\sin(e+fx)}{f^3}\right)$$

input
```
Integrate[(c + d*x)^2*(a + a*Cos[e + f*x]),x]
```

output
```
a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 + (2*d*(c + d*x)*Cos[e + f*x])/f^2 + ((c^
2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Sin[e + f*x])/f^3)
```

3.119.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a\cos(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^2\cos(e+fx)+a(c+dx)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ad(c+dx)\cos(e+fx)}{f^2}+\frac{a(c+dx)^2\sin(e+fx)}{f}+\frac{a(c+dx)^3}{3d}-\frac{2ad^2\sin(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + a*Cos[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) + (2*a*d*(c + d*x)*Cos[e + f*x])/f^2 - (2*a*d^2*Sin[
e + f*x])/f^3 + (a*(c + d*x)^2*Sin[e + f*x])/f
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.119.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{\left(\left((dx+c)^2 f^2-2d^2\right)\sin(fx+e)+\left((2d^2x+2cd)\cos(fx+e)+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)f^2-2cd\right)f\right)a}{f^3}$
risch	$\frac{a\,d^2x^3}{3}+acd\,x^2+a\,c^2x+\frac{a\,c^3}{3d}+\frac{2ad(dx+c)\cos(fx+e)}{f^2}+\frac{a\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\sin(fx+e)}{f^3}$
parts	$\frac{a(dx+c)^3}{3d}+\frac{a\left(\frac{d^2\left((fx+e)^2\sin(fx+e)-2\sin(fx+e)+2(fx+e)\cos(fx+e)\right)}{f^2}+\frac{2cd(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}-\frac{2d^2e(\cos(fx+e}{}\right.}{f}$
norman	$\frac{acd\,x^2+\frac{a\left(c^2f^2+2d^2\right)x}{f^2}+acd\,x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{a\left(c^2f^2-2d^2\right)x\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a\,d^2x^3}{3}-\frac{4acd\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{a\,d^2x^3(\text{t}}{}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$
derivativedivides	$\frac{a\,c^2\sin(fx+e)-\frac{2acde\sin(fx+e)}{f}+\frac{2acd(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{a\,d^2e^2\sin(fx+e)}{f^2}-\frac{2a\,d^2e(\cos(fx+e)+(fx+e)\sin(fx+e)}{f^2}}{}$
default	$\frac{a\,c^2\sin(fx+e)-\frac{2acde\sin(fx+e)}{f}+\frac{2acd(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+\frac{a\,d^2e^2\sin(fx+e)}{f^2}-\frac{2a\,d^2e(\cos(fx+e)+(fx+e)\sin(fx+e)}{f^2}}{}$

input
```
int((d*x+c)^2*(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
(((d*x+c)^2*f^2-2*d^2)*sin(f*x+e)+((2*d^2*x+2*c*d)*cos(f*x+e)+x*(1/3*x^2*d
^2+c*d*x+c^2)*f^2-2*c*d)*f)*a/f^3
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx$$
$$=\frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x+6\,(ad^2fx+acdf)\cos(fx+e)+3\,(ad^2f^2x^2+2\,acdf^2x+ac^2f^2-2\,ad^2}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x + 6*(a*d^2*f*x + a*c*
d*f)*cos(f*x + e) + 3*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2
)*sin(f*x + e))/f^3
```

3.119.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx$$
$$= \begin{cases} ac^2x + \frac{ac^2\sin(e+fx)}{f} + acdx^2 + \frac{2acdx\sin(e+fx)}{f} + \frac{2acd\cos(e+fx)}{f^2} + \frac{ad^2x^3}{3} + \frac{ad^2x^2\sin(e+fx)}{f} + \frac{2ad^2x\cos(e+fx)}{f^2} - \frac{2ad}{} \\ (a\cos(e)+a)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+a*cos(f*x+e)),x)
```

output
```
Piecewise((a*c**2*x + a*c**2*sin(e + f*x)/f + a*c*d*x**2 + 2*a*c*d*x*sin(e
 + f*x)/f + 2*a*c*d*cos(e + f*x)/f**2 + a*d**2*x**3/3 + a*d**2*x**2*sin(e
+ f*x)/f + 2*a*d**2*x*cos(e + f*x)/f**2 - 2*a*d**2*sin(e + f*x)/f**3, Ne(f
, 0)), ((a*cos(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.51

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx$$
$$= \frac{3(fx+e)ac^2 + \frac{(fx+e)^3ad^2}{f^2} - \frac{3(fx+e)^2ad^2e}{f^2} + \frac{3(fx+e)ad^2e^2}{f^2} + \frac{3(fx+e)^2acd}{f} - \frac{6(fx+e)acde}{f} + 3ac^2\sin(fx+e) + 3}{}$$

input
```
integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*(3*(f*x + e)*a*c^2 + (f*x + e)^3*a*d^2/f^2 - 3*(f*x + e)^2*a*d^2*e/f^2
 + 3*(f*x + e)*a*d^2*e^2/f^2 + 3*(f*x + e)^2*a*c*d/f - 6*(f*x + e)*a*c*d*e
/f + 3*a*c^2*sin(f*x + e) + 3*a*d^2*e^2*sin(f*x + e)/f^2 - 6*a*c*d*e*sin(f
*x + e)/f - 6*((f*x + e)*sin(f*x + e) + cos(f*x + e))*a*d^2*e/f^2 + 6*((f*
x + e)*sin(f*x + e) + cos(f*x + e))*a*c*d/f + 3*(2*(f*x + e)*cos(f*x + e)
+ ((f*x + e)^2 - 2)*sin(f*x + e))*a*d^2/f^2)/f
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.37

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{2\,(ad^2fx + acdf)\cos(fx+e)}{f^3} + \frac{(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2)\sin(fx+e)}{f^3}$$

input
```
integrate((d*x+c)^2*(a+a*cos(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e)
/f^3 + (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 - 2*a*d^2)*sin(f*x + e)/
f^3
```

3.119.9 Mupad [B] (verification not implemented)

Time = 14.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int (c+dx)^2(a+a\cos(e+fx))\,dx = \frac{a\,d^2\,x^3}{3} - \frac{\sin(e+f\,x)\,\left(2\,a\,d^2 - a\,c^2\,f^2\right)}{f^3} + a\,c^2\,x + a\,c\,d\,x^2 + \frac{2\,a\,d^2\,x\,\cos(e+f\,x)}{f^2} + \frac{a\,d^2\,x^2\,\sin(e+f\,x)}{f} + \frac{2\,a\,c\,d\,\cos(e+f\,x)}{f^2} + \frac{2\,a\,c\,d\,x\,\sin(e+f\,x)}{f}$$

input
```
int((a + a*cos(e + f*x))*(c + d*x)^2,x)
```

output
```
(a*d^2*x^3)/3 - (sin(e + f*x)*(2*a*d^2 - a*c^2*f^2))/f^3 + a*c^2*x + a*c*d
*x^2 + (2*a*d^2*x*cos(e + f*x))/f^2 + (a*d^2*x^2*sin(e + f*x))/f + (2*a*c*
d*cos(e + f*x))/f^2 + (2*a*c*d*x*sin(e + f*x))/f
```

3.120 $\int (c + dx)(a + a\cos(e + fx))\,dx$

3.120.1 Optimal result

Integrand size = 16, antiderivative size = 44

$$\int (c + dx)(a + a\cos(e + fx))\,dx = \frac{a(c + dx)^2}{2d} + \frac{ad\cos(e + fx)}{f^2} + \frac{a(c + dx)\sin(e + fx)}{f}$$

output
```
1/2*a*(d*x+c)^2/d+a*d*cos(f*x+e)/f^2+a*(d*x+c)*sin(f*x+e)/f
```

3.120.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int (c + dx)(a + a\cos(e + fx))\,dx = \frac{a(-2(e + fx)(de - 2cf - dfx) + 4d\cos(e + fx) + 4f(c + dx)\sin(e + fx))}{4f^2}$$

input
```
Integrate[(c + d*x)*(a + a*Cos[e + f*x]),x]
```

output
```
(a*(-2*(e + f*x)*(d*e - 2*c*f - d*f*x) + 4*d*Cos[e + f*x] + 4*f*(c + d*x)*
Sin[e + f*x]))/(4*f^2)
```

3.120.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a\cos(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)\cos(e+fx)+a(c+dx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)\sin(e+fx)}{f}+\frac{a(c+dx)^2}{2d}+\frac{ad\cos(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + a*Cos[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) + (a*d*Cos[e + f*x])/f^2 + (a*(c + d*x)*Sin[e + f*x]
)/f
```

3.120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.120.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result	siz
parallelrisch	$\frac{\left(f(dx+c)\sin(fx+e)+\cos(fx+e)d+x\left(\frac{dx}{2}+c\right)f^2+d\right)a}{f^2}$	40
risch	$\frac{ad\,x^2}{2}+acx+\frac{ad\cos(fx+e)}{f^2}+\frac{a(dx+c)\sin(fx+e)}{f}$	41
parts	$a\left(\frac{1}{2}d\,x^2+cx\right)+\frac{a\left(\frac{d(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+c\sin(fx+e)-\frac{de\sin(fx+e)}{f}\right)}{f}$	65
derivativedivides	$\frac{\sin(fx+e)ac-\frac{ade\sin(fx+e)}{f}+\frac{ad(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+ac(fx+e)-\frac{ade(fx+e)}{f}+\frac{ad(fx+e)^2}{2f}}{f}$	89
default	$\frac{\sin(fx+e)ac-\frac{ade\sin(fx+e)}{f}+\frac{ad(\cos(fx+e)+(fx+e)\sin(fx+e))}{f}+ac(fx+e)-\frac{ade(fx+e)}{f}+\frac{ad(fx+e)^2}{2f}}{f}$	89
norman	$\frac{acx+acx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{ad\,x^2}{2}-\frac{2ad\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{2ac\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{ad\,x^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{2adx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}}{1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)}$	11

input
```
int((d*x+c)*(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
(f*(d*x+c)*sin(f*x+e)+cos(f*x+e)*d+x*(1/2*d*x+c)*f^2+d)*a/f^2
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{adf^2x^2+2\,acf^2x+2\,ad\cos\left(fx+e\right)+2\left(adfx+acf\right)\sin\left(fx+e\right)}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x + 2*a*d*cos(f*x + e) + 2*(a*d*f*x + a*c*f)*
sin(f*x + e))/f^2
```

3.120.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \begin{cases} acx + \frac{ac\sin{(e+fx)}}{f} + \frac{adx^2}{2} + \frac{adx\sin{(e+fx)}}{f} + \frac{ad\cos{(e+fx)}}{f^2} & \text{for } f \neq 0 \\ (a\cos{(e)}+a)\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+a*cos(f*x+e)),x)
```

output
```
Piecewise((a*c*x + a*c*sin(e + f*x)/f + a*d*x**2/2 + a*d*x*sin(e + f*x)/f
+ a*d*cos(e + f*x)/f**2, Ne(f, 0)), ((a*cos(e) + a)*(c*x + d*x**2/2), True
))
```

3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.07

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{2\,(fx+e)ac + \frac{(fx+e)^2ad}{f} - \frac{2\,(fx+e)ade}{f} + 2\,ac\sin{(fx+e)} - \frac{2\,ade\sin(fx+e)}{f} + \frac{2\,((fx+e)\sin(fx+e)+\cos(fx+e))ad}{f}}{2\,f}$$

input
```
integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*(2*(f*x + e)*a*c + (f*x + e)^2*a*d/f - 2*(f*x + e)*a*d*e/f + 2*a*c*sin
(f*x + e) - 2*a*d*e*sin(f*x + e)/f + 2*((f*x + e)*sin(f*x + e) + cos(f*x +
 e))*a*d/f)/f
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{1}{2}adx^2+acx+\frac{ad\cos(fx+e)}{f^2}+\frac{(adfx+acf)\sin(fx+e)}{f^2}$$

input
```
integrate((d*x+c)*(a+a*cos(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x + a*d*cos(f*x + e)/f^2 + (a*d*f*x + a*c*f)*sin(f*x + e
)/f^2
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int (c+dx)(a+a\cos(e+fx))\,dx = \frac{\frac{a\,f\,(2\,c\,\sin(e+f\,x)+2\,d\,x\,\sin(e+f\,x))}{2}+a\,d\,\cos(e+f\,x)}{f^2} + \frac{a\,(d\,x^2+2\,c\,x)}{2}$$

input
```
int((a + a*cos(e + f*x))*(c + d*x),x)
```

output
```
((a*f*(2*c*sin(e + f*x) + 2*d*x*sin(e + f*x)))/2 + a*d*cos(e + f*x))/f^2 +
 (a*(2*c*x + d*x^2))/2
```

3.121 $\int \frac{a+a\cos(e+fx)}{c+dx}\,dx$

3.121.1 Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx=\frac{a\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}-\frac{a\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
a*Ci(c*f/d+f*x)*cos(-e+c*f/d)/d+a*ln(d*x+c)/d+a*Si(c*f/d+f*x)*sin(-e+c*f/d
)/d
```

3.121.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx =\frac{a\left(\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)+\log(c+dx)-\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

input
```
Integrate[(a + a*Cos[e + f*x])/(c + d*x),x]
```

output
```
(a*(Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] + Log[c + d*x] - Sin[e - (c*
f)/d]*SinIntegral[f*(c/d + x)]))/d
```

3.121.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(e+fx)+a}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(e+fx+\frac{\pi}{2}\right)+a}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a\cos(e+fx)}{c+dx}+\frac{a}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\cos\left(e-\frac{cf}{d}\right)}{d}-\frac{a\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

input
```
Int[(a + a*Cos[e + f*x])/(c + d*x),x]
```

output
```
(a*Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d - (
a*Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/d
```

3.121.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.121.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

method	result	size
parts	$\frac{a\ln(dx+c)}{d}+a\left(\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}\right)$	86
derivativedivides	$\frac{af\left(\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{af\ln(cf-de+d(fx+e))}{d}}{f}$	102
default	$\frac{af\left(\frac{\operatorname{Si}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}+\frac{\operatorname{Ci}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}\right)+\frac{af\ln(cf-de+d(fx+e))}{d}}{f}$	102
risch	$\frac{a\ln(dx+c)}{d}-\frac{a\,\mathrm{e}^{\frac{i(cf-de)}{d}}\operatorname{Ei}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d}-\frac{a\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\operatorname{Ei}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{2d}$	109

input
```
int((a+cos(f*x+e)*a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d+a*(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*
e)/d)*cos((c*f-d*e)/d)/d)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = \frac{a\cos\left(-\frac{de-cf}{d}\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)+a\sin\left(-\frac{de-cf}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)+a\log\left(dx+c\right)}{d}$$

input
```
integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
(a*cos(-(d*e - c*f)/d)*cos_integral((d*f*x + c*f)/d) + a*sin(-(d*e - c*f)/
d)*sin_integral((d*f*x + c*f)/d) + a*log(d*x + c))/d
```

3.121.6 Sympy [F]

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = a\left(\int \frac{\cos\left(e+fx\right)}{c+dx}\,dx + \int \frac{1}{c+dx}\,dx\right)$$

input
```
integrate((a+a*cos(f*x+e))/(d*x+c),x)
```

output
```
a*(Integral(cos(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

3.121.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.65

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = \frac{\frac{2\,af\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d} - \frac{\left(f\left(E_1\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_1\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)+f\left(i\,E_1\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)-i\,E_1\left(-i\cdots\right)\right)\right)}{d}}{2\,f}$$

input
```
integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(2*a*f*log(c + (f*x + e)*d/f - d*e/f)/d - (f*(exp_integral_e(1, (I*(f*
x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d*e +
 I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(I*exp_integral_e(1, (I*(f*x + e)*d -
I*d*e + I*c*f)/d) - I*exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*f)/d
))*sin(-(d*e - c*f)/d))*a/d)/f
```

3.121.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 673, normalized size of antiderivative = 10.35

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(2*a*log(abs(d*x + c))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + a*real_part(cos
_integral(f*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 + a*real_part(cos_in
tegral(-f*x - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d)^2 - 2*a*imag_part(cos_in
tegral(f*x + c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) + 2*a*imag_part(cos_integ
ral(-f*x - c*f/d))*tan(1/2*e)^2*tan(1/2*c*f/d) - 4*a*sin_integral((d*f*x +
 c*f)/d)*tan(1/2*e)^2*tan(1/2*c*f/d) + 2*a*imag_part(cos_integral(f*x + c*
f/d))*tan(1/2*e)*tan(1/2*c*f/d)^2 - 2*a*imag_part(cos_integral(-f*x - c*f/
d))*tan(1/2*e)*tan(1/2*c*f/d)^2 + 4*a*sin_integral((d*f*x + c*f)/d)*tan(1/
2*e)*tan(1/2*c*f/d)^2 + 2*a*log(abs(d*x + c))*tan(1/2*e)^2 - a*real_part(c
os_integral(f*x + c*f/d))*tan(1/2*e)^2 - a*real_part(cos_integral(-f*x - c
*f/d))*tan(1/2*e)^2 + 4*a*real_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*
tan(1/2*c*f/d) + 4*a*real_part(cos_integral(-f*x - c*f/d))*tan(1/2*e)*tan(
1/2*c*f/d) + 2*a*log(abs(d*x + c))*tan(1/2*c*f/d)^2 - a*real_part(cos_inte
gral(f*x + c*f/d))*tan(1/2*c*f/d)^2 - a*real_part(cos_integral(-f*x - c*f/
d))*tan(1/2*c*f/d)^2 - 2*a*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)
 + 2*a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*e) - 4*a*sin_integral
((d*f*x + c*f)/d)*tan(1/2*e) + 2*a*imag_part(cos_integral(f*x + c*f/d))*ta
n(1/2*c*f/d) - 2*a*imag_part(cos_integral(-f*x - c*f/d))*tan(1/2*c*f/d) +
4*a*sin_integral((d*f*x + c*f)/d)*tan(1/2*c*f/d) + 2*a*log(abs(d*x + c)) +
 a*real_part(cos_integral(f*x + c*f/d)) + a*real_part(cos_integral(-f*x...
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\cos(e+fx)}{c+dx}\,dx = \int \frac{a+a\,\cos\left(e+f\,x\right)}{c+d\,x}\,dx$$

input
```
int((a + a*cos(e + f*x))/(c + d*x),x)
```

output
```
int((a + a*cos(e + f*x))/(c + d*x), x)
```

3.122 $\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx$

3.122.1 Optimal result

Integrand size = 18, antiderivative size = 89

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)}-\frac{a\cos(e+fx)}{d(c+dx)} - \frac{af\,\text{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2} - \frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)-a*cos(f*x+e)/d/(d*x+c)-a*f*cos(-e+c*f/d)*Si(c*f/d+f*x)/d^2+a*
f*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^2
```

3.122.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{a\left(d(1+\cos(e+fx))+f(c+dx)\,\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)\sin\left(e-\frac{cf}{d}\right)+f(c+dx)\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(f\left(\frac{c}{d}\cdots\right.\right.\right.}{d^2(c+dx)}$$

input
```
Integrate[(a + a*Cos[e + f*x])/(c + d*x)^2,x]
```

output
```
-((a*(d*(1 + Cos[e + f*x]) + f*(c + d*x)*CosIntegral[f*(c/d + x)]*Sin[e -
(c*f)/d] + f*(c + d*x)*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)]))/(d^2*(c
 + d*x)))
```

3.122.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(e+fx)+a}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(e+fx+\frac{\pi}{2}\right)+a}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a\cos(e+fx)}{(c+dx)^2}+\frac{a}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{af\,\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{d^2}-\frac{af\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\cos(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

input
```
Int[(a + a*Cos[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) - (a*Cos[e + f*x])/(d*(c + d*x)) - (a*f*CosIntegral[(c*
f)/d + f*x]*Sin[e - (c*f)/d])/d^2 - (a*f*Cos[e - (c*f)/d]*SinIntegral[(c*f
)/d + f*x])/d^2
```

3.122.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.122.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.39

method	result
parts	$-\frac{a}{d(dx+c)}+af\left(-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}\right)$
derivativedivides	$\frac{af^2\left(-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-\frac{af^2}{(cf-de+d(fx+e))d}}{f}$
default	$\frac{af^2\left(-\frac{\cos(fx+e)}{(cf-de+d(fx+e))d}-\frac{\frac{\mathrm{Si}\left(fx+e+\frac{cf-de}{d}\right)\cos\left(\frac{cf-de}{d}\right)}{d}-\frac{\mathrm{Ci}\left(fx+e+\frac{cf-de}{d}\right)\sin\left(\frac{cf-de}{d}\right)}{d}}{d}\right)-\frac{af^2}{(cf-de+d(fx+e))d}}{f}$
risch	$-\frac{a}{d(dx+c)}+\frac{iaf\,e^{\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{2d^2}-\frac{ifa\,e^{-\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{2d^2}-\frac{a(-2dfx-2cf)}{2d(dx+c)(-}$

input
```
int((a+cos(f*x+e)*a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)+a*f*(-cos(f*x+e)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)
*cos((c*f-d*e)/d)/d-Ci(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d)
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{ad\cos\left(fx+e\right)-\left(adfx+acf\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right)+\left(adfx+acf\right)\cos\left(-\frac{de-cf}{d}\right)\operatorname{Si}\left(\frac{dfx+cf}{d}\right)+ad}{d^3x+cd^2}$$

input
```
integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(a*d*cos(f*x + e) - (a*d*f*x + a*c*f)*cos_integral((d*f*x + c*f)/d)*sin(-
(d*e - c*f)/d) + (a*d*f*x + a*c*f)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x
 + c*f)/d) + a*d)/(d^3*x + c*d^2)
```

3.122.6 Sympy [F]

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = a\left(\int \frac{\cos\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

input
```
integrate((a+a*cos(f*x+e))/(d*x+c)**2,x)
```

output
```
a*(Integral(cos(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c*
*2 + 2*c*d*x + d**2*x**2), x))
```

3.122.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.20

$$\int \frac{a+a\cos(e+fx)}{(c+dx)^2}\,dx = -\frac{\frac{2\,af^2}{(fx+e)d^2-d^2e+cdf}+\frac{\left(f^2\left(E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_2\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)-f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_2\left(-\right.\right.}{(fx+e)d^2-d^2e+cdf}}{2\,f}$$

input
```
integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) + (f^2*(exp_integral_e(2, (I
*(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d - I*d
*e + I*c*f)/d))*cos(-(d*e - c*f)/d) - f^2*(-I*exp_integral_e(2, (I*(f*x +
e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d - I*d*e + I
*c*f)/d))*sin(-(d*e - c*f)/d))*a/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(90) = 180$.

Time = 0.31 (sec) , antiderivative size = 535, normalized size of antiderivative = 6.01

$$\int \frac{a + a\cos(e + fx)}{(c + dx)^2}\, dx$$
$$= \frac{\left((dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right)f^2 \operatorname{Ci}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right) - de + cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right) - def^2 \operatorname{Ci}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} +\right.}{d}\right.\right.}{}$$
$$- \frac{a}{(dx+c)d}$$

input
```
integrate((a+a*cos(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos_integral(((d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-(d*e - c*f)/d) -
 d*e*f^2*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*sin(-(d*e - c*f)/d) + c*f^3*cos_integral(((d*x + c)*(d*e/(d*x +
 c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-(d*e - c*f)/d) - (d*x + c)*(
d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos(-(d*e - c*f)/d)*sin_integral(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + d*e*f^2*cos
(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) +
f) - d*e + c*f)/d) - c*f^3*cos(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(d*
e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - d*f^2*cos(-(d*x + c)*(d
*e/(d*x + c) - c*f/(d*x + c) + f)/d))*a*d^2/(((d*x + c)*d^4*(d*e/(d*x + c)
 - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f) - a/((d*x + c)*d)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\cos(e + fx)}{(c + dx)^2}\,dx = \int \frac{a + a\,\cos\left(e + f\,x\right)}{\left(c + d\,x\right)^2}\,dx$$

input
```
int((a + a*cos(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + a*cos(e + f*x))/(c + d*x)^2, x)
```

3.123 $\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$

3.123.1 Optimal result

Integrand size = 20, antiderivative size = 237

$$\begin{aligned}\int (c+dx)^3(a+a\cos(e+fx))^2\,dx =& -\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d} \\ &-\frac{12a^2d^3\cos(e+fx)}{f^4}+\frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2} \\ &-\frac{3a^2d^3\cos^2(e+fx)}{8f^4}+\frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2} \\ &-\frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3} \\ &+\frac{2a^2(c+dx)^3\sin(e+fx)}{f} \\ &-\frac{3a^2d^2(c+dx)\cos(e+fx)\sin(e+fx)}{4f^3} \\ &+\frac{a^2(c+dx)^3\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

output
```
-3/4*a^2*c*d^2*x/f^2-3/8*a^2*d^3*x^2/f^2+3/8*a^2*(d*x+c)^4/d-12*a^2*d^3*co
s(f*x+e)/f^4+6*a^2*d*(d*x+c)^2*cos(f*x+e)/f^2-3/8*a^2*d^3*cos(f*x+e)^2/f^4
+3/4*a^2*d*(d*x+c)^2*cos(f*x+e)^2/f^2-12*a^2*d^2*(d*x+c)*sin(f*x+e)/f^3+2*
a^2*(d*x+c)^3*sin(f*x+e)/f-3/4*a^2*d^2*(d*x+c)*cos(f*x+e)*sin(f*x+e)/f^3+1
/2*a^2*(d*x+c)^3*cos(f*x+e)*sin(f*x+e)/f
```

3.123.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.92

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$$
$$=\frac{a^2(96d(c^2f^2+2cdf^2x+d^2(-2+f^2x^2))\cos(e+fx)+3d(2c^2f^2+4cdf^2x+d^2(-1+2f^2x^2))\cos(2(e+f$$

input
```
Integrate[(c + d*x)^3*(a + a*Cos[e + f*x])^2,x]
```

output
```
(a^2*(96*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-2 + f^2*x^2))*Cos[e + f*x] + 3*d
*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(-1 + 2*f^2*x^2))*Cos[2*(e + f*x)] + 2*f*(
3*f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*(c + d*x)*(c^2*f^
2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Sin[e + f*x] + (c + d*x)*(2*c^2*f^2
+ 4*c*d*f^2*x + d^2*(-3 + 2*f^2*x^2))*Sin[2*(e + f*x)])))/(16*f^4)
```

3.123.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a\cos(e+fx)+a)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^3\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)^3\cos^2(e+fx)+2a^2(c+dx)^3\cos(e+fx)+a^2(c+dx)^3\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{12a^2d^2(c+dx)\sin(e+fx)}{f^3} - \frac{3a^2d^2(c+dx)\sin(e+fx)\cos(e+fx)}{4f^3} + \frac{3a^2d(c+dx)^2\cos^2(e+fx)}{4f^2} + \frac{6a^2d(c+dx)^2\cos(e+fx)}{f^2} + \frac{2a^2(c+dx)^3\sin(e+fx)}{f} + \frac{a^2(c+dx)^3\sin(e+fx)\cos(e+fx)}{2f} - \frac{3a^2d(c+dx)^2}{8f^2} + \frac{3a^2(c+dx)^4}{8d} - \frac{3a^2d^3\cos^2(e+fx)}{8f^4} - \frac{12a^2d^3\cos(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + a*Cos[e + f*x])^2,x]
```

output
```
(-3*a^2*d*(c + d*x)^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*d) - (12*a^2*d^3*C
os[e + f*x])/f^4 + (6*a^2*d*(c + d*x)^2*Cos[e + f*x])/f^2 - (3*a^2*d^3*Cos
[e + f*x]^2)/(8*f^4) + (3*a^2*d*(c + d*x)^2*Cos[e + f*x]^2)/(4*f^2) - (12*
a^2*d^2*(c + d*x)*Sin[e + f*x])/f^3 + (2*a^2*(c + d*x)^3*Sin[e + f*x])/f -
 (3*a^2*d^2*(c + d*x)*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) + (a^2*(c + d*x)^
3*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.123.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{\left((dx+c)\left((dx+c)^2 f^2-\frac{3d^2}{2}\right) f\sin(2fx+2e)+\frac{3\left((dx+c)^2 f^2-\frac{d^2}{2}\right) d\cos(2fx+2e)}{2}+8(dx+c)\left((dx+c)^2 f^2-6d^2\right) f\sin(fx+e)+\right.}{4f^4}$
risch	$\frac{3a^2d^3x^4}{8}+\frac{3a^2c\,d^2x^3}{2}+\frac{9a^2d\,c^2x^2}{4}+\frac{3a^2c^3x}{2}+\frac{3a^2c^4}{8d}+\frac{6a^2d\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\cos(fx+e)}{f^4}+\frac{2a^2}{}$
norman	$\frac{\frac{12a^2c^2d\,f^2-24a^2d^3}{f^4}+\frac{3a^2d^3x^4}{8}+\frac{\left(18a^2c^2d\,f^2-45a^2d^3\right)\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2f^4}+\frac{3a^2c\,d^2x^3}{2}+\frac{3a^2d^3x^4\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{4}+\frac{3a^2d^3x^4\left(\tan^4\right.}{8}}{}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^3*(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*((d*x+c)*((d*x+c)^2*f^2-3/2*d^2)*f*sin(2*f*x+2*e)+3/2*((d*x+c)^2*f^2-1
/2*d^2)*d*cos(2*f*x+2*e)+8*(d*x+c)*((d*x+c)^2*f^2-6*d^2)*f*sin(f*x+e)+24*(
(d*x+c)^2*f^2-2*d^2)*d*cos(f*x+e)+(6*c*d^2*x^3+9*x^2*c^2*d+3/2*d^3*x^4+6*x
*c^3)*f^4+45/2*c^2*d*f^2-189/4*d^3)*a^2/f^4
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.56

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$$
$$=\frac{3\,a^2d^3f^4x^4+12\,a^2cd^2f^4x^3+3\left(6\,a^2c^2df^4-a^2d^3f^2\right)x^2+3\left(2\,a^2d^3f^2x^2+4\,a^2cd^2f^2x+2\,a^2c^2df^2-a^2d^3\right)\mathrm{c}}{}$$

input
```
integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*(3*a^2*d^3*f^4*x^4 + 12*a^2*c*d^2*f^4*x^3 + 3*(6*a^2*c^2*d*f^4 - a^2*d
^3*f^2)*x^2 + 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 -
 a^2*d^3)*cos(f*x + e)^2 + 6*(2*a^2*c^3*f^4 - a^2*c*d^2*f^2)*x + 48*(a^2*d
^3*f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2 - 2*a^2*d^3)*cos(f*x + e) +
 2*(8*a^2*d^3*f^3*x^3 + 24*a^2*c*d^2*f^3*x^2 + 8*a^2*c^3*f^3 - 48*a^2*c*d^
2*f + 24*(a^2*c^2*d*f^3 - 2*a^2*d^3*f)*x + (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^
2*f^3*x^2 + 2*a^2*c^3*f^3 - 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 - a^2*d^3*f
)*x)*cos(f*x + e))*sin(f*x + e))/f^4
```

3.123.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(243) = 486$.

Time = 0.44 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.29

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^3x\sin^2{(e+fx)}}{2} + \frac{a^2c^3x\cos^2{(e+fx)}}{2} + a^2c^3x + \frac{a^2c^3\sin{(e+fx)}\cos{(e+fx)}}{2f} + \frac{2a^2c^3\sin{(e+fx)}}{f} + \frac{3a^2c^2dx^2\sin^2{(e+fx)}}{4} + \frac{3a^2c \\ (a\cos{(e)}+a)^2\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+a*cos(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**3*x*sin(e + f*x)**2/2 + a**2*c**3*x*cos(e + f*x)**2/2 +
 a**2*c**3*x + a**2*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c**3*sin
(e + f*x)/f + 3*a**2*c**2*d*x**2*sin(e + f*x)**2/4 + 3*a**2*c**2*d*x**2*co
s(e + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 + 3*a**2*c**2*d*x*sin(e + f*x)*cos(
e + f*x)/(2*f) + 6*a**2*c**2*d*x*sin(e + f*x)/f - 3*a**2*c**2*d*sin(e + f*
x)**2/(4*f**2) + 6*a**2*c**2*d*cos(e + f*x)/f**2 + a**2*c*d**2*x**3*sin(e
+ f*x)**2/2 + a**2*c*d**2*x**3*cos(e + f*x)**2/2 + a**2*c*d**2*x**3 + 3*a*
*2*c*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 6*a**2*c*d**2*x**2*sin(e
+ f*x)/f - 3*a**2*c*d**2*x*sin(e + f*x)**2/(4*f**2) + 3*a**2*c*d**2*x*cos(
e + f*x)**2/(4*f**2) + 12*a**2*c*d**2*x*cos(e + f*x)/f**2 - 3*a**2*c*d**2*
sin(e + f*x)*cos(e + f*x)/(4*f**3) - 12*a**2*c*d**2*sin(e + f*x)/f**3 + a*
*2*d**3*x**4*sin(e + f*x)**2/8 + a**2*d**3*x**4*cos(e + f*x)**2/8 + a**2*d
**3*x**4/4 + a**2*d**3*x**3*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*d**3*
x**3*sin(e + f*x)/f - 3*a**2*d**3*x**2*sin(e + f*x)**2/(8*f**2) + 3*a**2*d
**3*x**2*cos(e + f*x)**2/(8*f**2) + 6*a**2*d**3*x**2*cos(e + f*x)/f**2 - 3
*a**2*d**3*x*sin(e + f*x)*cos(e + f*x)/(4*f**3) - 12*a**2*d**3*x*sin(e + f
*x)/f**3 + 3*a**2*d**3*sin(e + f*x)**2/(8*f**4) - 12*a**2*d**3*cos(e + f*x
)/f**4, Ne(f, 0)), ((a*cos(e) + a)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x
**3 + d**3*x**4/4), True))
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(223) = 446$.

Time = 0.40 (sec) , antiderivative size = 949, normalized size of antiderivative = 4.00

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*(4*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^3 + 16*(f*x + e)*a^2*c^3 +
4*(f*x + e)^4*a^2*d^3/f^3 - 16*(f*x + e)^3*a^2*d^3*e/f^3 + 24*(f*x + e)^2*
a^2*d^3*e^2/f^3 - 4*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*d^3*e^3/f^3 - 16*
(f*x + e)*a^2*d^3*e^3/f^3 + 16*(f*x + e)^3*a^2*c*d^2/f^2 - 48*(f*x + e)^2*
a^2*c*d^2*e/f^2 + 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c*d^2*e^2/f^2 +
48*(f*x + e)*a^2*c*d^2*e^2/f^2 + 24*(f*x + e)^2*a^2*c^2*d/f - 12*(2*f*x +
2*e + sin(2*f*x + 2*e))*a^2*c^2*d*e/f - 48*(f*x + e)*a^2*c^2*d*e/f + 32*a^
2*c^3*sin(f*x + e) - 32*a^2*d^3*e^3*sin(f*x + e)/f^3 + 96*a^2*c*d^2*e^2*si
n(f*x + e)/f^2 - 96*a^2*c^2*d*e*sin(f*x + e)/f + 6*(2*(f*x + e)^2 + 2*(f*x
 + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d^3*e^2/f^3 + 96*((f*x + e)
*sin(f*x + e) + cos(f*x + e))*a^2*d^3*e^2/f^3 - 12*(2*(f*x + e)^2 + 2*(f*x
 + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*c*d^2*e/f^2 - 192*((f*x + e
)*sin(f*x + e) + cos(f*x + e))*a^2*c*d^2*e/f^2 + 6*(2*(f*x + e)^2 + 2*(f*x
 + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*c^2*d/f + 96*((f*x + e)*sin
(f*x + e) + cos(f*x + e))*a^2*c^2*d/f - 2*(4*(f*x + e)^3 + 6*(f*x + e)*cos
(2*f*x + 2*e) + 3*(2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^3*e/f^3 - 96
*(2*(f*x + e)*cos(f*x + e) + ((f*x + e)^2 - 2)*sin(f*x + e))*a^2*d^3*e/f^3
 + 2*(4*(f*x + e)^3 + 6*(f*x + e)*cos(2*f*x + 2*e) + 3*(2*(f*x + e)^2 - 1)
*sin(2*f*x + 2*e))*a^2*c*d^2/f^2 + 96*(2*(f*x + e)*cos(f*x + e) + ((f*x +
e)^2 - 2)*sin(f*x + e))*a^2*c*d^2/f^2 + (2*(f*x + e)^4 + 3*(2*(f*x + e)...
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.41

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx = \frac{3}{8}\,a^2d^3x^4 + \frac{3}{2}\,a^2cd^2x^3 + \frac{9}{4}\,a^2c^2dx^2 + \frac{3}{2}\,a^2c^3x + \frac{3\,(2\,a^2d^3f^2x^2 + 4\,a^2cd^2f^2x + 2\,a^2c^2df^2 - a^2d^3)\cos(2\,fx + 2\,e)}{16\,f^4} + \frac{6\,(a^2d^3f^2x^2 + 2\,a^2cd^2f^2x + a^2c^2df^2 - 2\,a^2d^3)\cos(fx + e)}{f^4} + \frac{(2\,a^2d^3f^3x^3 + 6\,a^2cd^2f^3x^2 + 6\,a^2c^2df^3x + 2\,a^2c^3f^3 - 3\,a^2d^3fx - 3\,a^2cd^2f)\sin(2\,fx + 2\,e)}{8\,f^4} + \frac{2\,(a^2d^3f^3x^3 + 3\,a^2cd^2f^3x^2 + 3\,a^2c^2df^3x + a^2c^3f^3 - 6\,a^2d^3fx - 6\,a^2cd^2f)\sin(fx + e)}{f^4}$$

input
```
integrate((d*x+c)^3*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

output
```
3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x +
3/16*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2 - a^2*d^3)*c
os(2*f*x + 2*e)/f^4 + 6*(a^2*d^3*f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f
^2 - 2*a^2*d^3)*cos(f*x + e)/f^4 + 1/8*(2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^
3*x^2 + 6*a^2*c^2*d*f^3*x + 2*a^2*c^3*f^3 - 3*a^2*d^3*f*x - 3*a^2*c*d^2*f)
*sin(2*f*x + 2*e)/f^4 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c
^2*d*f^3*x + a^2*c^3*f^3 - 6*a^2*d^3*f*x - 6*a^2*c*d^2*f)*sin(f*x + e)/f^4
```

3.123.9 Mupad [B] (verification not implemented)

Time = 15.28 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.91

$$\int (c+dx)^3(a+a\cos(e+fx))^2\,dx$$
$$= \frac{16\,a^2\,c^3\,f^3\,\sin\left(e+f\,x\right)-\frac{3\,a^2\,d^3\,\cos(2\,e+2\,f\,x)}{2}-96\,a^2\,d^3\,\cos\left(e+f\,x\right)+12\,a^2\,c^3\,f^4\,x+2\,a^2\,c^3\,f^3\,\sin\left(2\,e+2\right.}{}$$

input
```
int((a + a*cos(e + f*x))^2*(c + d*x)^3,x)
```

output
```
(16*a^2*c^3*f^3*sin(e + f*x) - (3*a^2*d^3*cos(2*e + 2*f*x))/2 - 96*a^2*d^3
*cos(e + f*x) + 12*a^2*c^3*f^4*x + 2*a^2*c^3*f^3*sin(2*e + 2*f*x) + 3*a^2*
d^3*f^4*x^4 - 96*a^2*c*d^2*f*sin(e + f*x) - 96*a^2*d^3*f*x*sin(e + f*x) +
3*a^2*d^3*f^2*x^2*cos(2*e + 2*f*x) + 2*a^2*d^3*f^3*x^3*sin(2*e + 2*f*x) +
48*a^2*c^2*d*f^2*cos(e + f*x) - 3*a^2*c*d^2*f*sin(2*e + 2*f*x) - 3*a^2*d^3
*f*x*sin(2*e + 2*f*x) + 3*a^2*c^2*d*f^2*cos(2*e + 2*f*x) + 18*a^2*c^2*d*f^
4*x^2 + 12*a^2*c*d^2*f^4*x^3 + 48*a^2*d^3*f^2*x^2*cos(e + f*x) + 16*a^2*d^
3*f^3*x^3*sin(e + f*x) + 6*a^2*c*d^2*f^2*x*cos(2*e + 2*f*x) + 6*a^2*c^2*d*
f^3*x*sin(2*e + 2*f*x) + 48*a^2*c*d^2*f^3*x^2*sin(e + f*x) + 6*a^2*c*d^2*f
^3*x^2*sin(2*e + 2*f*x) + 96*a^2*c*d^2*f^2*x*cos(e + f*x) + 48*a^2*c^2*d*f
^3*x*sin(e + f*x))/(8*f^4)
```

3.124 $\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$

3.124.1 Optimal result

Integrand size = 20, antiderivative size = 168

$$\begin{aligned}\int (c+dx)^2(a+a\cos(e+fx))^2\,dx = &-\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}\\&+\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}\\&-\frac{4a^2d^2\sin(e+fx)}{f^3}+\frac{2a^2(c+dx)^2\sin(e+fx)}{f}\\&-\frac{a^2d^2\cos(e+fx)\sin(e+fx)}{4f^3}\\&+\frac{a^2(c+dx)^2\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

output
```
-1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d+4*a^2*d*(d*x+c)*cos(f*x+e)/f^2+1/2*
a^2*d*(d*x+c)*cos(f*x+e)^2/f^2-4*a^2*d^2*sin(f*x+e)/f^3+2*a^2*(d*x+c)^2*si
n(f*x+e)/f-1/4*a^2*d^2*cos(f*x+e)*sin(f*x+e)/f^3+1/2*a^2*(d*x+c)^2*cos(f*x
+e)*sin(f*x+e)/f
```

3.124.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.15

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$=\frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3+32df(c+dx)\cos(e+fx)+2df(c+dx)\cos(2(e+fx))-32d^2\sin(e}{}$$

input
```
Integrate[(c + d*x)^2*(a + a*Cos[e + f*x])^2,x]
```

output
```
(a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 + 32*d*f*(c + d*x)*Cos
[e + f*x] + 2*d*f*(c + d*x)*Cos[2*(e + f*x)] - 32*d^2*Sin[e + f*x] + 16*c^
2*f^2*Sin[e + f*x] + 32*c*d*f^2*x*Sin[e + f*x] + 16*d^2*f^2*x^2*Sin[e + f*
x] - d^2*Sin[2*(e + f*x)] + 2*c^2*f^2*Sin[2*(e + f*x)] + 4*c*d*f^2*x*Sin[2
*(e + f*x)] + 2*d^2*f^2*x^2*Sin[2*(e + f*x)]))/(8*f^3)
```

3.124.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a\cos(e+fx)+a)^2\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (c+dx)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2dx$$
$$\Big\downarrow \text{ 3798}$$
$$\int \left(a^2(c+dx)^2\cos^2(e+fx)+2a^2(c+dx)^2\cos(e+fx)+a^2(c+dx)^2\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2d(c+dx)\cos^2(e+fx)}{2f^2}+\frac{4a^2d(c+dx)\cos(e+fx)}{f^2}+\frac{2a^2(c+dx)^2\sin(e+fx)}{f}+\frac{a^2(c+dx)^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}-\frac{4a^2d^2\sin(e+fx)}{f^3}-\frac{a^2d^2\sin(e+fx)\cos(e+fx)}{4f^3}-\frac{a^2d^2x}{4f^2}$$

input
```
Int[(c + d*x)^2*(a + a*Cos[e + f*x])^2,x]
```

output
```
-1/4*(a^2*d^2*x)/f^2 + (a^2*(c + d*x)^3)/(2*d) + (4*a^2*d*(c + d*x)*Cos[e
+ f*x])/f^2 + (a^2*d*(c + d*x)*Cos[e + f*x]^2)/(2*f^2) - (4*a^2*d^2*Sin[e
+ f*x])/f^3 + (2*a^2*(c + d*x)^2*Sin[e + f*x])/f - (a^2*d^2*Cos[e + f*x]*S
in[e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.124.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{\left(\left((dx+c)^2 f^2-\frac{d^2}{2}\right)\sin(2fx+2e)+df(dx+c)\cos(2fx+2e)+8\left((dx+c)^2 f^2-2d^2\right)\sin(fx+e)+6\left(\frac{8d(dx+c)\cos(fx+e)}{3}+x\left(\frac{1}{3}\right.\right.\right.}{4f^3}$
risch	$\frac{a^2 d^2 x^3}{2}+\frac{3a^2 cd\,x^2}{2}+\frac{3a^2 c^2 x}{2}+\frac{a^2 c^3}{2d}+\frac{4a^2 d(dx+c)\cos(fx+e)}{f^2}+\frac{2a^2\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2\right)\sin(fx+e)}{f^3}$
norman	$\frac{\frac{8a^2cd}{f^2}+a^2d^2x^3\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{a^2d^2x^3}{2}+\frac{6a^2cd\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{3a^2cd\,x^2}{2}+\frac{a^2d^2x^3\left(\tan^4\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{2}+\frac{3a^2\left(2c^2f^2-5d^2\right)(\tan}{2f^3}}{}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{a^2\left(\frac{d^2\left((fx+e)^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)+\frac{(fx+e)\left(\cos^2(fx+e)\right)}{2}-\frac{\cos(fx+e)\sin(fx+e)}{4}-\frac{fx}{4}-\frac{e}{4}-\frac{(fx+e}{3}\right.}{f^2}\right.}{}$
derivativedivides	$\frac{a^2c^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cde\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a^2cd\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)\right.}{f}}{}$
default	$\frac{a^2c^2\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{2a^2cde\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{2a^2cd\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)\right.}{f}}{}$

input
```
int((d*x+c)^2*(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(((d*x+c)^2*f^2-1/2*d^2)*sin(2*f*x+2*e)+d*f*(d*x+c)*cos(2*f*x+2*e)+8*(
(d*x+c)^2*f^2-2*d^2)*sin(f*x+e)+6*(8/3*d*(d*x+c)*cos(f*x+e)+x*(1/3*x^2*d^2
+c*d*x+c^2)*f^2-17/6*c*d)*f)*a^2/f^3
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.26

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$=\frac{2\,a^2d^2f^3x^3+6\,a^2cdf^3x^2+2\,(a^2d^2fx+a^2cdf)\cos{(fx+e)}^2+(6\,a^2c^2f^3-a^2d^2f)x+16\,(a^2d^2fx+a^2cdf)}{}$$

input
```
integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 + 2*(a^2*d^2*f*x + a^2*c*d*f)*c
os(f*x + e)^2 + (6*a^2*c^2*f^3 - a^2*d^2*f)*x + 16*(a^2*d^2*f*x + a^2*c*d*
f)*cos(f*x + e) + (8*a^2*d^2*f^2*x^2 + 16*a^2*c*d*f^2*x + 8*a^2*c^2*f^2 -
16*a^2*d^2 + (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^
2)*cos(f*x + e))*sin(f*x + e))/f^3
```

3.124.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(163) = 326$.

Time = 0.33 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.71

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$= \begin{cases} \frac{a^2c^2x\sin^2(e+fx)}{2} + \frac{a^2c^2x\cos^2(e+fx)}{2} + a^2c^2x + \frac{a^2c^2\sin(e+fx)\cos(e+fx)}{2f} + \frac{2a^2c^2\sin(e+fx)}{f} + \frac{a^2cdx^2\sin^2(e+fx)}{2} + \frac{a^2cdx^2}{} \\ (a\cos(e)+a)^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+a*cos(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**2*x*sin(e + f*x)**2/2 + a**2*c**2*x*cos(e + f*x)**2/2 +
 a**2*c**2*x + a**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c**2*sin
(e + f*x)/f + a**2*c*d*x**2*sin(e + f*x)**2/2 + a**2*c*d*x**2*cos(e + f*x)
**2/2 + a**2*c*d*x**2 + a**2*c*d*x*sin(e + f*x)*cos(e + f*x)/f + 4*a**2*c*
d*x*sin(e + f*x)/f - a**2*c*d*sin(e + f*x)**2/(2*f**2) + 4*a**2*c*d*cos(e
+ f*x)/f**2 + a**2*d**2*x**3*sin(e + f*x)**2/6 + a**2*d**2*x**3*cos(e + f*
x)**2/6 + a**2*d**2*x**3/3 + a**2*d**2*x**2*sin(e + f*x)*cos(e + f*x)/(2*f
) + 2*a**2*d**2*x**2*sin(e + f*x)/f - a**2*d**2*x*sin(e + f*x)**2/(4*f**2)
 + a**2*d**2*x*cos(e + f*x)**2/(4*f**2) + 4*a**2*d**2*x*cos(e + f*x)/f**2
- a**2*d**2*sin(e + f*x)*cos(e + f*x)/(4*f**3) - 4*a**2*d**2*sin(e + f*x)/
f**3, Ne(f, 0)), ((a*cos(e) + a)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), Tru
e))
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(158) = 316$.

Time = 0.32 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.94

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$= \frac{6\,(2\,fx+2\,e+\sin\left(2\,fx+2\,e\right))a^2c^2+24\,(fx+e)a^2c^2+\frac{8\,(fx+e)^3a^2d^2}{f^2}-\frac{24\,(fx+e)^2a^2d^2e}{f^2}+\frac{6\,(2\,fx+2\,e+\sin(2\,fx+2}{f^2}}{}$$

input
```
integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/24*(6*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c^2 + 24*(f*x + e)*a^2*c^2 +
8*(f*x + e)^3*a^2*d^2/f^2 - 24*(f*x + e)^2*a^2*d^2*e/f^2 + 6*(2*f*x + 2*e
+ sin(2*f*x + 2*e))*a^2*d^2*e^2/f^2 + 24*(f*x + e)*a^2*d^2*e^2/f^2 + 24*(f
*x + e)^2*a^2*c*d/f - 12*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c*d*e/f - 48
*(f*x + e)*a^2*c*d*e/f + 48*a^2*c^2*sin(f*x + e) + 48*a^2*d^2*e^2*sin(f*x
+ e)/f^2 - 96*a^2*c*d*e*sin(f*x + e)/f - 6*(2*(f*x + e)^2 + 2*(f*x + e)*si
n(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d^2*e/f^2 - 96*((f*x + e)*sin(f*x +
 e) + cos(f*x + e))*a^2*d^2*e/f^2 + 6*(2*(f*x + e)^2 + 2*(f*x + e)*sin(2*f
*x + 2*e) + cos(2*f*x + 2*e))*a^2*c*d/f + 96*((f*x + e)*sin(f*x + e) + cos
(f*x + e))*a^2*c*d/f + (4*(f*x + e)^3 + 6*(f*x + e)*cos(2*f*x + 2*e) + 3*(
2*(f*x + e)^2 - 1)*sin(2*f*x + 2*e))*a^2*d^2/f^2 + 48*(2*(f*x + e)*cos(f*x
 + e) + ((f*x + e)^2 - 2)*sin(f*x + e))*a^2*d^2/f^2)/f
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.21

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$= \frac{1}{2}\,a^2d^2x^3+\frac{3}{2}\,a^2cdx^2+\frac{3}{2}\,a^2c^2x+\frac{(a^2d^2fx+a^2cdf)\cos\left(2\,fx+2\,e\right)}{4\,f^3}$$
$$+\frac{4\,(a^2d^2fx+a^2cdf)\cos\left(fx+e\right)}{f^3}$$
$$+\frac{(2\,a^2d^2f^2x^2+4\,a^2cdf^2x+2\,a^2c^2f^2-a^2d^2)\sin\left(2\,fx+2\,e\right)}{8\,f^3}$$
$$+\frac{2\,(a^2d^2f^2x^2+2\,a^2cdf^2x+a^2c^2f^2-2\,a^2d^2)\sin\left(fx+e\right)}{f^3}$$

input
```
integrate((d*x+c)^2*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x + 1/4*(a^2*d^2*f*x + a^2
*c*d*f)*cos(2*f*x + 2*e)/f^3 + 4*(a^2*d^2*f*x + a^2*c*d*f)*cos(f*x + e)/f^
3 + 1/8*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - a^2*d^2)*si
n(2*f*x + 2*e)/f^3 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2 -
2*a^2*d^2)*sin(f*x + e)/f^3
```

3.124.9 Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+a\cos(e+fx))^2\,dx$$
$$= \frac{8\,a^2\,c^2\,f^2\,\sin\left(e+f\,x\right)-\frac{a^2\,d^2\,\sin(2\,e+2\,f\,x)}{2}-16\,a^2\,d^2\,\sin\left(e+f\,x\right)+6\,a^2\,c^2\,f^3\,x+a^2\,c^2\,f^2\,\sin\left(2\,e+2\,f\,x\right)+}{}$$

input
```
int((a + a*cos(e + f*x))^2*(c + d*x)^2,x)
```

output
```
(8*a^2*c^2*f^2*sin(e + f*x) - (a^2*d^2*sin(2*e + 2*f*x))/2 - 16*a^2*d^2*si
n(e + f*x) + 6*a^2*c^2*f^3*x + a^2*c^2*f^2*sin(2*e + 2*f*x) + 2*a^2*d^2*f^
3*x^3 + a^2*c*d*f*cos(2*e + 2*f*x) + 16*a^2*d^2*f*x*cos(e + f*x) + a^2*d^2
*f^2*x^2*sin(2*e + 2*f*x) + 6*a^2*c*d*f^3*x^2 + a^2*d^2*f*x*cos(2*e + 2*f*
x) + 16*a^2*c*d*f*cos(e + f*x) + 8*a^2*d^2*f^2*x^2*sin(e + f*x) + 16*a^2*c
*d*f^2*x*sin(e + f*x) + 2*a^2*c*d*f^2*x*sin(2*e + 2*f*x))/(4*f^3)
```

3.125 $\int (c+dx)(a+a\cos(e+fx))^2\,dx$

3.125.1 Optimal result

Integrand size = 18, antiderivative size = 118

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx = \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}+\frac{2a^2d\cos(e+fx)}{f^2}+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)\sin(e+fx)}{f}+\frac{a^2(c+dx)\cos(e+fx)\sin(e+fx)}{2f}$$

output
```
1/2*a^2*c*x+1/4*a^2*d*x^2+1/2*a^2*(d*x+c)^2/d+2*a^2*d*cos(f*x+e)/f^2+1/4*a
^2*d*cos(f*x+e)^2/f^2+2*a^2*(d*x+c)*sin(f*x+e)/f+1/2*a^2*(d*x+c)*cos(f*x+e
)*sin(f*x+e)/f
```

3.125.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx = \frac{a^2(-6(e+fx)(-2cf+d(e-fx))+16d\cos(e+fx)+d\cos(2(e+fx))+16f(c+dx)\sin(e+fx)+2f}{8f^2}$$

input
```
Integrate[(c + d*x)*(a + a*Cos[e + f*x])^2,x]
```

output
```
(a^2*(-6*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*d*Cos[e + f*x] + d*Cos[2*(e
 + f*x)] + 16*f*(c + d*x)*Sin[e + f*x] + 2*f*(c + d*x)*Sin[2*(e + f*x)]))/
(8*f^2)
```

3.125.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a\cos(e+fx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)\cos^2(e+fx)+2a^2(c+dx)\cos(e+fx)+a^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2(c+dx)\sin(e+fx)}{f}+\frac{a^2(c+dx)\sin(e+fx)\cos(e+fx)}{2f}+\frac{3a^2(c+dx)^2}{4d}+\frac{a^2d\cos^2(e+fx)}{4f^2}+\frac{2a^2d\cos(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + a*Cos[e + f*x])^2,x]
```

output
```
(3*a^2*(c + d*x)^2)/(4*d) + (2*a^2*d*Cos[e + f*x])/f^2 + (a^2*d*Cos[e + f*
x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sin[e + f*x])/f + (a^2*(c + d*x)*Cos[e +
f*x]*Sin[e + f*x])/(2*f)
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

3.125.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.78

method	result
risch	$\frac{3a^2d\,x^2}{4}+\frac{3a^2cx}{2}+\frac{2a^2d\cos(fx+e)}{f^2}+\frac{2a^2(dx+c)\sin(fx+e)}{f}+\frac{a^2d\cos(2fx+2e)}{8f^2}+\frac{a^2(dx+c)\sin(2fx+2e)}{4f}$
parts	$a^2\left(\frac{1}{2}d\,x^2+cx\right)+\frac{a^2\left(\frac{d\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{(fx+e)^2}{4}-\frac{\left(\sin^2(fx+e)\right)}{4}\right)}{f}+c\left(\frac{\cos(fx+e)\sin(fx+e)}{2}\right.\right.}{f}$
norman	$\frac{\frac{4a^2d}{f^2}+\frac{3a^2d\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f^2}+\frac{3a^2cx}{2}+\frac{3a^2d\,x^2}{4}+\frac{5a^2c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{3a^2c\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{f}+3a^2cx\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\frac{3a^2cx(\tan}{}}{\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right.}$
derivativedivides	$\frac{a^2c\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2de\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2d\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-(fx\right.}{f}}{}$
default	$\frac{a^2c\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-\frac{a^2de\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}+\frac{a^2d\left((fx+e)\left(\frac{\cos(fx+e)\sin(fx+e)}{2}+\frac{fx}{2}+\frac{e}{2}\right)-(fx\right.}{f}}{}$

input
```
int((d*x+c)*(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4*a^2*d*x^2+3/2*a^2*c*x+2*a^2*d*cos(f*x+e)/f^2+2*a^2*(d*x+c)*sin(f*x+e)/
f+1/8*a^2*d/f^2*cos(2*f*x+2*e)+1/4*a^2/f*(d*x+c)*sin(2*f*x+2*e)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$
$$= \frac{3\,a^2df^2x^2 + 6\,a^2cf^2x + a^2d\cos\left(fx+e\right)^2 + 8\,a^2d\cos\left(fx+e\right) + 2\left(4\,a^2dfx + 4\,a^2cf + \left(a^2dfx + a^2cf\right)\cos\right.}{4\,f^2}$$

input
```
integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*(3*a^2*d*f^2*x^2 + 6*a^2*c*f^2*x + a^2*d*cos(f*x + e)^2 + 8*a^2*d*cos(
f*x + e) + 2*(4*a^2*d*f*x + 4*a^2*c*f + (a^2*d*f*x + a^2*c*f)*cos(f*x + e)
)*sin(f*x + e))/f^2
```

3.125.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.86

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$
$$= \begin{cases} \frac{a^2cx\sin^2\left(e+fx\right)}{2} + \frac{a^2cx\cos^2\left(e+fx\right)}{2} + a^2cx + \frac{a^2c\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} + \frac{2a^2c\sin\left(e+fx\right)}{f} + \frac{a^2dx^2\sin^2\left(e+fx\right)}{4} + \frac{a^2dx^2\cos^2\left(e\right.}{4} \\ \left(a\cos\left(e\right)+a\right)^2\left(cx+\frac{dx^2}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)*(a+a*cos(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c*x*sin(e + f*x)**2/2 + a**2*c*x*cos(e + f*x)**2/2 + a**2*
c*x + a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*c*sin(e + f*x)/f + a
**2*d*x**2*sin(e + f*x)**2/4 + a**2*d*x**2*cos(e + f*x)**2/4 + a**2*d*x**2
/2 + a**2*d*x*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*a**2*d*x*sin(e + f*x)/f
- a**2*d*sin(e + f*x)**2/(4*f**2) + 2*a**2*d*cos(e + f*x)/f**2, Ne(f, 0)),
 ((a*cos(e) + a)**2*(c*x + d*x**2/2), True))
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.67

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$

$$= \frac{2\,(2\,fx+2\,e+\sin\left(2\,fx+2\,e\right))a^2c+8\,(fx+e)a^2c+\frac{4\,(fx+e)^2a^2d}{f}-\frac{2\,(2\,fx+2\,e+\sin(2\,fx+2\,e))a^2de}{f}-\frac{8\,(fx+e)a^2de}{f}}{}$$

input
```
integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/8*(2*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*c + 8*(f*x + e)*a^2*c + 4*(f*x
 + e)^2*a^2*d/f - 2*(2*f*x + 2*e + sin(2*f*x + 2*e))*a^2*d*e/f - 8*(f*x +
e)*a^2*d*e/f + 16*a^2*c*sin(f*x + e) - 16*a^2*d*e*sin(f*x + e)/f + (2*(f*x
 + e)^2 + 2*(f*x + e)*sin(2*f*x + 2*e) + cos(2*f*x + 2*e))*a^2*d/f + 16*((
f*x + e)*sin(f*x + e) + cos(f*x + e))*a^2*d/f)/f
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx = \frac{3}{4}\,a^2dx^2+\frac{3}{2}\,a^2cx+\frac{a^2d\cos\left(2\,fx+2\,e\right)}{8\,f^2} + \frac{2\,a^2d\cos\left(fx+e\right)}{f^2}+\frac{\left(a^2dfx+a^2cf\right)\sin\left(2\,fx+2\,e\right)}{4\,f^2} + \frac{2\,\left(a^2dfx+a^2cf\right)\sin\left(fx+e\right)}{f^2}$$

input
```
integrate((d*x+c)*(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

output
```
3/4*a^2*d*x^2 + 3/2*a^2*c*x + 1/8*a^2*d*cos(2*f*x + 2*e)/f^2 + 2*a^2*d*cos
(f*x + e)/f^2 + 1/4*(a^2*d*f*x + a^2*c*f)*sin(2*f*x + 2*e)/f^2 + 2*(a^2*d*
f*x + a^2*c*f)*sin(f*x + e)/f^2
```

3.125.9 Mupad [B] (verification not implemented)

Time = 14.94 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int (c+dx)(a+a\cos(e+fx))^2\,dx$$
$$= \frac{3\,a^2\,d\,f^2\,x^2 - 16\,a^2\,d\sin\left(\frac{e}{2}+\frac{f\,x}{2}\right)^2 - a^2\,d\sin\left(e+f\,x\right)^2 + 8\,a^2\,c\,f\,\sin\left(e+f\,x\right) + a^2\,c\,f\,\sin\left(2\,e+2\,f\,x\right) +}{4\,f^2}$$

input
```
int((a + a*cos(e + f*x))^2*(c + d*x),x)
```

output
```
(3*a^2*d*f^2*x^2 - 16*a^2*d*sin(e/2 + (f*x)/2)^2 - a^2*d*sin(e + f*x)^2 +
8*a^2*c*f*sin(e + f*x) + a^2*c*f*sin(2*e + 2*f*x) + 6*a^2*c*f^2*x + a^2*d*
f*x*sin(2*e + 2*f*x) + 8*a^2*d*f*x*sin(e + f*x))/(4*f^2)
```

3.126 $\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx$

3.126.1 Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = \frac{2a^2\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a^2\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2cf}{d}+2fx\right)}{2d} + \frac{3a^2\log(c+dx)}{2d} - \frac{2a^2\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(\frac{cf}{d}+fx\right)}{d} - \frac{a^2\sin\left(2e-\frac{2cf}{d}\right)\text{Si}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

output
```
1/2*a^2*Ci(2*c*f/d+2*f*x)*cos(-2*e+2*c*f/d)/d+2*a^2*Ci(c*f/d+f*x)*cos(-e+c
*f/d)/d+3/2*a^2*ln(d*x+c)/d+1/2*a^2*Si(2*c*f/d+2*f*x)*sin(-2*e+2*c*f/d)/d+
2*a^2*Si(c*f/d+f*x)*sin(-e+c*f/d)/d
```

3.126.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = \frac{a^2\left(4\cos\left(e-\frac{cf}{d}\right)\text{CosIntegral}\left(f\left(\frac{c}{d}+x\right)\right)+\cos\left(2e-\frac{2cf}{d}\right)\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)+3\log(c+dx)-4\sin\cdots\right.}{2d}$$

input
```
Integrate[(a + a*Cos[e + f*x])^2/(c + d*x),x]
```

output
```
(a^2*(4*Cos[e - (c*f)/d]*CosIntegral[f*(c/d + x)] + Cos[2*e - (2*c*f)/d]*C
osIntegral[(2*f*(c + d*x))/d] + 3*Log[c + d*x] - 4*Sin[e - (c*f)/d]*SinInt
egral[f*(c/d + x)] - Sin[2*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d]))
/(2*d)
```

3.126.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(e+fx)+a)^2}{c+dx}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{c+dx}dx$$

$$\Big\downarrow \text{ 3799}$$

$$4a^2\int \frac{\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{ 3042}$$

$$4a^2\int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^4}{c+dx}dx$$

$$\Big\downarrow \text{ 3793}$$

$$4a^2\int \left(\frac{\cos(e+fx)}{2(c+dx)}+\frac{\cos(2e+2fx)}{8(c+dx)}+\frac{3}{8(c+dx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$4a^2\left(\frac{\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\cos\left(e-\frac{cf}{d}\right)}{2d}+\frac{\text{CosIntegral}\left(2xf+\frac{2cf}{d}\right)\cos\left(2e-\frac{2cf}{d}\right)}{8d}-\frac{\sin\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{2d}\right.$$

input
```
Int[(a + a*Cos[e + f*x])^2/(c + d*x),x]
```

output
```
4*a^2*((Cos[e - (c*f)/d]*CosIntegral[(c*f)/d + f*x])/(2*d) + (Cos[2*e - (2
*c*f)/d]*CosIntegral[(2*c*f)/d + 2*f*x])/(8*d) + (3*Log[c + d*x])/(8*d) -
(Sin[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(2*d) - (Sin[2*e - (2*c*f)/d
]*SinIntegral[(2*c*f)/d + 2*f*x])/(8*d))
```

3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.126.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{f a^2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+\frac{3f a^2\ln(cf-de+d(fx+e))}{2d}+2f a^2\left(\right.}{f}$
default	$\frac{\frac{f a^2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}\right)}{4}+\frac{3f a^2\ln(cf-de+d(fx+e))}{2d}+2f a^2\left(\right.}{f}$
parts	$\frac{a^2\ln(dx+c)}{d}+\frac{a^2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{2d}+\frac{a^2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{2d}+\frac{a^2\ln(cf-d}{2}$
risch	$-\frac{a^2 e^{\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{d}-\frac{a^2 e^{-\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{d}+\frac{3a^2\ln(dx+c)}{2d}-\frac{a^2 e^{\frac{2i(cf-de)}{d}}}{}$

input
```
int((a+cos(f*x+e)*a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*f*a^2*(2*Si(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*(c*f-d*e)/d)/d+2*Ci(2*
f*x+2*e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d)+3/2*f*a^2*ln(c*f-d*e+d*(f*x+e
))/d+2*f*a^2*(Si(f*x+e+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d+Ci(f*x+e+(c*f-d*e)/
d)*cos((c*f-d*e)/d)/d))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx$$
$$=\frac{a^2\cos\left(-\frac{2(de-cf)}{d}\right)\mathrm{Ci}\left(\frac{2(dfx+cf)}{d}\right)+4\,a^2\cos\left(-\frac{de-cf}{d}\right)\mathrm{Ci}\left(\frac{dfx+cf}{d}\right)+a^2\sin\left(-\frac{2(de-cf)}{d}\right)\mathrm{Si}\left(\frac{2(dfx+cf)}{d}\right)+4\,a}{2\,d}$$

input
```
integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(a^2*cos(-2*(d*e - c*f)/d)*cos_integral(2*(d*f*x + c*f)/d) + 4*a^2*cos
(-(d*e - c*f)/d)*cos_integral((d*f*x + c*f)/d) + a^2*sin(-2*(d*e - c*f)/d)
*sin_integral(2*(d*f*x + c*f)/d) + 4*a^2*sin(-(d*e - c*f)/d)*sin_integral(
(d*f*x + c*f)/d) + 3*a^2*log(d*x + c))/d
```

3.126.6 Sympy [F]

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx = a^2\left(\int \frac{2\cos{(e+fx)}}{c+dx}\,dx + \int \frac{\cos^2{(e+fx)}}{c+dx}\,dx + \int \frac{1}{c+dx}\,dx\right)$$

input
```
integrate((a+a*cos(f*x+e))**2/(d*x+c),x)
```

output
```
a**2*(Integral(2*cos(e + f*x)/(c + d*x), x) + Integral(cos(e + f*x)**2/(c
+ d*x), x) + Integral(1/(c + d*x), x))
```

3.126.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.34

$$\int \frac{(a+a\cos(e+fx))^2}{c+dx}\,dx$$
$$= \frac{\frac{4a^2f\log\left(c+\frac{(fx+e)d}{f}-\frac{de}{f}\right)}{d} - \frac{4\Big(f\Big(E_1\Big(\frac{i(fx+e)d-i\,de+i\,cf}{d}\Big)+E_1\Big(-\frac{i(fx+e)d-i\,de+i\,cf}{d}\Big)\Big)\cos\Big(-\frac{de-cf}{d}\Big)+f\Big(i\,E_1\Big(\frac{i(fx+e)d-i\,de+i\,cf}{d}\Big)-i\,E_1\Big(\cdots}{d}}{\cdots}$$

input
```
integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/4*(4*a^2*f*log(c + (f*x + e)*d/f - d*e/f)/d - 4*(f*(exp_integral_e(1, (I
*(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(1, -(I*(f*x + e)*d - I*d
*e + I*c*f)/d))*cos(-(d*e - c*f)/d) + f*(I*exp_integral_e(1, (I*(f*x + e)*
d - I*d*e + I*c*f)/d) - I*exp_integral_e(1, -(I*(f*x + e)*d - I*d*e + I*c*
f)/d))*sin(-(d*e - c*f)/d))*a^2/d - (f*(exp_integral_e(1, 2*(-I*(f*x + e)*
d + I*d*e - I*c*f)/d) + exp_integral_e(1, -2*(-I*(f*x + e)*d + I*d*e - I*c
*f)/d))*cos(-2*(d*e - c*f)/d) - f*(I*exp_integral_e(1, 2*(-I*(f*x + e)*d +
 I*d*e - I*c*f)/d) - I*exp_integral_e(1, -2*(-I*(f*x + e)*d + I*d*e - I*c*
f)/d))*sin(-2*(d*e - c*f)/d) - 2*f*log((f*x + e)*d - d*e + c*f))*a^2/d)/f
```

3.126.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.43 (sec) , antiderivative size = 6693, normalized size of antiderivative = 46.16

$$\int \frac{(a + a\cos(e + fx))^2}{c + dx}\, dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(6*a^2*log(abs(d*x + c))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*
f/d)^2 + a^2*real_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(e)^
2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(cos_integral(f*x + c*f/d
))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*real_part(c
os_integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/
d)^2 + a^2*real_part(cos_integral(-2*f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2
*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 8*a^2*imag_part(cos_integral(f*x + c*f/d)
)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d) + 8*a^2*imag_part(cos_
integral(-f*x - c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*tan(1/2*c*f/d)
- 16*a^2*sin_integral((d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)^2*
tan(1/2*c*f/d) - 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)
^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 + 2*a^2*imag_part(cos_integral(-2*
f*x - 2*c*f/d))*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c*f/d)^2 - 4*a^2*
sin_integral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)^2*tan(c*f/d)*tan(1/2*c
*f/d)^2 + 2*a^2*imag_part(cos_integral(2*f*x + 2*c*f/d))*tan(1/2*e)^2*tan(
e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 - 2*a^2*imag_part(cos_integral(-2*f*x - 2
*c*f/d))*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2 + 4*a^2*sin_int
egral(2*(d*f*x + c*f)/d)*tan(1/2*e)^2*tan(e)*tan(c*f/d)^2*tan(1/2*c*f/d)^2
 + 8*a^2*imag_part(cos_integral(f*x + c*f/d))*tan(1/2*e)*tan(e)^2*tan(c*f/
d)^2*tan(1/2*c*f/d)^2 - 8*a^2*imag_part(cos_integral(-f*x - c*f/d))*tan...
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(e + fx))^2}{c + dx}\, dx = \int \frac{(a + a\,\cos(e + f\,x))^2}{c + d\,x}\, dx$$

input
```
int((a + a*cos(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + a*cos(e + f*x))^2/(c + d*x), x)
```

3.127 $\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2} dx$

3.127.1 Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2} dx = -\frac{4a^2\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} - \frac{a^2 f \operatorname{CosIntegral}\left(\frac{2cf}{d}+2fx\right)\sin\left(2e-\frac{2cf}{d}\right)}{d^2} - \frac{2a^2 f \operatorname{CosIntegral}\left(\frac{cf}{d}+fx\right)\sin\left(e-\frac{cf}{d}\right)}{d^2} - \frac{2a^2 f\cos\left(e-\frac{cf}{d}\right)\operatorname{Si}\left(\frac{cf}{d}+fx\right)}{d^2} - \frac{a^2 f\cos\left(2e-\frac{2cf}{d}\right)\operatorname{Si}\left(\frac{2cf}{d}+2fx\right)}{d^2}$$

output
```
-4*a^2*cos(1/2*f*x+1/2*e)^4/d/(d*x+c)-2*a^2*f*cos(-e+c*f/d)*Si(c*f/d+f*x)/
d^2-a^2*f*cos(-2*e+2*c*f/d)*Si(2*c*f/d+2*f*x)/d^2+a^2*f*Ci(2*c*f/d+2*f*x)*
sin(-2*e+2*c*f/d)/d^2+2*a^2*f*Ci(c*f/d+f*x)*sin(-e+c*f/d)/d^2
```

3.127.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx =$$

$$-\frac{a^2\left(3d+4d\cos(e+fx)+d\cos(2(e+fx))+2f(c+dx)\,\text{CosIntegral}\left(\frac{2f(c+dx)}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)+4f(c\cdots\right.}{}$$

input
```
Integrate[(a + a*Cos[e + f*x])^2/(c + d*x)^2,x]
```

output
```
-1/2*(a^2*(3*d + 4*d*Cos[e + f*x] + d*Cos[2*(e + f*x)] + 2*f*(c + d*x)*Cos
Integral[(2*f*(c + d*x))/d]*Sin[2*e - (2*c*f)/d] + 4*f*(c + d*x)*CosIntegr
al[f*(c/d + x)]*Sin[e - (c*f)/d] + 4*c*f*Cos[e - (c*f)/d]*SinIntegral[f*(c
/d + x)] + 4*d*f*x*Cos[e - (c*f)/d]*SinIntegral[f*(c/d + x)] + 2*c*f*Cos[2
*e - (2*c*f)/d]*SinIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cos[2*e - (2*c*f)
/d]*SinIntegral[(2*f*(c + d*x))/d]))/(d^2*(c + d*x))
```

3.127.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3799, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(e+fx)+a)^2}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \; 3799$$

$$4a^2\int \frac{\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \; 3042$$

$$4a^2 \int \frac{\sin\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^4}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3794}$$

$$4a^2\left(\frac{2f\int\left(-\frac{\sin(e+fx)}{4(c+dx)}-\frac{\sin(2e+2fx)}{8(c+dx)}\right)dx}{d}-\frac{\cos^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{2009}$$

$$4a^2\left(\frac{2f\left(-\frac{\text{CosIntegral}\left(2xf+\frac{2cf}{d}\right)\sin\left(2e-\frac{2cf}{d}\right)}{8d}-\frac{\text{CosIntegral}\left(xf+\frac{cf}{d}\right)\sin\left(e-\frac{cf}{d}\right)}{4d}-\frac{\cos\left(e-\frac{cf}{d}\right)\text{Si}\left(xf+\frac{cf}{d}\right)}{4d}-\frac{\cos\left(2e-\frac{2cf}{d}\right)\text{Si}\left(2xf\right.}{8d}\right.}{d}\right.$$

input
```
Int[(a + a*Cos[e + f*x])^2/(c + d*x)^2,x]
```

output
```
4*a^2*(-(Cos[e/2 + (f*x)/2]^4/(d*(c + d*x))) + (2*f*(-1/8*(CosIntegral[(2*
c*f)/d + 2*f*x]*Sin[2*e - (2*c*f)/d])/d - (CosIntegral[(c*f)/d + f*x]*Sin[
e - (c*f)/d])/(4*d) - (Cos[e - (c*f)/d]*SinIntegral[(c*f)/d + f*x])/(4*d)
- (Cos[2*e - (2*c*f)/d]*SinIntegral[(2*c*f)/d + 2*f*x])/(8*d)))/d)
```

3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.127.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.74

method	result
derivativedivides	$\frac{\frac{a^2 f^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}\right)}{d}\right)}{4}-\frac{\cdots}{2(cf-\cdots}}{f}$
default	$\frac{\frac{a^2 f^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf-2de}{d}\right)}{d}\right)}{d}\right)}{4}-\frac{\cdots}{2(cf-\cdots}}{f}$
parts	$-\frac{a^2}{d(dx+c)}+\frac{a^2\left(\frac{f^2\left(-\frac{2\cos(2fx+2e)}{(cf-de+d(fx+e))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2fx+2e+\frac{2cf-2de}{d}\right)\cos\left(\frac{2cf-2de}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2fx+2e+\frac{2cf-2de}{d}\right)\sin\left(\frac{2cf\cdots}{\cdots}\right)\cdots}{d}\right)}{d}\cdots\right)}{4}\cdots\right)}{f}$
risch	$\frac{i a^2 f\,\mathrm{e}^{\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(ifx+ie+\frac{i(cf-de)}{d}\right)}{d^2}-\frac{i f a^2\,\mathrm{e}^{-\frac{i(cf-de)}{d}}\,\mathrm{Ei}_1\left(-ifx-ie-\frac{icf-ide}{d}\right)}{d^2}-\frac{3a^2}{2d(dx+c)}+\frac{i a^2 f\,\mathrm{e}^{\frac{2i(cf-d\cdots}{d}}\cdots}{\cdots}$

input
```
int((a+cos(f*x+e)*a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/4*a^2*f^2*(-2*cos(2*f*x+2*e)/(c*f-d*e+d*(f*x+e))/d-2*(2*Si(2*f*x+2*
e+2*(c*f-d*e)/d)*cos(2*(c*f-d*e)/d)/d-2*Ci(2*f*x+2*e+2*(c*f-d*e)/d)*sin(2*
(c*f-d*e)/d)/d)/d)-3/2*a^2*f^2/(c*f-d*e+d*(f*x+e))/d+2*a^2*f^2*(-cos(f*x+e
)/(c*f-d*e+d*(f*x+e))/d-(Si(f*x+e+(c*f-d*e)/d)*cos((c*f-d*e)/d)/d-Ci(f*x+e
+(c*f-d*e)/d)*sin((c*f-d*e)/d)/d)/d))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.38

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx =$$
$$-\frac{a^2d\cos\left(fx+e\right)^2+2\,a^2d\cos\left(fx+e\right)+a^2d-2\left(a^2dfx+a^2cf\right)\operatorname{Ci}\left(\frac{dfx+cf}{d}\right)\sin\left(-\frac{de-cf}{d}\right)-\left(a^2dfx+a^2\right.}{}$$

input
```
integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(a^2*d*cos(f*x + e)^2 + 2*a^2*d*cos(f*x + e) + a^2*d - 2*(a^2*d*f*x + a^2
*c*f)*cos_integral((d*f*x + c*f)/d)*sin(-(d*e - c*f)/d) - (a^2*d*f*x + a^2
*c*f)*cos_integral(2*(d*f*x + c*f)/d)*sin(-2*(d*e - c*f)/d) + (a^2*d*f*x +
 a^2*c*f)*cos(-2*(d*e - c*f)/d)*sin_integral(2*(d*f*x + c*f)/d) + 2*(a^2*d
*f*x + a^2*c*f)*cos(-(d*e - c*f)/d)*sin_integral((d*f*x + c*f)/d))/(d^3*x
+ c*d^2)
```

3.127.6 Sympy [F]

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx = a^2\left(\int \frac{2\cos\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{\cos^2\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

input
```
integrate((a+a*cos(f*x+e))**2/(d*x+c)**2,x)
```

output
```
a**2*(Integral(2*cos(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(
cos(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*
d*x + d**2*x**2), x))
```

3.127.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.34

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx =$$

$$-\frac{\frac{4\,a^2f^2}{(fx+e)d^2-d^2e+cdf}+\frac{4\left(f^2\left(E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+E_2\left(-\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)\right)\cos\left(-\frac{de-cf}{d}\right)-f^2\left(-i\,E_2\left(\frac{i\,(fx+e)d-i\,de+i\,cf}{d}\right)+i\,E_2\left(\right.\right.\right.}{(fx+e)d^2-d^2e+cdf}}{}$$

input
```
integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*a^2*f^2/((f*x + e)*d^2 - d^2*e + c*d*f) + 4*(f^2*(exp_integral_e(2
, (I*(f*x + e)*d - I*d*e + I*c*f)/d) + exp_integral_e(2, -(I*(f*x + e)*d -
 I*d*e + I*c*f)/d))*cos(-(d*e - c*f)/d) - f^2*(-I*exp_integral_e(2, (I*(f*
x + e)*d - I*d*e + I*c*f)/d) + I*exp_integral_e(2, -(I*(f*x + e)*d - I*d*e
 + I*c*f)/d))*sin(-(d*e - c*f)/d))*a^2/((f*x + e)*d^2 - d^2*e + c*d*f) + (
f^2*(exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) + exp_integra
l_e(2, -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*cos(-2*(d*e - c*f)/d) - f^2
*(I*exp_integral_e(2, 2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d) - I*exp_integr
al_e(2, -2*(-I*(f*x + e)*d + I*d*e - I*c*f)/d))*sin(-2*(d*e - c*f)/d) + 2*
f^2)*a^2/((f*x + e)*d^2 - d^2*e + c*d*f))/f
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. 2(156) = 312.

Time = 0.45 (sec) , antiderivative size = 1049, normalized size of antiderivative = 6.60

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(4*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos_integral(
((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-(d*e -
 c*f)/d) - 4*a^2*d*e*f^2*cos_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x
 + c) + f) - d*e + c*f)/d)*sin(-(d*e - c*f)/d) + 4*a^2*c*f^3*cos_integral(
((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-(d*e -
 c*f)/d) + 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*cos_int
egral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin
(-2*(d*e - c*f)/d) - 2*a^2*d*e*f^2*cos_integral(2*((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*sin(-2*(d*e - c*f)/d) + 2*a^2*c*f^3
*cos_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f
)/d)*sin(-2*(d*e - c*f)/d) - 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c
) + f)*f^2*cos(-2*(d*e - c*f)/d)*sin_integral(2*((d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f) - d*e + c*f)/d) + 2*a^2*d*e*f^2*cos(-2*(d*e - c*f)/d)
*sin_integral(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f
)/d) - 2*a^2*c*f^3*cos(-2*(d*e - c*f)/d)*sin_integral(2*((d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) - 4*(d*x + c)*a^2*(d*e/(d*x +
 c) - c*f/(d*x + c) + f)*f^2*cos(-(d*e - c*f)/d)*sin_integral(((d*x + c)*(
d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d) + 4*a^2*d*e*f^2*cos(-(d
*e - c*f)/d)*sin_integral(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
 d*e + c*f)/d) - 4*a^2*c*f^3*cos(-(d*e - c*f)/d)*sin_integral(((d*x + c...
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(e+fx))^2}{(c+dx)^2}\,dx=\int \frac{(a+a\,\cos\left(e+f\,x\right))^2}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((a + a*cos(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + a*cos(e + f*x))^2/(c + d*x)^2, x)
```

3.128 $\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$

3.128.1 Optimal result

Integrand size = 20, antiderivative size = 134

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = -\frac{i(c+dx)^3}{af}+\frac{6d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3} + \frac{12d^3\operatorname{PolyLog}\left(3,-e^{i(e+fx)}\right)}{af^4}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

output
```
-I*(d*x+c)^3/a/f+6*d*(d*x+c)^2*ln(1+exp(I*(f*x+e)))/a/f^2-12*I*d^2*(d*x+c)
*polylog(2,-exp(I*(f*x+e)))/a/f^3+12*d^3*polylog(3,-exp(I*(f*x+e)))/a/f^4+
(d*x+c)^3*tan(1/2*f*x+1/2*e)/a/f
```

3.128.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-\frac{i\cos\left(\frac{1}{2}(e+fx)\right)\left(f^2(c+dx)^2\left(f(c+dx)+6id\log\left(1+e^{i(e+fx)}\right)\right)+12d^2f(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)+12id^3\operatorname{PolyLog}(\right.}{f^3}\right.}{af(1+\cos(e+fx))}$$

input
```
Integrate[(c + d*x)^3/(a + a*Cos[e + f*x]),x]
```

output
```
(2*Cos[(e + f*x)/2]*(((-I)*Cos[(e + f*x)/2]*(f^2*(c + d*x)^2*(f*(c + d*x)
+ (6*I)*d*Log[1 + E^(I*(e + f*x))]) + 12*d^2*f*(c + d*x)*PolyLog[2, -E^(I*
(e + f*x))] + (12*I)*d^3*PolyLog[3, -E^(I*(e + f*x))]))/f^3 + (c + d*x)^3*
Sin[(e + f*x)/2]))/(a*f*(1 + Cos[e + f*x]))
```

3.128.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {3042, 3799, 3042, 4672, 25, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a\cos(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{a\sin\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^3\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{6d\int -(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}+\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int (c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 4202$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{i(e+fx)}(c+dx)^2}{1+e^{i(e+fx)}}dx\right)}{f}}{2a}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\int(c+dx)\log\left(1+e^{i(e+fx)}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \; 3011$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)}{f}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)dx}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \; 2720$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)}{f}-\frac{d\int e^{-i(e+fx)}\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)de^{i(e+fx)}}{f^2}\right)}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \; 7143$$

$$\frac{\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(e+fx)}\right)}{f^2}\right)}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^3/(a + a*Cos[e + f*x]),x]
```

output
```
((-6*d*(((I/3)*(c + d*x)^3)/d - (2*I)*(((-I)*(c + d*x)^2*Log[1 + E^(I*(e +
 f*x))])/f + ((2*I)*d*((I*(c + d*x)*PolyLog[2, -E^(I*(e + f*x))])/f - (d*P
olyLog[3, -E^(I*(e + f*x))])/f^2))/f)))/f + (2*(c + d*x)^3*Tan[e/2 + (f*x)
/2])/f)/(2*a)
```

3.128. $\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$

3.128.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

3.128. $\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.128.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(122) = 244.

Time = 1.62 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.72

method	result
risch	$-\frac{12id^3 \operatorname{Li}_2\left(-e^{i(fx+e)}\right)x}{a f^3}+\frac{12d^2c\ln\left(e^{i(fx+e)}+1\right)x}{a f^2}-\frac{6id^2c e^2}{a f^3}+\frac{4id^3e^3}{a f^4}+\frac{12d^2ce\ln\left(e^{i(fx+e)}\right)}{a f^3}-\frac{12id^2cex}{a f^2}-\frac{12id^2c \operatorname{Li}_2(}{a f}$

input
```
int((d*x+c)^3/(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
-12*I/a/f^3*d^3*polylog(2,-exp(I*(f*x+e)))*x+12/a/f^2*d^2*c*ln(exp(I*(f*x+
e))+1)*x-6*I/a/f^3*d^2*c*e^2+4*I/a/f^4*d^3*e^3+12/a/f^3*d^2*c*e*ln(exp(I*(
f*x+e)))-12*I/a/f^2*d^2*c*e*x-12*I/a/f^3*d^2*c*polylog(2,-exp(I*(f*x+e)))+
2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))+1)+6*I/a/f^3*d
^3*e^2*x-2*I/a/f*d^3*x^3-6/a/f^4*d^3*e^2*ln(exp(I*(f*x+e)))-6*I/a/f*d^2*c*
x^2+6/a/f^2*d^3*ln(exp(I*(f*x+e))+1)*x^2+12*d^3*polylog(3,-exp(I*(f*x+e)))
/a/f^4-6/a/f^2*d*c^2*ln(exp(I*(f*x+e)))+6/a/f^2*d*c^2*ln(exp(I*(f*x+e))+1)
```

3.128.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(119) = 238.

Time = 0.31 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.15

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = -\frac{6\left(-i\,d^3fx-i\,cd^2f+\left(-i\,d^3fx-i\,cd^2f\right)\cos\left(fx+e\right)\right)\operatorname{Li}_2(-\cos\left(fx+e\right)+i\,\sin\left(fx+e\right))+6\left(i\,d^3fx\right.}{}$$

input
```
integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
-(6*(-I*d^3*f*x - I*c*d^2*f + (-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e))*dilog
(-cos(f*x + e) + I*sin(f*x + e)) + 6*(I*d^3*f*x + I*c*d^2*f + (I*d^3*f*x +
 I*c*d^2*f)*cos(f*x + e))*dilog(-cos(f*x + e) - I*sin(f*x + e)) - 3*(d^3*f
^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*
f^2)*cos(f*x + e))*log(cos(f*x + e) + I*sin(f*x + e) + 1) - 3*(d^3*f^2*x^2
 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*c
os(f*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) - 6*(d^3*cos(f*x + e)
+ d^3)*polylog(3, -cos(f*x + e) + I*sin(f*x + e)) - 6*(d^3*cos(f*x + e) +
d^3)*polylog(3, -cos(f*x + e) - I*sin(f*x + e)) - (d^3*f^3*x^3 + 3*c*d^2*f
^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3)*sin(f*x + e))/(a*f^4*cos(f*x + e) + a*f^
4)
```

3.128.6 Sympy [F]

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx$$
$$= \frac{\int \frac{c^3}{\cos(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cos(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cos(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cos(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**3/(a+a*cos(f*x+e)),x)
```

output
```
(Integral(c**3/(cos(e + f*x) + 1), x) + Integral(d**3*x**3/(cos(e + f*x) +
 1), x) + Integral(3*c*d**2*x**2/(cos(e + f*x) + 1), x) + Integral(3*c**2*
d*x/(cos(e + f*x) + 1), x))/a
```

3.128.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(119) = 238$.

Time = 0.45 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.98

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
-(6*((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f*x +
e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e))*c*
d^2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 + 2*a*f^2*cos(f*x + e)
+ a*f^2) - 3*((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(c
os(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x
 + e))*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*cos(f*x + e)
 + a*f) - c^3*sin(f*x + e)/(a*(cos(f*x + e) + 1)) - 3*c*d^2*e^2*sin(f*x +
e)/(a*f^2*(cos(f*x + e) + 1)) + 3*c^2*d*e*sin(f*x + e)/(a*f*(cos(f*x + e)
+ 1)) + (2*d^3*e^3 - 6*((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f
*x + e) + ((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*f)*(f*x + e))*cos(
f*x + e) - (-I*(f*x + e)^2*d^3 - I*d^3*e^2 + 2*(I*d^3*e - I*c*d^2*f)*(f*x
+ e))*sin(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) + 2*((f*x + e)
^3*d^3 + 3*(f*x + e)*d^3*e^2 - 3*(d^3*e - c*d^2*f)*(f*x + e)^2)*cos(f*x +
e) + 12*((f*x + e)*d^3 - d^3*e + c*d^2*f + ((f*x + e)*d^3 - d^3*e + c*d^2*
f)*cos(f*x + e) + (I*(f*x + e)*d^3 - I*d^3*e + I*c*d^2*f)*sin(f*x + e))*di
log(-e^(I*f*x + I*e)) + 3*(I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(-I*d^3*e + I
*c*d^2*f)*(f*x + e) + (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(-I*d^3*e + I*c*d
^2*f)*(f*x + e))*cos(f*x + e) - ((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*
d^2*f)*(f*x + e))*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*co
s(f*x + e) + 1) + 12*(I*d^3*cos(f*x + e) - d^3*sin(f*x + e) + I*d^3)*po...
```

3.128.8 Giac [F]

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx=\int \frac{(dx+c)^3}{a\cos(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+a*cos(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(a*cos(f*x + e) + a), x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+a\cos(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+a\,\cos\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^3/(a + a*cos(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + a*cos(e + f*x)), x)
```

3.129 $\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx$

3.129.1 Optimal result

Integrand size = 20, antiderivative size = 101

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = -\frac{i(c+dx)^2}{af}+\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{af^2} - \frac{4id^2\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{af^3}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

output
```
-I*(d*x+c)^2/a/f+4*d*(d*x+c)*ln(1+exp(I*(f*x+e)))/a/f^2-4*I*d^2*polylog(2,
-exp(I*(f*x+e)))/a/f^3+(d*x+c)^2*tan(1/2*f*x+1/2*e)/a/f
```

3.129.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-4id^2\cos\left(\frac{1}{2}(e+fx)\right)\text{PolyLog}\left(2,-e^{i(e+fx)}\right)+f(c+dx)\left(\cos\left(\frac{1}{2}(e+fx)\right)\left(-if(c+d\right.\right.\right.}{af^3(1+\cos(e+fx))}$$

input
```
Integrate[(c + d*x)^2/(a + a*Cos[e + f*x]),x]
```

output
```
(2*Cos[(e + f*x)/2]*((-4*I)*d^2*Cos[(e + f*x)/2]*PolyLog[2, -E^(I*(e + f*x
))] + f*(c + d*x)*(Cos[(e + f*x)/2]*((-I)*f*(c + d*x) + 4*d*Log[1 + E^(I*(
e + f*x))]) + f*(c + d*x)*Sin[(e + f*x)/2])))/(a*f^3*(1 + Cos[e + f*x]))
```

3.129.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3799, 3042, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a\cos(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{a\sin\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{4d\int -\left((c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)dx}{f}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{4202}$$

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{i(e+fx)}(c+dx)}{1+e^{i(e+fx)}}dx\right)}{f}}{2a}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(e+fx)}\right)dx}{f}-\frac{i(c+dx)\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(e+fx)}\log\left(1+e^{i(e+fx)}\right)de^{i(e+fx)}}{f^2}-\frac{i(c+dx)\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{i(c+dx)\log\left(1+e^{i(e+fx)}\right)}{f}-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{f^2}\right)\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^2/(a + a*Cos[e + f*x]),x]
```

output
```
((-4*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-I)*(c + d*x)*Log[1 + E^(I*(e + f
*x))])/f - (d*PolyLog[2, -E^(I*(e + f*x))])/f^2)))/f + (2*(c + d*x)^2*Tan[
e/2 + (f*x)/2])/f)/(2*a)
```

3.129.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.129. $\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.129.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(91) = 182$.

Time = 1.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.95

method	result
risch	$\frac{2i\left(x^{2} d^{2}+2 c d x+c^{2}\right)}{f a \left({\mathrm e}^{i \left(f x +e \right)}+1\right)}-\frac{4 d c \ln \left({\mathrm e}^{i \left(f x +e \right)}\right)}{a \,f^{2}}+\frac{4 d c \ln \left({\mathrm e}^{i \left(f x +e \right)}+1\right)}{a \,f^{2}}-\frac{2 i d^{2} x^{2}}{a f}-\frac{4 i d^{2} e x}{a \,f^{2}}-\frac{2 i d^{2} e^{2}}{a \,f^{3}}+\frac{4 d^{2} \ln \left({\mathrm e}^{i \left(f x +e \right)}+1\right) x}{a \,f^{2}}-$ 4ic

input
```
int((d*x+c)^2/(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))+1)-4/a/f^2*d*c*ln(exp(I*(f*x
+e)))+4/a/f^2*d*c*ln(exp(I*(f*x+e))+1)-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-2
*I/a/f^3*d^2*e^2+4/a/f^2*d^2*ln(exp(I*(f*x+e))+1)*x-4*I*d^2*polylog(2,-exp
(I*(f*x+e)))/a/f^3+4/a/f^3*d^2*e*ln(exp(I*(f*x+e)))
```

3.129.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(88) = 176.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.24

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = -\frac{2\left(-i\,d^2\cos\left(fx+e\right)-i\,d^2\right)\mathrm{Li}_2\left(-\cos\left(fx+e\right)+i\,\sin\left(fx+e\right)\right)+2\left(i\,d^2\cos\left(fx+e\right)+i\,d^2\right)\mathrm{Li}_2(-\cos(}{}$$

input
```
integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
-(2*(-I*d^2*cos(f*x + e) - I*d^2)*dilog(-cos(f*x + e) + I*sin(f*x + e)) +
2*(I*d^2*cos(f*x + e) + I*d^2)*dilog(-cos(f*x + e) - I*sin(f*x + e)) - 2*(
d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) + I*sin
(f*x + e) + 1) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e))*log(
cos(f*x + e) - I*sin(f*x + e) + 1) - (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)
*sin(f*x + e))/(a*f^3*cos(f*x + e) + a*f^3)
```

3.129.6 Sympy [F]

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \frac{\int \frac{c^2}{\cos(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cos(e+fx)+1}\,dx + \int \frac{2cdx}{\cos(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**2/(a+a*cos(f*x+e)),x)
```

output
```
(Integral(c**2/(cos(e + f*x) + 1), x) + Integral(d**2*x**2/(cos(e + f*x) +
 1), x) + Integral(2*c*d*x/(cos(e + f*x) + 1), x))/a
```

3.129.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(88) = 176.

Time = 0.37 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.81

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx$$
$$= \frac{2\left(c^2f^2+2\left(d^2fx+cdf+\left(d^2fx+cdf\right)\cos\left(fx+e\right)-\left(-i\,d^2fx-i\,cdf\right)\sin\left(fx+e\right)\right)\arctan\left(\sin\left(fx+e\right)\right.}{}$$

input
```
integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
2*(c^2*f^2 + 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e) - (-I*d^2
*f*x - I*c*d*f)*sin(f*x + e))*arctan2(sin(f*x + e), cos(f*x + e) + 1) - (d
^2*f^2*x^2 + 2*c*d*f^2*x)*cos(f*x + e) - 2*(d^2*cos(f*x + e) + I*d^2*sin(f
*x + e) + d^2)*dilog(-e^(I*f*x + I*e)) - (I*d^2*f*x + I*c*d*f + (I*d^2*f*x
 + I*c*d*f)*cos(f*x + e) - (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e
)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) - (I*d^2*f^2*x^2 + 2*I*c*d*f^2*
x)*sin(f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) - I*a*f^3)
```

3.129.8 Giac [F]

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \int \frac{(dx+c)^2}{a\cos\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(a*cos(f*x + e) + a), x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+a\cos(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+a\,\cos(e+f\,x)}\,dx$$

input
```
int((c + d*x)^2/(a + a*cos(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + a*cos(e + f*x)), x)
```

3.130 $\int \frac{c+dx}{a+a\cos(e+fx)}\,dx$

3.130.1 Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx=\frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

output
```
2*d*ln(cos(1/2*f*x+1/2*e))/a/f^2+(d*x+c)*tan(1/2*f*x+1/2*e)/a/f
```

3.130.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx =\frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(2d\cos\left(\frac{1}{2}(e+fx)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+f(c+dx)\sin\left(\frac{1}{2}(e+fx)\right)\right)}{af^2(1+\cos(e+fx))}$$

input
```
Integrate[(c + d*x)/(a + a*Cos[e + f*x]),x]
```

output
```
(2*Cos[(e + f*x)/2]*(2*d*Cos[(e + f*x)/2]*Log[Cos[(e + f*x)/2]] + f*(c + d
*x)*Sin[(e + f*x)/2]))/(a*f^2*(1 + Cos[e + f*x]))
```

3.130.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 3799, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a\cos(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a\sin\left(e+fx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2d\int -\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}}{2a}$$

input
```
Int[(c + d*x)/(a + a*Cos[e + f*x]),x]
```

output
```
((4*d*Log[Cos[e/2 + (f*x)/2]])/f^2 + (2*(c + d*x)*Tan[e/2 + (f*x)/2])/f)/(
2*a)
```

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.130.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$\frac{-d\ln\left(\sec^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)f(dx+c)}{a\,f^2}$	40
norman	$\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{af}-\frac{d\ln\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a\,f^2}$	60
risch	$-\frac{2idx}{af}-\frac{2ide}{a\,f^2}+\frac{2i(dx+c)}{fa\left(e^{i(fx+e)}+1\right)}+\frac{2d\ln\left(e^{i(fx+e)}+1\right)}{a\,f^2}$	72

input
```
int((d*x+c)/(a+cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
(-d*ln(sec(1/2*f*x+1/2*e)^2)+tan(1/2*f*x+1/2*e)*f*(d*x+c))/a/f^2
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \frac{(d\cos(fx+e)+d)\log\left(\frac{1}{2}\cos(fx+e)+\frac{1}{2}\right)+(dfx+cf)\sin(fx+e)}{af^2\cos(fx+e)+af^2}$$

input
```
integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
((d*cos(f*x + e) + d)*log(1/2*cos(f*x + e) + 1/2) + (d*f*x + c*f)*sin(f*x
+ e))/(a*f^2*cos(f*x + e) + a*f^2)
```

3.130.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \begin{cases} \frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{dx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2} & \text{for } f \neq 0 \\ \frac{cx+\frac{dx^2}{2}}{a\cos(e)+a} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)/(a+a*cos(f*x+e)),x)
```

output
```
Piecewise((c*tan(e/2 + f*x/2)/(a*f) + d*x*tan(e/2 + f*x/2)/(a*f) - d*log(t
an(e/2 + f*x/2)**2 + 1)/(a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cos(e) +
 a), True))
```

3.130.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(41) = 82$.

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.27

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx$$
$$= \frac{\frac{\left(\left(\cos(fx+e)^2+\sin(fx+e)^2+2\cos(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2+2\cos(fx+e)+1\right)+2(fx+e)\sin(fx+e)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2+2af\cos(fx+e)+af}+\frac{c\sin(fx+e)}{a(\cos(fx+e)+1)}-}{f}$$

input
```
integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
(((cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1)*log(cos(f*x + e)^
2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) + 2*(f*x + e)*sin(f*x + e))*d/(a*
f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 + 2*a*f*cos(f*x + e) + a*f) + c*sin(
f*x + e)/(a*(cos(f*x + e) + 1)) - d*e*sin(f*x + e)/(a*f*(cos(f*x + e) + 1)
))/f
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(41) = 82$.

Time = 0.33 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.96

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx =$$
$$-\frac{dfx\tan\left(\frac{1}{2}fx\right)+dfx\tan\left(\frac{1}{2}e\right)-d\log\left(\frac{4\left(\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2-2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+1\right)}{\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx\right)^2+\tan\left(\frac{1}{2}e\right)^2+1}\right)\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+}{af^2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)-}$$

input
```
integrate((d*x+c)/(a+a*cos(f*x+e)),x, algorithm="giac")
```

output
```
-(d*f*x*tan(1/2*f*x) + d*f*x*tan(1/2*e) - d*log(4*(tan(1/2*f*x)^2*tan(1/2*
e)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1
/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x)*tan(1/2*e) + c*f*tan(1/2*f*x)
+ c*f*tan(1/2*e) + d*log(4*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*t
an(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^
2 + 1)))/(a*f^2*tan(1/2*f*x)*tan(1/2*e) - a*f^2)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 15.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{c+dx}{a+a\cos(e+fx)}\,dx = \frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\right)}{a\,f^2} + \frac{(c+d\,x)\,2\mathrm{i}}{a\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)} - \frac{d\,x\,2\mathrm{i}}{a\,f}$$

input
```
int((c + d*x)/(a + a*cos(e + f*x)),x)
```

output
```
(2*d*log(exp(e*1i)*exp(f*x*1i) + 1))/(a*f^2) + ((c + d*x)*2i)/(a*f*(exp(e*
1i + f*x*1i) + 1)) - (d*x*2i)/(a*f)
```

3.131 $\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$

3.131.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \mathrm{Int}\left(\frac{1}{(c+dx)(a+a\cos(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

3.131.2 Mathematica [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])), x]
```

3.131.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\cos(e+fx)+a)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a\cos(e+fx)+a)}dx$$

input
```
Int[1/((c + d*x)*(a + a*Cos[e + f*x])),x]
```

output
```
$Aborted
```

3.131.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.131.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a+\cos(fx+e)\,a)}dx$$

input
```
int(1/(d*x+c)/(a+cos(f*x+e)*a),x)
```

output
```
int(1/(d*x+c)/(a+cos(f*x+e)*a),x)
```

3.131.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (a*d*x + a*c)*cos(f*x + e)), x)
```

3.131.6 Sympy [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \frac{\int \frac{1}{c\cos(e+fx)+c+dx\cos(e+fx)+dx}\,dx}{a}$$

input
```
integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x)
```

output
```
Integral(1/(c*cos(e + f*x) + c + d*x*cos(e + f*x) + d*x), x)/a
```

3.131.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 285, normalized size of antiderivative = 14.25

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f*
x + a*c*d*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e))*integr
ate(sin(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*a
*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)
*sin(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)), x
) + sin(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (a
*d*f*x + a*c*f)*sin(f*x + e)^2 + 2*(a*d*f*x + a*c*f)*cos(f*x + e))
```

3.131.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cos(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(a*cos(f*x + e) + a)), x)
```

3.131.9 Mupad [N/A]

Not integrable

Time = 14.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))}\,dx=\int \frac{1}{(a+a\,\cos\left(e+f\,x\right))\,\left(c+d\,x\right)} \,dx$$

input
```
int(1/((a + a*cos(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + a*cos(e + f*x))*(c + d*x)), x)
```

3.132 $\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$

3.132.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+a\cos(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a+a*cos(f*x+e)),x)
```

3.132.2 Mathematica [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])), x]
```

3.132.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\cos(e+fx)+a)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a\cos(e+fx)+a)}dx$$

input
```
Int[1/((c + d*x)^2*(a + a*Cos[e + f*x])),x]
```

output
```
$Aborted
```

3.132.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.132.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2 (a+\cos(fx+e)\, a)} dx$$

input
```
int(1/(d*x+c)^2/(a+cos(f*x+e)*a),x)
```

output
```
int(1/(d*x+c)^2/(a+cos(f*x+e)*a),x)
```

3.132.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\, dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)}\, dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2
)*cos(f*x + e)), x)
```

3.132.6 Sympy [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\, dx = \frac{\int \frac{1}{c^2\cos(e+fx)+c^2+2cdx\cos(e+fx)+2cdx+d^2x^2\cos(e+fx)+d^2x^2}\, dx}{a}$$

input
```
integrate(1/(d*x+c)**2/(a+a*cos(f*x+e)),x)
```

output
```
Integral(1/(c**2*cos(e + f*x) + c**2 + 2*c*d*x*cos(e + f*x) + 2*c*d*x + d*
*2*x**2*cos(e + f*x) + d**2*x**2), x)/a
```

3.132.7 Maxima [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 442, normalized size of antiderivative = 22.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*f
*x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)
*sin(f*x + e)^2 + 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*cos(f*x + e)
)*integrate(sin(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x +
a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f*
x + e)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(f
*x + e)^2 + 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*co
s(f*x + e)), x) + sin(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*
d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d
*f*x + a*c^2*f)*sin(f*x + e)^2 + 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*c
os(f*x + e))
```

3.132.8 Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cos(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(a*cos(f*x + e) + a)), x)
```

3.132.9 Mupad [N/A]

Not integrable

Time = 14.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))}\,dx = \int \frac{1}{(a+a\,\cos\left(e+f\,x\right))\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*cos(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + a*cos(e + f*x))*(c + d*x)^2), x)
```

3.133 $\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx$

3.133.1 Optimal result

Integrand size = 20, antiderivative size = 271

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = &-\frac{i(c+dx)^3}{3a^2f}+\frac{2d(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{a^2f^2}\\ &+\frac{4d^3\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4}-\frac{4id^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{a^2f^3}\\ &+\frac{4d^3\,\mathrm{PolyLog}\left(3,-e^{i(e+fx)}\right)}{a^2f^4}-\frac{d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2}\\ &+\frac{2d^2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}+\frac{(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}\\ &+\frac{(c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}\end{aligned}$$

output
```
-1/3*I*(d*x+c)^3/a^2/f+2*d*(d*x+c)^2*ln(1+exp(I*(f*x+e)))/a^2/f^2+4*d^3*ln
(cos(1/2*f*x+1/2*e))/a^2/f^4-4*I*d^2*(d*x+c)*polylog(2,-exp(I*(f*x+e)))/a^
2/f^3+4*d^3*polylog(3,-exp(I*(f*x+e)))/a^2/f^4-1/2*d*(d*x+c)^2*sec(1/2*f*x
+1/2*e)^2/a^2/f^2+2*d^2*(d*x+c)*tan(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^3*t
an(1/2*f*x+1/2*e)/a^2/f+1/6*(d*x+c)^3*sec(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2
*e)/a^2/f
```

3.133.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-3df^2(c+dx)^2\cos\left(\frac{1}{2}(e+fx)\right)+f^3(c+dx)^3\sin\left(\frac{1}{2}(e+fx)\right)+12d^2\cos^3\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Cos[e + f*x])^2,x]
```

output
```
(2*Cos[(e + f*x)/2]*(-3*d*f^2*(c + d*x)^2*Cos[(e + f*x)/2] + f^3*(c + d*x)
^3*Sin[(e + f*x)/2] + 12*d^2*Cos[(e + f*x)/2]^3*(2*d*Log[Cos[(e + f*x)/2]]
 + f*(c + d*x)*Tan[(e + f*x)/2]) - 2*Cos[(e + f*x)/2]^3*(I*f^3*(c + d*x)^3
 - 6*d*(f^2*(c + d*x)^2*Log[1 + E^(I*(e + f*x))] - (2*I)*d*f*(c + d*x)*Pol
yLog[2, -E^(I*(e + f*x))] + 2*d^2*PolyLog[3, -E^(I*(e + f*x))]) - f^3*(c +
 d*x)^3*Tan[(e + f*x)/2])))/(3*a^2*f^4*(1 + Cos[e + f*x])^2)
```

3.133.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3799, 3042, 4674, 3042, 4672, 25, 3042, 3956, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\cos(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^3\sec^4\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^3 \csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^4 dx}{4a^2}$$

$$\Big\downarrow \; 4674$$

$$\frac{\frac{4d^2 \int (c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f^2}+\frac{2}{3}\int (c+dx)^3\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx-\frac{2d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f^2}+\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{4d^2 \int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx}{f^2}+\frac{2}{3}\int (c+dx)^3\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx-\frac{2d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f^2}+\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}-\right.}{3f}}{4a^2}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{4d^2\left(\frac{2d\int -\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{6d\int -(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}+\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}\right)-\frac{2d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right.}{f^2}}{4a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{4d^2\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int (c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)-\frac{2d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f^2}+}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{4d^2\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int \tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int (c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)-\frac{2d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f^2}+}{4a^2}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int (c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{4d^2\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}\right)}{f^2}-\frac{2d(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f^2}-}{4a^2}$$

$$\Big\downarrow \; 4202$$

3.133. $\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{i(e+fx)}(c+dx)^2}{1+e^{i(e+fx)}}dx\right)}{f}\right)+\frac{4d^2\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}\right)}{f^2}-\frac{2d(c+dx)^2\sec^2}{f^2}}{4a^2}$$

$\downarrow$ 2620

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\int(c+dx)\log\left(1+e^{i(e+fx)}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}\right)+\frac{4d^2\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\right)}{f^2}}{4a^2}$$

$\downarrow$ 3011

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)}{f}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)dx}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}\right)}{4a^2}$$

$\downarrow$ 2720

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)}{f}-\frac{d\int e^{-i(e+fx)}\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)de^{i(e+fx)}}{f^2}\right)}{f}-\frac{i(c+dx)^2\log}{}\right)\right)}{f}\right)}{4a^2}$$

$\downarrow$ 7143

$$\frac{\frac{4d^2\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx)}\right)}{f}-\frac{d\operatorname{PolyL}}{}\right)}{f}\right)\right)}{f}\right)}{4a^2}$$

3.133. $\int\frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx$

input
```
Int[(c + d*x)^3/(a + a*Cos[e + f*x])^2,x]
```

output
```
((-2*d*(c + d*x)^2*Sec[e/2 + (f*x)/2]^2)/f^2 + (2*(c + d*x)^3*Sec[e/2 + (f
*x)/2]^2*Tan[e/2 + (f*x)/2])/(3*f) + (4*d^2*((4*d*Log[Cos[e/2 + (f*x)/2]])
/f^2 + (2*(c + d*x)*Tan[e/2 + (f*x)/2])/f))/f^2 + (2*((-6*d*(((I/3)*(c + d
*x)^3)/d - (2*I)*(((-I)*(c + d*x)^2*Log[1 + E^(I*(e + f*x))])/f + ((2*I)*d
*((I*(c + d*x)*PolyLog[2, -E^(I*(e + f*x))])/f - (d*PolyLog[3, -E^(I*(e +
f*x))])/f^2))/f)))/f + (2*(c + d*x)^3*Tan[e/2 + (f*x)/2])/f))/3)/(4*a^2)
```

3.133.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.133. $\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx$

3.133.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(231) = 462.

Time = 2.92 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.50

method	result
risch	$-\frac{2id^3x^3}{3a^2f}-\frac{4id^2cex}{a^2f^2}+\frac{4d^2c\ln\left(e^{i(fx+e)}+1\right)x}{a^2f^2}-\frac{4id^2c\ \mathrm{Li}_2\left(-e^{i(fx+e)}\right)}{a^2f^3}+\frac{4d^2ce\ln\left(e^{i(fx+e)}\right)}{a^2f^3}-\frac{2id^2c\,e^2}{a^2f^3}-\frac{2id^2c\,x^2}{a^2f}+\frac{4id}{3a^2}$

input
```
int((d*x+c)^3/(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/a^2/f*d^3*x^3-4*I/a^2/f^2*d^2*c*e*x+4/a^2/f^2*d^2*c*ln(exp(I*(f*x+e
))+1)*x-4*I/a^2/f^3*d^2*c*polylog(2,-exp(I*(f*x+e)))+4/a^2/f^3*d^2*c*e*ln(
exp(I*(f*x+e)))-2*I/a^2/f^3*d^2*c*e^2-2*I/a^2/f*d^2*c*x^2+4/3*I/a^2/f^4*d^
3*e^3+2/3*I*(3*I*c^2*d*f*exp(I*(f*x+e))+3*d^3*f^2*x^3*exp(I*(f*x+e))+6*I*c
*d^2*f*x*exp(2*I*(f*x+e))+3*I*d^3*f*x^2*exp(I*(f*x+e))+9*c*d^2*f^2*x^2*exp
(I*(f*x+e))+d^3*x^3*f^2+3*I*d^3*f*x^2*exp(2*I*(f*x+e))+3*I*c^2*d*f*exp(2*I
*(f*x+e))+9*c^2*d*f^2*x*exp(I*(f*x+e))+3*c*d^2*f^2*x^2+6*I*c*d^2*f*x*exp(I
*(f*x+e))+3*c^3*f^2*exp(I*(f*x+e))+3*c^2*d*f^2*x+6*d^3*x*exp(2*I*(f*x+e))+
c^3*f^2+6*c*d^2*exp(2*I*(f*x+e))+12*d^3*x*exp(I*(f*x+e))+12*c*d^2*exp(I*(f
*x+e))+6*d^3*x+6*d^2*c)/f^3/a^2/(exp(I*(f*x+e))+1)^3+2*I/a^2/f^3*d^3*e^2*x
+2/a^2/f^2*d^3*ln(exp(I*(f*x+e))+1)*x^2-2/a^2/f^2*d*c^2*ln(exp(I*(f*x+e)))
+2/a^2/f^2*d*c^2*ln(exp(I*(f*x+e))+1)-2/a^2/f^4*d^3*e^2*ln(exp(I*(f*x+e)))
-4*I/a^2/f^3*d^3*polylog(2,-exp(I*(f*x+e)))*x+4*d^3*polylog(3,-exp(I*(f*x+
e)))/a^2/f^4-4/a^2/f^4*d^3*ln(exp(I*(f*x+e)))+4/a^2/f^4*d^3*ln(exp(I*(f*x+
e))+1)
```

3.133.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(228) = 456.

Time = 0.31 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.85

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx =$$

$$-\frac{3\,d^3f^2x^2+6\,cd^2f^2x+3\,c^2df^2+3\left(d^3f^2x^2+2\,cd^2f^2x+c^2df^2\right)\cos\left(fx+e\right)+6\left(-i\,d^3fx-i\,cd^2f+(-\right.}{}$$

input
```
integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/3*(3*d^3*f^2*x^2 + 6*c*d^2*f^2*x + 3*c^2*d*f^2 + 3*(d^3*f^2*x^2 + 2*c*d
^2*f^2*x + c^2*d*f^2)*cos(f*x + e) + 6*(-I*d^3*f*x - I*c*d^2*f + (-I*d^3*f
*x - I*c*d^2*f)*cos(f*x + e)^2 + 2*(-I*d^3*f*x - I*c*d^2*f)*cos(f*x + e))*
dilog(-cos(f*x + e) + I*sin(f*x + e)) + 6*(I*d^3*f*x + I*c*d^2*f + (I*d^3*
f*x + I*c*d^2*f)*cos(f*x + e)^2 + 2*(I*d^3*f*x + I*c*d^2*f)*cos(f*x + e))*
dilog(-cos(f*x + e) - I*sin(f*x + e)) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c
^2*d*f^2 + 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f
*x + e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f*x +
e))*log(cos(f*x + e) + I*sin(f*x + e) + 1) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*
x + c^2*d*f^2 + 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*
cos(f*x + e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + 2*d^3)*cos(f
*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) - 6*(d^3*cos(f*x + e)^2 +
2*d^3*cos(f*x + e) + d^3)*polylog(3, -cos(f*x + e) + I*sin(f*x + e)) - 6*(
d^3*cos(f*x + e)^2 + 2*d^3*cos(f*x + e) + d^3)*polylog(3, -cos(f*x + e) -
I*sin(f*x + e)) - (2*d^3*f^3*x^3 + 6*c*d^2*f^3*x^2 + 2*c^3*f^3 + 6*c*d^2*f
 + 6*(c^2*d*f^3 + d^3*f)*x + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + c^3*f^3 + 6*
c*d^2*f + 3*(c^2*d*f^3 + 2*d^3*f)*x)*cos(f*x + e))*sin(f*x + e))/(a^2*f^4*
cos(f*x + e)^2 + 2*a^2*f^4*cos(f*x + e) + a^2*f^4)
```

3.133.6 Sympy [F]

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \frac{\int \frac{c^3}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**3/(a+a*cos(f*x+e))**2,x)
```

output
```
(Integral(c**3/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(d**3*
x**3/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(
cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(3*c**2*d*x/(cos(e + f
*x)**2 + 2*cos(e + f*x) + 1), x))/a**2
```

3.133.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3275 vs. 2(228) = 456.

Time = 1.00 (sec) , antiderivative size = 3275, normalized size of antiderivative = 12.08

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/6*(12*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*e) + cos(f*x + e))*co
s(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f
*x + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (2*(3*cos(2*f*x + 2*
e) + 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x + 3*e)^2 + 6*(3*cos(
f*x + e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9*cos(f*x + e)^2 +
 6*(sin(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + sin(3*f*x + 3*e)^2
 + 9*sin(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e) + 9*sin(f*x + e
)^2 + 6*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x
+ e) + 1) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x + 2*e) - sin
(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e) + e - 2*si
n(f*x + e))*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x + e)^2 + 2
*cos(f*x + e))*c*d^2*e/(a^2*f^2*cos(3*f*x + 3*e)^2 + 9*a^2*f^2*cos(2*f*x +
 2*e)^2 + 9*a^2*f^2*cos(f*x + e)^2 + a^2*f^2*sin(3*f*x + 3*e)^2 + 9*a^2*f^
2*sin(2*f*x + 2*e)^2 + 18*a^2*f^2*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f^
2*sin(f*x + e)^2 + 6*a^2*f^2*cos(f*x + e) + a^2*f^2 + 2*(3*a^2*f^2*cos(2*f
*x + 2*e) + 3*a^2*f^2*cos(f*x + e) + a^2*f^2)*cos(3*f*x + 3*e) + 6*(3*a^2*
f^2*cos(f*x + e) + a^2*f^2)*cos(2*f*x + 2*e) + 6*(a^2*f^2*sin(2*f*x + 2*e)
 + a^2*f^2*sin(f*x + e))*sin(3*f*x + 3*e)) - 6*(2*(3*(f*x + e)*sin(f*x + e
) + cos(2*f*x + 2*e) + cos(f*x + e))*cos(3*f*x + 3*e) + 2*(9*(f*x + e)*sin
(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f*x + 2*e) + 6*cos(2*f*x + 2*e)^2...
```

3.133.8 Giac [F]

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(a\cos(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(a*cos(f*x + e) + a)^2, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+a\cos(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(a + a*cos(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

3.134 $\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$

3.134.1 Optimal result

Integrand size = 20, antiderivative size = 212

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx =& -\frac{i(c+dx)^2}{3a^2f}+\frac{4d(c+dx)\log\left(1+e^{i(e+fx)}\right)}{3a^2f^2}\\&-\frac{4id^2\,\mathrm{PolyLog}\left(2,-e^{i(e+fx)}\right)}{3a^2f^3}-\frac{d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}\\&+\frac{2d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3}+\frac{(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}\\&+\frac{(c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}\end{aligned}$$

output
```
-1/3*I*(d*x+c)^2/a^2/f+4/3*d*(d*x+c)*ln(1+exp(I*(f*x+e)))/a^2/f^2-4/3*I*d^
2*polylog(2,-exp(I*(f*x+e)))/a^2/f^3-1/3*d*(d*x+c)*sec(1/2*f*x+1/2*e)^2/a^
2/f^2+2/3*d^2*tan(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^2*tan(1/2*f*x+1/2*e)/
a^2/f+1/6*(d*x+c)^2*sec(1/2*f*x+1/2*e)^2*tan(1/2*f*x+1/2*e)/a^2/f
```

3.134.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$$
$$= \frac{2\cos\left(\frac{1}{2}(e+fx)\right)\left(-2df(c+dx)\cos\left(\frac{1}{2}(e+fx)\right)-2if(c+dx)\cos^3\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)+4id\log\left(1-\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + a*Cos[e + f*x])^2,x]
```

output
```
(2*Cos[(e + f*x)/2]*(-2*d*f*(c + d*x)*Cos[(e + f*x)/2] - (2*I)*f*(c + d*x)
*Cos[(e + f*x)/2]^3*(f*(c + d*x) + (4*I)*d*Log[1 + E^(I*(e + f*x))]) - (8*
I)*d^2*Cos[(e + f*x)/2]^3*PolyLog[2, -E^(I*(e + f*x))] + (2*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(1 + f^2*x^2)) + (c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2)
)*Cos[e + f*x])*Sin[(e + f*x)/2]))/(3*a^2*f^3*(1 + Cos[e + f*x])^2)
```

3.134.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3799, 3042, 4674, 3042, 4254, 24, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\cos(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(c+dx)^2}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \ 3799$$

$$\frac{\int (c+dx)^2\sec^4\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{4a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int (c+dx)^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^4dx}{4a^2}$$

$$\Big\downarrow \; 4674$$

$$\frac{\frac{2}{3}\int (c+dx)^2\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx+\frac{4d^2\int \sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{3f^2}-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2}{3}\int (c+dx)^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx+\frac{4d^2\int \csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx}{3f^2}-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}}{4a^2}$$

$$\Big\downarrow \; 4254$$

$$\frac{\frac{2}{3}\int (c+dx)^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx-\frac{8d^2\int 1d\left(-\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f^3}-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}}{4a^2}$$

$$\Big\downarrow \; 24$$

$$\frac{\frac{2}{3}\int (c+dx)^2\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{8d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^3}}{4a^2}$$

$$\Big\downarrow \; 4672$$

$$\frac{\frac{2}{3}\left(\frac{4d\int -\left((c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)dx}{f}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}\right)-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{8d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^3}}{4a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{8d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^3}}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\int (c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{8d^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^3}}{4a^2}$$

$$\Big\downarrow \; 4202$$

3.134. $\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{i(e+fx)}(c+dx)}{1+e^{i(e+fx)}}dx\right)}{f}\right)-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+}{4a^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(e+fx)}\right)dx}{f}-\frac{i(c+dx)\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}\right)-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2}{}}{4a^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(e+fx)}\log\left(1+e^{i(e+fx)}\right)de^{i(e+fx)}}{f^2}-\frac{i(c+dx)\log\left(1+e^{i(e+fx)}\right)}{f}\right)\right)}{f}\right)-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\right.}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{i(c+dx)\log\left(1+e^{i(e+fx)}\right)}{f}-\frac{d\,\text{PolyLog}\left(2,-e^{i(e+fx)}\right)}{f^2}\right)\right)}{f}\right)-\frac{4d(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+d}{}}{4a^2}$$

input

```
Int[(c + d*x)^2/(a + a*Cos[e + f*x])^2,x]
```

output

```
((-4*d*(c + d*x)*Sec[e/2 + (f*x)/2]^2)/(3*f^2) + (8*d^2*Tan[e/2 + (f*x)/2]
)/(3*f^3) + (2*(c + d*x)^2*Sec[e/2 + (f*x)/2]^2*Tan[e/2 + (f*x)/2])/(3*f)
 + (2*((-4*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-I)*(c + d*x)*Log[1 + E^(I*(
e + f*x))])/f - (d*PolyLog[2, -E^(I*(e + f*x))])/f^2)))/f + (2*(c + d*x)^2
*Tan[e/2 + (f*x)/2])/f))/3)/(4*a^2)
```

3.134. $\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$

3.134.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.134. $\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

3.134.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(172) = 344.

Time = 2.55 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.69

method	result
risch	$\frac{2i\left(2id^2fx\,\mathrm{e}^{2i(fx+e)}+3d^2f^2x^2\mathrm{e}^{i(fx+e)}+2icdf\,\mathrm{e}^{2i(fx+e)}+2id^2fx\,\mathrm{e}^{i(fx+e)}+6cd\,f^2x\,\mathrm{e}^{i(fx+e)}+d^2x^2f^2+2icdf\,\mathrm{e}^{i(fx+e)}+3c^2f^2\mathrm{e}^{i(fx+e}\right.}{3f^3a^2\left(\mathrm{e}^{i(fx+e)}+1\right)^3}$

input
```
int((d*x+c)^2/(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*I*(2*I*d^2*f*x*exp(2*I*(f*x+e))+3*d^2*f^2*x^2*exp(I*(f*x+e))+2*I*c*d*f
*exp(2*I*(f*x+e))+2*I*d^2*f*x*exp(I*(f*x+e))+6*c*d*f^2*x*exp(I*(f*x+e))+d^
2*x^2*f^2+2*I*c*d*f*exp(I*(f*x+e))+3*c^2*f^2*exp(I*(f*x+e))+2*c*d*f^2*x+c^
2*f^2+2*d^2*exp(2*I*(f*x+e))+4*d^2*exp(I*(f*x+e))+2*d^2)/f^3/a^2/(exp(I*(f
*x+e))+1)^3-4/3/a^2*d/f^2*c*ln(exp(I*(f*x+e)))+4/3/a^2*d/f^2*c*ln(exp(I*(f
*x+e))+1)-2/3*I/a^2*d^2/f*x^2-4/3*I/a^2*d^2/f^2*e*x-2/3*I/a^2*d^2/f^3*e^2+
4/3/a^2*d^2/f^2*ln(exp(I*(f*x+e))+1)*x-4/3*I*d^2*polylog(2,-exp(I*(f*x+e))
)/a^2/f^3+4/3/a^2*d^2/f^3*e*ln(exp(I*(f*x+e)))
```

3.134.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(169) = 338$.

Time = 0.27 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx =$$
$$-\frac{2\,d^2fx+2\,cdf+2\,(d^2fx+cdf)\cos(fx+e)+2\,(-i\,d^2\cos(fx+e)^2-2i\,d^2\cos(fx+e)-i\,d^2)\mathrm{Li}_2(-}{}$$

input
```
integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/3*(2*d^2*f*x + 2*c*d*f + 2*(d^2*f*x + c*d*f)*cos(f*x + e) + 2*(-I*d^2*c
os(f*x + e)^2 - 2*I*d^2*cos(f*x + e) - I*d^2)*dilog(-cos(f*x + e) + I*sin(
f*x + e)) + 2*(I*d^2*cos(f*x + e)^2 + 2*I*d^2*cos(f*x + e) + I*d^2)*dilog(
-cos(f*x + e) - I*sin(f*x + e)) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*c
os(f*x + e)^2 + 2*(d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) + I*sin
(f*x + e) + 1) - 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(f*x + e)^2 + 2
*(d^2*f*x + c*d*f)*cos(f*x + e))*log(cos(f*x + e) - I*sin(f*x + e) + 1) -
(2*d^2*f^2*x^2 + 4*c*d*f^2*x + 2*c^2*f^2 + 2*d^2 + (d^2*f^2*x^2 + 2*c*d*f^
2*x + c^2*f^2 + 2*d^2)*cos(f*x + e))*sin(f*x + e))/(a^2*f^3*cos(f*x + e)^2
 + 2*a^2*f^3*cos(f*x + e) + a^2*f^3)
```

3.134.6 Sympy [F]

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \frac{\int \frac{c^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx+\int \frac{d^2x^2}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx+\int \frac{2cdx}{\cos^2(e+fx)+2\cos(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**2/(a+a*cos(f*x+e))**2,x)
```

output
```
(Integral(c**2/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(d**2*
x**2/(cos(e + f*x)**2 + 2*cos(e + f*x) + 1), x) + Integral(2*c*d*x/(cos(e
+ f*x)**2 + 2*cos(e + f*x) + 1), x))/a**2
```

3.134.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 776 vs. $2(169) = 338$.

Time = 0.61 (sec) , antiderivative size = 776, normalized size of antiderivative = 3.66

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx$$
$$= \frac{2\left(c^2f^2+2d^2+2\left(d^2fx+cdf+\left(d^2fx+cdf\right)\cos\left(3fx+3e\right)+3\left(d^2fx+cdf\right)\cos\left(2fx+2e\right)+3\left(d^2fx\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*(c^2*f^2 + 2*d^2 + 2*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cos(3*f*x + 3*
e) + 3*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e) + 3*(d^2*f*x + c*d*f)*cos(f*x +
e) - (-I*d^2*f*x - I*c*d*f)*sin(3*f*x + 3*e) - 3*(-I*d^2*f*x - I*c*d*f)*si
n(2*f*x + 2*e) - 3*(-I*d^2*f*x - I*c*d*f)*sin(f*x + e))*arctan2(sin(f*x +
e), cos(f*x + e) + 1) - (d^2*f^2*x^2 + 2*c*d*f^2*x)*cos(3*f*x + 3*e) - (3*
d^2*f^2*x^2 - 2*I*c*d*f - 2*d^2 + 2*(3*c*d*f^2 - I*d^2*f)*x)*cos(2*f*x + 2
*e) + (3*c^2*f^2 + 2*I*d^2*f*x + 2*I*c*d*f + 4*d^2)*cos(f*x + e) - 2*(d^2*
cos(3*f*x + 3*e) + 3*d^2*cos(2*f*x + 2*e) + 3*d^2*cos(f*x + e) + I*d^2*sin
(3*f*x + 3*e) + 3*I*d^2*sin(2*f*x + 2*e) + 3*I*d^2*sin(f*x + e) + d^2)*dil
og(-e^(I*f*x + I*e)) - (I*d^2*f*x + I*c*d*f + (I*d^2*f*x + I*c*d*f)*cos(3*
f*x + 3*e) + 3*(I*d^2*f*x + I*c*d*f)*cos(2*f*x + 2*e) + 3*(I*d^2*f*x + I*c
*d*f)*cos(f*x + e) - (d^2*f*x + c*d*f)*sin(3*f*x + 3*e) - 3*(d^2*f*x + c*d
*f)*sin(2*f*x + 2*e) - 3*(d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^
2 + sin(f*x + e)^2 + 2*cos(f*x + e) + 1) - (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)
*sin(3*f*x + 3*e) - (3*I*d^2*f^2*x^2 + 2*c*d*f - 2*I*d^2 + 2*(3*I*c*d*f^2
+ d^2*f)*x)*sin(2*f*x + 2*e) - (-3*I*c^2*f^2 + 2*d^2*f*x + 2*c*d*f - 4*I*d
^2)*sin(f*x + e))/(-3*I*a^2*f^3*cos(3*f*x + 3*e) - 9*I*a^2*f^3*cos(2*f*x +
 2*e) - 9*I*a^2*f^3*cos(f*x + e) + 3*a^2*f^3*sin(3*f*x + 3*e) + 9*a^2*f^3*
sin(2*f*x + 2*e) + 9*a^2*f^3*sin(f*x + e) - 3*I*a^2*f^3)
```

3.134.8 Giac [F]

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(a\cos(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(a*cos(f*x + e) + a)^2, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+a\cos(e+fx))^2}\,dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(a + a*cos(e + f*x))^2,x)
```

output
```
\text{Hanged}
```

3.135 $\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx$

3.135.1 Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \frac{2d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2} - \frac{d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

output
```
2/3*d*ln(cos(1/2*f*x+1/2*e))/a^2/f^2-1/6*d*sec(1/2*f*x+1/2*e)^2/a^2/f^2+1/
3*(d*x+c)*tan(1/2*f*x+1/2*e)/a^2/f+1/6*(d*x+c)*sec(1/2*f*x+1/2*e)^2*tan(1/
2*f*x+1/2*e)/a^2/f
```

3.135.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.92

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx$$

$$= \frac{\cos\left(\frac{1}{2}(e+fx)\right)\left(2d\cos\left(\frac{3}{2}(e+fx)\right)\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)+2d\cos\left(\frac{1}{2}(e+fx)\right)\left(-1+3\log\left(\cos\left(\frac{1}{2}(e+f\right.\right.\right.\right.}{3a^2f^2(1+\cos(e+fx))^2}$$

input
```
Integrate[(c + d*x)/(a + a*Cos[e + f*x])^2,x]
```

output
```
(Cos[(e + f*x)/2]*(2*d*Cos[(3*(e + f*x))/2]*Log[Cos[(e + f*x)/2]] + 2*d*Co
s[(e + f*x)/2]*(-1 + 3*Log[Cos[(e + f*x)/2]]) + f*(c + d*x)*(3*Sin[(e + f*
x)/2] + Sin[(3*(e + f*x))/2])))/(3*a^2*f^2*(1 + Cos[e + f*x])^2)
```

3.135.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3799, 3042, 4673, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\cos(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)\sec^4\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^4dx}{4a^2}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{2}{3}\int (c+dx)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{2d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)^2dx+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{2d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2}{3}\left(\frac{2d\int -\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}\right)+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{2d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{2d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int\tan\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{2d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \; 3956$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4d\log\left(\cos\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}\right)+\frac{2(c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{2d\sec^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

input
```
Int[(c + d*x)/(a + a*Cos[e + f*x])^2,x]
```

output
```
((-2*d*Sec[e/2 + (f*x)/2]^2)/(3*f^2) + (2*(c + d*x)*Sec[e/2 + (f*x)/2]^2*T
an[e/2 + (f*x)/2])/(3*f) + (2*((4*d*Log[Cos[e/2 + (f*x)/2]])/f^2 + (2*(c +
 d*x)*Tan[e/2 + (f*x)/2])/f))/3)/(4*a^2)
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.135.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

method	result	size
parallelrisch	$\frac{-2d\ln\left(\sec^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\left(f(dx+c)\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-d\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+3(dx+c)f\right)}{6a^{2}f^{2}}$	74
default	$\frac{\frac{c\left(\frac{\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6}+\frac{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2}\right)}{f}-\frac{d\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f^{2}}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2f}+\frac{dx\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6f}-\frac{d\ln\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3f^{2}}}{a^{2}}$	109
norman	$\frac{\frac{c\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{c\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}-\frac{d\left(\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6a\,f^{2}}+\frac{dx\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{2af}+\frac{dx\left(\tan^{3}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{6af}}{a}-\frac{d\ln\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{3a^{2}f^{2}}$	128
risch	$-\frac{2idx}{3a^{2}f}-\frac{2ide}{3a^{2}f^{2}}-\frac{2\left(-3idfx\,\mathrm{e}^{i(fx+e)}-3icf\,\mathrm{e}^{i(fx+e)}-idfx-icf+\mathrm{e}^{2i(fx+e)}d+d\,\mathrm{e}^{i(fx+e)}\right)}{3f^{2}a^{2}\left(\mathrm{e}^{i(fx+e)}+1\right)^{3}}+\frac{2d\ln\left(\mathrm{e}^{i(fx+e)}+1\right)}{3a^{2}f^{2}}$	129

input
```
int((d*x+c)/(a+cos(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-2*d*ln(sec(1/2*f*x+1/2*e)^2)+tan(1/2*f*x+1/2*e)*(f*(d*x+c)*tan(1/2*f
*x+1/2*e)^2-d*tan(1/2*f*x+1/2*e)+3*(d*x+c)*f))/a^2/f^2
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.96

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = -\frac{d\cos(fx+e) - \left(d\cos(fx+e)^2 + 2\,d\cos(fx+e) + d\right)\log\left(\frac{1}{2}\cos(fx+e) + \frac{1}{2}\right) - (2\,dfx + 2\,cf + (df}{3\left(a^2f^2\cos(fx+e)^2 + 2\,a^2f^2\cos(fx+e) + a^2f^2\right)}$$

input
```
integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/3*(d*cos(f*x + e) - (d*cos(f*x + e)^2 + 2*d*cos(f*x + e) + d)*log(1/2*c
os(f*x + e) + 1/2) - (2*d*f*x + 2*c*f + (d*f*x + c*f)*cos(f*x + e))*sin(f*
x + e) + d)/(a^2*f^2*cos(f*x + e)^2 + 2*a^2*f^2*cos(f*x + e) + a^2*f^2)
```

3.135.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.19

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \begin{cases} \frac{c\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{c\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f} + \frac{dx\tan^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{dx\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f} - \frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{3a^2f^2} - \frac{d\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} & \text{for } f \neq \\ \frac{cx+\frac{dx^2}{2}}{(a\cos(e)+a)^2} & \text{otherwi} \end{cases}$$

input
```
integrate((d*x+c)/(a+a*cos(f*x+e))**2,x)
```

output
```
Piecewise((c*tan(e/2 + f*x/2)**3/(6*a**2*f) + c*tan(e/2 + f*x/2)/(2*a**2*f
) + d*x*tan(e/2 + f*x/2)**3/(6*a**2*f) + d*x*tan(e/2 + f*x/2)/(2*a**2*f) -
 d*log(tan(e/2 + f*x/2)**2 + 1)/(3*a**2*f**2) - d*tan(e/2 + f*x/2)**2/(6*a
**2*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cos(e) + a)**2, True))
```

3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(95) = 190$.

Time = 0.35 (sec) , antiderivative size = 763, normalized size of antiderivative = 6.20

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx =$$

$$-\frac{2\left(2\left(3\left(fx+e\right)\sin(fx+e)+\cos(2fx+2e)+\cos(fx+e)\right)\cos(3fx+3e)+2\left(9\left(fx+e\right)\sin(fx+e)+6\cos(fx+e)+1\right)\cos(2fx+2e)+6\cos(2fx+2e\cdots}{\cdots}$$

input
```
integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(2*(3*(f*x + e)*sin(f*x + e) + cos(2*f*x + 2*e) + cos(f*x + e))*co
s(3*f*x + 3*e) + 2*(9*(f*x + e)*sin(f*x + e) + 6*cos(f*x + e) + 1)*cos(2*f
*x + 2*e) + 6*cos(2*f*x + 2*e)^2 + 6*cos(f*x + e)^2 - (2*(3*cos(2*f*x + 2*
e) + 3*cos(f*x + e) + 1)*cos(3*f*x + 3*e) + cos(3*f*x + 3*e)^2 + 6*(3*cos(
f*x + e) + 1)*cos(2*f*x + 2*e) + 9*cos(2*f*x + 2*e)^2 + 9*cos(f*x + e)^2 +
 6*(sin(2*f*x + 2*e) + sin(f*x + e))*sin(3*f*x + 3*e) + sin(3*f*x + 3*e)^2
 + 9*sin(2*f*x + 2*e)^2 + 18*sin(2*f*x + 2*e)*sin(f*x + e) + 9*sin(f*x + e
)^2 + 6*cos(f*x + e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*cos(f*x
+ e) + 1) - 2*(f*x + 3*(f*x + e)*cos(f*x + e) + e - sin(2*f*x + 2*e) - sin
(f*x + e))*sin(3*f*x + 3*e) - 6*(f*x + 3*(f*x + e)*cos(f*x + e) + e - 2*si
n(f*x + e))*sin(2*f*x + 2*e) + 6*sin(2*f*x + 2*e)^2 + 6*sin(f*x + e)^2 + 2
*cos(f*x + e))*d/(a^2*f*cos(3*f*x + 3*e)^2 + 9*a^2*f*cos(2*f*x + 2*e)^2 +
9*a^2*f*cos(f*x + e)^2 + a^2*f*sin(3*f*x + 3*e)^2 + 9*a^2*f*sin(2*f*x + 2*
e)^2 + 18*a^2*f*sin(2*f*x + 2*e)*sin(f*x + e) + 9*a^2*f*sin(f*x + e)^2 + 6
*a^2*f*cos(f*x + e) + a^2*f + 2*(3*a^2*f*cos(2*f*x + 2*e) + 3*a^2*f*cos(f*
x + e) + a^2*f)*cos(3*f*x + 3*e) + 6*(3*a^2*f*cos(f*x + e) + a^2*f)*cos(2*
f*x + 2*e) + 6*(a^2*f*sin(2*f*x + 2*e) + a^2*f*sin(f*x + e))*sin(3*f*x + 3
*e)) - c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3)/a^2 + d*e*(3*sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^3/(co
s(f*x + e) + 1)^3)/(a^2*f))/f
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(95) = 190$.

Time = 0.49 (sec) , antiderivative size = 661, normalized size of antiderivative = 5.37

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

output
```
-1/6*(3*d*f*x*tan(1/2*f*x)^3*tan(1/2*e)^2 + 3*d*f*x*tan(1/2*f*x)^2*tan(1/2
*e)^3 - 2*d*log(4*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e)
 + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*t
an(1/2*f*x)^3*tan(1/2*e)^3 + 3*c*f*tan(1/2*f*x)^3*tan(1/2*e)^2 + 3*c*f*tan
(1/2*f*x)^2*tan(1/2*e)^3 + d*tan(1/2*f*x)^3*tan(1/2*e)^3 + d*f*x*tan(1/2*f
*x)^3 - 3*d*f*x*tan(1/2*f*x)^2*tan(1/2*e) - 3*d*f*x*tan(1/2*f*x)*tan(1/2*e
)^2 + 6*d*log(4*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e) +
 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan
(1/2*f*x)^2*tan(1/2*e)^2 + d*f*x*tan(1/2*e)^3 + c*f*tan(1/2*f*x)^3 - 3*c*f
*tan(1/2*f*x)^2*tan(1/2*e) + d*tan(1/2*f*x)^3*tan(1/2*e) - 3*c*f*tan(1/2*f
*x)*tan(1/2*e)^2 - d*tan(1/2*f*x)^2*tan(1/2*e)^2 + c*f*tan(1/2*e)^3 + d*ta
n(1/2*f*x)*tan(1/2*e)^3 + 3*d*f*x*tan(1/2*f*x) + 3*d*f*x*tan(1/2*e) - 6*d*
log(4*(tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1
/2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x)*
tan(1/2*e) + 3*c*f*tan(1/2*f*x) - d*tan(1/2*f*x)^2 + 3*c*f*tan(1/2*e) + d*
tan(1/2*f*x)*tan(1/2*e) - d*tan(1/2*e)^2 + 2*d*log(4*(tan(1/2*f*x)^2*tan(1
/2*e)^2 - 2*tan(1/2*f*x)*tan(1/2*e) + 1)/(tan(1/2*f*x)^2*tan(1/2*e)^2 + ta
n(1/2*f*x)^2 + tan(1/2*e)^2 + 1)) - d)/(a^2*f^2*tan(1/2*f*x)^3*tan(1/2*e)^
3 - 3*a^2*f^2*tan(1/2*f*x)^2*tan(1/2*e)^2 + 3*a^2*f^2*tan(1/2*f*x)*tan(1/2
*e) - a^2*f^2)
```

3.135.9 Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.42

$$\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx = \frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}+1\right)}{3\,a^2\,f^2} + \frac{(c\,f+d\,f\,x-d\,1\mathrm{i})\ 2\mathrm{i}}{3\,a^2\,f^2\ \left(2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)} - \frac{d\,x\,2\mathrm{i}}{3\,a^2\,f} - \frac{2\,d}{3\,a^2\,f^2\ \left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+1\right)} + \frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,(c+d\,x)\ 4\mathrm{i}}{3\,a^2\,f\ \left(3\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+3\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}+1\right)}$$

input
```
int((c + d*x)/(a + a*cos(e + f*x))^2,x)
```

output
```
(2*d*log(exp(e*1i)*exp(f*x*1i) + 1))/(3*a^2*f^2) + ((c*f - d*1i + d*f*x)*2
i)/(3*a^2*f^2*(2*exp(e*1i + f*x*1i) + exp(e*2i + f*x*2i) + 1)) - (d*x*2i)/
(3*a^2*f) - (2*d)/(3*a^2*f^2*(exp(e*1i + f*x*1i) + 1)) + (exp(e*1i + f*x*1
i)*(c + d*x)*4i)/(3*a^2*f*(3*exp(e*1i + f*x*1i) + 3*exp(e*2i + f*x*2i) + e
xp(e*3i + f*x*3i) + 1))
```

3.135. $\int \frac{c+dx}{(a+a\cos(e+fx))^2}\,dx$

3.136 $\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$

3.136.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+a\cos(e+fx))^2},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a+a*cos(f*x+e))^2,x)
```

3.136.2 Mathematica [N/A]

Not integrable

Time = 11.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + a*Cos[e + f*x])^2), x]
```

3.136.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\cos(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)(a\cos(e+fx)+a)^2}dx$$

input
```
Int[1/((c + d*x)*(a + a*Cos[e + f*x])^2),x]
```

output
```
$Aborted
```

3.136.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.136.4 Maple [N/A] (verified)

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+\cos\left(fx+e\right)a\right)^{2}}dx$$

input
```
int(1/(d*x+c)/(a+cos(f*x+e)*a)^2,x)
```

output
```
int(1/(d*x+c)/(a+cos(f*x+e)*a)^2,x)
```

3.136.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cos\left(fx+e\right)+a)^{2}}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (a^2*d*x + a^2*c)*cos(f*x + e)^2 + 2*(a^2*d*
x + a^2*c)*cos(f*x + e)), x)
```

3.136.6 Sympy [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \frac{\int \frac{1}{c\cos^{2}\left(e+fx\right)+2c\cos\left(e+fx\right)+c+dx\cos^{2}\left(e+fx\right)+2dx\cos\left(e+fx\right)+dx}\,dx}{a^{2}}$$

input
```
integrate(1/(d*x+c)/(a+a*cos(f*x+e))**2,x)
```

output
```
Integral(1/(c*cos(e + f*x)**2 + 2*c*cos(e + f*x) + c + d*x*cos(e + f*x)**2
 + 2*d*x*cos(e + f*x) + d*x), x)/a**2
```

3.136.7 Maxima [N/A]

Not integrable

Time = 10.73 (sec) , antiderivative size = 2913, normalized size of antiderivative = 145.65

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*(6*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + 6*(d^2*f*x + c*d*f)*cos(f*x
+ e)^2 + 6*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 4*d^2*sin(f*x + e) + 6*(
d^2*f*x + c*d*f)*sin(f*x + e)^2 - 2*(2*d^2*sin(2*f*x + 2*e) - (d^2*f*x + c
*d*f)*cos(2*f*x + 2*e) - (d^2*f*x + c*d*f)*cos(f*x + e) + (3*d^2*f^2*x^2 +
 6*c*d*f^2*x + 3*c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) + 2*(d^2*
f*x + c*d*f + 6*(d^2*f*x + c*d*f)*cos(f*x + e) - 3*(3*d^2*f^2*x^2 + 6*c*d*
f^2*x + 3*c^2*f^2 + 2*d^2)*sin(f*x + e))*cos(2*f*x + 2*e) + 2*(d^2*f*x + c
*d*f)*cos(f*x + e) + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*
d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2
*d*f^3*x + a^2*c^3*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*
d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e)^2 + 9*(a^2
*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(
f*x + e)^2 + (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x +
a^2*c^3*f^3)*sin(3*f*x + 3*e)^2 + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2
 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^3*f^3*x
^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*sin(2*f*x + 2*
e)*sin(f*x + e) + 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f
^3*x + a^2*c^3*f^3)*sin(f*x + e)^2 + 2*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*
x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f
^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*cos(2*f*x + 2*e) + 3*(a^2*d^3...
```

3.136.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cos(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(a*cos(f*x + e) + a)^2), x)
```

3.136.9 Mupad [N/A]

Not integrable

Time = 14.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cos(e+fx))^2} \, dx = \int \frac{1}{{\left(a+a\,\cos\left(e+f\,x\right)\right)}^2\,\left(c+d\,x\right)} \,d x$$

input
```
int(1/((a + a*cos(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + a*cos(e + f*x))^2*(c + d*x)), x)
```

3.137 $\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$

3.137.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+a\cos(e+fx))^2},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x)
```

3.137.2 Mathematica [N/A]

Not integrable

Time = 12.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2), x]
```

3.137.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\cos(e+fx)+a)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)^2\left(a\sin\left(e+fx+\frac{\pi}{2}\right)+a\right)^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)^2(a\cos(e+fx)+a)^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + a*Cos[e + f*x])^2),x]
```

output
```
$Aborted
```

3.137.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.137.4 Maple [N/A] (verified)

Not integrable

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2 (a+\cos(fx+e) a)^2}dx$$

input
```
int(1/(d*x+c)^2/(a+cos(f*x+e)*a)^2,x)
```

output
```
int(1/(d*x+c)^2/(a+cos(f*x+e)*a)^2,x)
```

3.137.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.95

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (a^2*d^2*x^2 + 2*a^2*c*d
*x + a^2*c^2)*cos(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*cos
(f*x + e)), x)
```

3.137.6 Sympy [N/A]

Not integrable

Time = 4.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \frac{\int \frac{1}{c^2\cos^2(e+fx)+2c^2\cos(e+fx)+c^2+2cdx\cos^2(e+fx)+4cdx\cos(e+fx)+2cdx+d^2x^2\cos^2(e+fx)+2d^2x^2\cos(e+fx)+d^2x^2}\,dx}{a^2}$$

input
```
integrate(1/(d*x+c)**2/(a+a*cos(f*x+e))**2,x)
```

output
```
Integral(1/(c**2*cos(e + f*x)**2 + 2*c**2*cos(e + f*x) + c**2 + 2*c*d*x*co
s(e + f*x)**2 + 4*c*d*x*cos(e + f*x) + 2*c*d*x + d**2*x**2*cos(e + f*x)**2
 + 2*d**2*x**2*cos(e + f*x) + d**2*x**2), x)/a**2
```

3.137.7 Maxima [N/A]

Not integrable

Time = 19.79 (sec) , antiderivative size = 3521, normalized size of antiderivative = 176.05

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\cos{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*(12*(d^2*f*x + c*d*f)*cos(2*f*x + 2*e)^2 + 12*(d^2*f*x + c*d*f)*cos(f*
x + e)^2 + 12*(d^2*f*x + c*d*f)*sin(2*f*x + 2*e)^2 + 12*d^2*sin(f*x + e) +
 12*(d^2*f*x + c*d*f)*sin(f*x + e)^2 - 2*(6*d^2*sin(2*f*x + 2*e) - 2*(d^2*
f*x + c*d*f)*cos(2*f*x + 2*e) - 2*(d^2*f*x + c*d*f)*cos(f*x + e) + 3*(d^2*
f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 4*d^2)*sin(f*x + e))*cos(3*f*x + 3*e) +
2*(2*d^2*f*x + 2*c*d*f + 12*(d^2*f*x + c*d*f)*cos(f*x + e) - 9*(d^2*f^2*x^
2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2)*sin(f*x + e))*cos(2*f*x + 2*e) + 4*(d^2
*f*x + c*d*f)*cos(f*x + e) + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*
a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4 +
 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4
*f^3)*cos(3*f*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*cos(2*f*x + 2*e)^2 +
9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c
^3*d*f^3*x + a^2*c^4*f^3)*cos(f*x + e)^2 + (a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*
f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(3*f
*x + 3*e)^2 + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3
*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*sin(2*f*x + 2*e)^2 + 18*(a^2*d^4*f
^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x +
 a^2*c^4*f^3)*sin(2*f*x + 2*e)*sin(f*x + e) + 9*(a^2*d^4*f^3*x^4 + 4*a^2*c
*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)...
```

3.137.8 Giac [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\cos(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cos(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(a*cos(f*x + e) + a)^2), x)
```

3.137.9 Mupad [N/A]

Not integrable

Time = 14.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cos(e+fx))^2}\,dx = \int \frac{1}{(a+a\,\cos(e+f\,x))^2\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + a*cos(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + a*cos(e + f*x))^2*(c + d*x)^2), x)
```

3.138 $\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx$

3.138.1 Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = -\frac{i(c+dx)^3}{af} - \frac{(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{6d(c+dx)^2\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{12id^2(c+dx)\operatorname{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3} + \frac{12d^3\operatorname{PolyLog}\left(3,e^{i(e+fx)}\right)}{af^4}$$

output
```
-I*(d*x+c)^3/a/f-(d*x+c)^3*cot(1/2*f*x+1/2*e)/a/f+6*d*(d*x+c)^2*ln(1-exp(I
*(f*x+e)))/a/f^2-12*I*d^2*(d*x+c)*polylog(2,exp(I*(f*x+e)))/a/f^3+12*d^3*p
olylog(3,exp(I*(f*x+e)))/a/f^4
```

3.138.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \frac{2\sin\left(\frac{1}{2}(e+fx)\right)\left(f^3(c+dx)^3\csc\left(\frac{e}{2}\right)\sin\left(\frac{fx}{2}\right)+2\left(-\frac{if^3(c+dx)^3}{-1+e^{ie}}+3df^2(c+dx)^2\log\left(1-e^{-i(e+fx)}\right)+6id^2\right.\right.}{f^4(a-a\cos(e+fx))}$$

input
```
Integrate[(c + d*x)^3/(a - a*Cos[e + f*x]),x]
```

output
```
(2*Sin[(e + f*x)/2]*(f^3*(c + d*x)^3*Csc[e/2]*Sin[(f*x)/2] + 2*(((-I)*f^3*
(c + d*x)^3)/(-1 + E^(I*e)) + 3*d*f^2*(c + d*x)^2*Log[1 - E^((-I)*(e + f*x
))] + (6*I)*d^2*f*(c + d*x)*PolyLog[2, E^((-I)*(e + f*x))] + 6*d^3*PolyLog
[3, E^((-I)*(e + f*x))])*Sin[(e + f*x)/2]))/(f^4*(a - a*Cos[e + f*x]))
```

3.138.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{a-a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^3\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^3\csc\left(\frac{e}{2}+\frac{fx}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{6d\int (c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{6d\int -(c+dx)^2\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)dx}{f}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{6d\int (c+dx)^2\tan\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$\Big\downarrow$ 4202

$$\frac{-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{i(e+fx+\pi)}(c+dx)^2}{1+e^{i(e+fx+\pi)}}dx\right)}{f}}{2a}$$

$\Big\downarrow$ 2620

$$\frac{-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\int (c+dx)\log\left(1+e^{i(e+fx+\pi)}\right)dx}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx+\pi)}\right)}{f}\right)\right)}{f}}{2a}$$

$\Big\downarrow$ 3011

$$\frac{-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx+\pi)}\right)}{f}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(e+fx+\pi)}\right)dx}{f}\right)}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx+\cdots}\right.}{f}\right.\right.}{f}}{2a}$$

$\Big\downarrow$ 2720

$$\frac{-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx+\pi)}\right)}{f}-\frac{d\int e^{-i(e+fx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(e+fx+\pi)}\right)de^{i(e+fx+\pi)}}{f^2}\right)}{f}-\frac{i(\cdots}{}\right.\right.}{f}}{2a}$$

$\Big\downarrow$ 7143

$$\frac{-\frac{2(c+dx)^3\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{2id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(e+fx+\pi)}\right)}{f}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(e+fx+\pi)}\right)}{f^2}\right)}{f}-\frac{i(c+dx)^2\log\left(1+e^{i(e+fx+\pi)}\right)}{f}\right)\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^3/(a - a*Cos[e + f*x]),x]
```

output
```
((-2*(c + d*x)^3*Cot[e/2 + (f*x)/2])/f - (6*d*(((I/3)*(c + d*x)^3)/d - (2*
I)*(((-I)*(c + d*x)^2*Log[1 + E^(I*(e + Pi + f*x))])/f + ((2*I)*d*((I*(c +
 d*x)*PolyLog[2, -E^(I*(e + Pi + f*x))])/f - (d*PolyLog[3, -E^(I*(e + Pi +
 f*x))])/f^2))/f)))/f)/(2*a)
```

3.138. $\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx$

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

3.138. $\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx$

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.138.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(121) = 242$.

Time = 1.34 (sec) , antiderivative size = 468, normalized size of antiderivative = 3.52

method	result
risch	$-\frac{6id^2cx^2}{af}-\frac{6d^3\ln\left(1-e^{i(fx+e)}\right)e^2}{a\,f^4}-\frac{6id^2c\,e^2}{a\,f^3}+\frac{4id^3e^3}{a\,f^4}-\frac{6d^3e^2\ln\left(e^{i(fx+e)}\right)}{a\,f^4}+\frac{6d^3e^2\ln\left(e^{i(fx+e)}-1\right)}{a\,f^4}-\frac{12d^2ce\ln\left(e^{i(fx+\right.}}{a\,f^3}$

input
```
int((d*x+c)^3/(a-cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
-6*I*d^2/a/f*c*x^2-6*d^3/a/f^4*ln(1-exp(I*(f*x+e)))*e^2-6*I*d^2/a/f^3*c*e^
2+4*I*d^3/a/f^4*e^3-6*d^3/a/f^4*e^2*ln(exp(I*(f*x+e)))+6*d^3/a/f^4*e^2*ln(
exp(I*(f*x+e))-1)-12*d^2/a/f^3*c*e*ln(exp(I*(f*x+e))-1)+12*d^2/a/f^3*c*e*l
n(exp(I*(f*x+e)))-12*I*d^2/a/f^3*c*polylog(2,exp(I*(f*x+e)))-2*I*(d^3*x^3+
3*c*d^2*x^2+3*c^2*d*x+c^3)/f/a/(exp(I*(f*x+e))-1)+12*d^2/a/f^3*c*ln(1-exp(
I*(f*x+e)))*e-2*I*d^3/a/f*x^3-12*I*d^3/a/f^3*polylog(2,exp(I*(f*x+e)))*x-1
2*I*d^2/a/f^2*c*e*x+6*I*d^3/a/f^3*e^2*x+12*d^2/a/f^2*c*ln(1-exp(I*(f*x+e))
)*x+6*d^3/a/f^2*ln(1-exp(I*(f*x+e)))*x^2+12*d^3*polylog(3,exp(I*(f*x+e)))/
a/f^4-6*d/a/f^2*c^2*ln(exp(I*(f*x+e)))+6*d/a/f^2*c^2*ln(exp(I*(f*x+e))-1)
```

3.138.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(118) = 236.

Time = 0.30 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.51

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = -\frac{d^3f^3x^3+3\,cd^2f^3x^2+3\,c^2df^3x+c^3f^3-6\,d^3\text{polylog}(3,\cos(fx+e)+i\,\sin(fx+e))\sin(fx+e)-6\,d}{}$$

input
```
integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
-(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + c^3*f^3 - 6*d^3*polylog(
3, cos(f*x + e) + I*sin(f*x + e))*sin(f*x + e) - 6*d^3*polylog(3, cos(f*x
+ e) - I*sin(f*x + e))*sin(f*x + e) + 6*(I*d^3*f*x + I*c*d^2*f)*dilog(cos(
f*x + e) + I*sin(f*x + e))*sin(f*x + e) + 6*(-I*d^3*f*x - I*c*d^2*f)*dilog
(cos(f*x + e) - I*sin(f*x + e))*sin(f*x + e) - 3*(d^3*e^2 - 2*c*d^2*e*f +
c^2*d*f^2)*log(-1/2*cos(f*x + e) + 1/2*I*sin(f*x + e) + 1/2)*sin(f*x + e)
- 3*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*log(-1/2*cos(f*x + e) - 1/2*I*sin(
f*x + e) + 1/2)*sin(f*x + e) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 +
2*c*d^2*e*f)*log(-cos(f*x + e) + I*sin(f*x + e) + 1)*sin(f*x + e) - 3*(d^3
*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*log(-cos(f*x + e) - I*si
n(f*x + e) + 1)*sin(f*x + e) + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^
3*x + c^3*f^3)*cos(f*x + e))/(a*f^4*sin(f*x + e))
```

3.138.6 Sympy [F]

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = -\frac{\int \frac{c^3}{\cos(e+fx)-1}\,dx+\int \frac{d^3x^3}{\cos(e+fx)-1}\,dx+\int \frac{3cd^2x^2}{\cos(e+fx)-1}\,dx+\int \frac{3c^2dx}{\cos(e+fx)-1}\,dx}{a}$$

input
```
integrate((d*x+c)**3/(a-a*cos(f*x+e)),x)
```

output
```
-(Integral(c**3/(cos(e + f*x) - 1), x) + Integral(d**3*x**3/(cos(e + f*x)
- 1), x) + Integral(3*c*d**2*x**2/(cos(e + f*x) - 1), x) + Integral(3*c**2
*d*x/(cos(e + f*x) - 1), x))/a
```

3.138.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 967 vs. 2(118) = 236.

Time = 0.45 (sec) , antiderivative size = 967, normalized size of antiderivative = 7.27

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
-(6*((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x +
e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*c*
d^2*e/(a*f^2*cos(f*x + e)^2 + a*f^2*sin(f*x + e)^2 - 2*a*f^2*cos(f*x + e)
+ a*f^2) - 3*((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(c
os(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x
 + e))*c^2*d/(a*f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*cos(f*x + e)
 + a*f) + c^3*(cos(f*x + e) + 1)/(a*sin(f*x + e)) + 3*c*d^2*e^2*(cos(f*x +
 e) + 1)/(a*f^2*sin(f*x + e)) - 3*c^2*d*e*(cos(f*x + e) + 1)/(a*f*sin(f*x
+ e)) - (2*d^3*e^3 + 6*(d^3*e^2*cos(f*x + e) + I*d^3*e^2*sin(f*x + e) - d^
3*e^2)*arctan2(sin(f*x + e), cos(f*x + e) - 1) + 6*((f*x + e)^2*d^3 - 2*(d
^3*e - c*d^2*f)*(f*x + e) - ((f*x + e)^2*d^3 - 2*(d^3*e - c*d^2*f)*(f*x +
e))*cos(f*x + e) - (I*(f*x + e)^2*d^3 + 2*(-I*d^3*e + I*c*d^2*f)*(f*x + e)
)*sin(f*x + e))*arctan2(sin(f*x + e), -cos(f*x + e) + 1) - 2*((f*x + e)^3*
d^3 + 3*(f*x + e)*d^3*e^2 - 3*(d^3*e - c*d^2*f)*(f*x + e)^2)*cos(f*x + e)
+ 12*((f*x + e)*d^3 - d^3*e + c*d^2*f - ((f*x + e)*d^3 - d^3*e + c*d^2*f)*
cos(f*x + e) - (I*(f*x + e)*d^3 - I*d^3*e + I*c*d^2*f)*sin(f*x + e))*dilog
(e^(I*f*x + I*e)) - 3*(-I*(f*x + e)^2*d^3 - I*d^3*e^2 + 2*(I*d^3*e - I*c*d
^2*f)*(f*x + e) + (I*(f*x + e)^2*d^3 + I*d^3*e^2 + 2*(-I*d^3*e + I*c*d^2*f
)*(f*x + e))*cos(f*x + e) - ((f*x + e)^2*d^3 + d^3*e^2 - 2*(d^3*e - c*d^2*
f)*(f*x + e))*sin(f*x + e))*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos...
```

3.138.8 Giac [F]

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \int -\frac{(dx+c)^3}{a\cos(fx+e)-a}\,dx$$

input
```
integrate((d*x+c)^3/(a-a*cos(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^3/(a*cos(f*x + e) - a), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a-a\cos(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a-a\,\cos\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^3/(a - a*cos(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a - a*cos(e + f*x)), x)
```

3.139 $\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx$

3.139.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = -\frac{i(c+dx)^2}{af} - \frac{(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{4d(c+dx)\log\left(1-e^{i(e+fx)}\right)}{af^2} - \frac{4id^2\operatorname{PolyLog}\left(2,e^{i(e+fx)}\right)}{af^3}$$

output
```
-I*(d*x+c)^2/a/f-(d*x+c)^2*cot(1/2*f*x+1/2*e)/a/f+4*d*(d*x+c)*ln(1-exp(I*(
f*x+e)))/a/f^2-4*I*d^2*polylog(2,exp(I*(f*x+e)))/a/f^3
```

3.139.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 292 vs. 2(102) = 204.

Time = 6.46 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.86

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = \frac{2\csc\left(\frac{e}{2}\right)\sin\left(\frac{1}{2}(e+fx)\right)\left(f^2(c+dx)^2\sin\left(\frac{fx}{2}\right) - 2cdf\left(fx\cos\left(\frac{e}{2}\right) - 2\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin\left(\frac{e}{2}\right)\right)\sin\left(\frac{1}{2}\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a - a*Cos[e + f*x]),x]
```

output
```
(2*Csc[e/2]*Sin[(e + f*x)/2]*(f^2*(c + d*x)^2*Sin[(f*x)/2] - 2*c*d*f*(f*x*
Cos[e/2] - 2*Log[Sin[(e + f*x)/2]]*Sin[e/2])*Sin[(e + f*x)/2] + d^2*(-(E^(
I*ArcTan[Tan[e/2]])*f^2*x^2*Cos[e/2]*Sqrt[Sec[e/2]^2]) - 4*((-1/2*I)*f*x*(
Pi - 2*ArcTan[Tan[e/2]]) - Pi*Log[1 + E^((-I)*f*x)] - (f*x + 2*ArcTan[Tan[
e/2]])*Log[1 - E^(I*(f*x + 2*ArcTan[Tan[e/2]]))] + Pi*Log[Cos[(f*x)/2]] +
2*ArcTan[Tan[e/2]]*Log[Sin[(f*x)/2 + ArcTan[Tan[e/2]]]] + I*PolyLog[2, E^(
I*(f*x + 2*ArcTan[Tan[e/2]]))])*Sin[e/2])*Sin[(e + f*x)/2]))/(f^3*(a - a*C
os[e + f*x]))
```

3.139.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {3042, 3799, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{a-a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^2\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)^2\csc\left(\frac{e}{2}+\frac{fx}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{4d\int (c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{4d\int -\left((c+dx)\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{4d\int(c+dx)\tan\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \; 4202$$

$$\frac{-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{i(e+fx+\pi)}(c+dx)}{1+e^{i(e+fx+\pi)}}dx\right)}{f}}{2a}$$

$$\Big\downarrow \; 2620$$

$$\frac{-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(e+fx+\pi)}\right)dx}{f}-\frac{i(c+dx)\log\left(1+e^{i(e+fx+\pi)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \; 2715$$

$$\frac{-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(e+fx+\pi)}\log\left(1+e^{i(e+fx+\pi)}\right)de^{i(e+fx+\pi)}}{f^2}-\frac{i(c+dx)\log\left(1+e^{i(e+fx+\pi)}\right)}{f}\right)\right)}{f}}{2a}$$

$$\Big\downarrow \; 2838$$

$$\frac{-\frac{2(c+dx)^2\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{i(c+dx)\log\left(1+e^{i(e+fx+\pi)}\right)}{f}-\frac{d\,\text{PolyLog}\left(2,-e^{i(e+fx+\pi)}\right)}{f^2}\right)\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^2/(a - a*Cos[e + f*x]),x]
```

output
```
((-2*(c + d*x)^2*Cot[e/2 + (f*x)/2])/f - (4*d*(((I/2)*(c + d*x)^2)/d - (2*
I)*(((-I)*(c + d*x)*Log[1 + E^(I*(e + Pi + f*x))])/f - (d*PolyLog[2, -E^(I
*(e + Pi + f*x))])/f^2)))/f)/(2*a)
```

3.139. $\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx$

3.139.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.139. $\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx$

3.139.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(92) = 184.

Time = 1.24 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.42

method	result
risch	$-\frac{2i(x^2d^2+2cdx+c^2)}{fa(e^{i(fx+e)}-1)}-\frac{4dc\ln(e^{i(fx+e)})}{a\,f^2}+\frac{4dc\ln(e^{i(fx+e)}-1)}{a\,f^2}-\frac{2id^2x^2}{af}-\frac{4id^2ex}{a\,f^2}-\frac{2id^2e^2}{a\,f^3}+\frac{4d^2\ln(1-e^{i(fx+e)})x}{a\,f^2}+$

input
```
int((d*x+c)^2/(a-cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(exp(I*(f*x+e))-1)-4/a/f^2*d*c*ln(exp(I*(f*
x+e)))+4*d/a/f^2*c*ln(exp(I*(f*x+e))-1)-2*I/a/f*d^2*x^2-4*I/a/f^2*d^2*e*x-
2*I/a/f^3*d^2*e^2+4*d^2/a/f^2*ln(1-exp(I*(f*x+e)))*x+4*d^2/a/f^3*ln(1-exp(
I*(f*x+e)))*e-4*I*d^2*polylog(2,exp(I*(f*x+e)))/a/f^3+4/a/f^3*d^2*e*ln(exp
(I*(f*x+e)))-4*d^2/a/f^3*e*ln(exp(I*(f*x+e))-1)
```

3.139.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(89) = 178.

Time = 0.28 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.77

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx =$$
$$-\frac{d^2f^2x^2+2\,cdf^2x+c^2f^2+2i\,d^2\mathrm{Li}_2(\cos(fx+e)+i\,\sin(fx+e))\sin(fx+e)-2i\,d^2\mathrm{Li}_2(\cos(fx+e)-\cdots}{}$$

input
```
integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
-(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*I*d^2*dilog(cos(f*x + e) + I*sin
(f*x + e))*sin(f*x + e) - 2*I*d^2*dilog(cos(f*x + e) - I*sin(f*x + e))*sin
(f*x + e) + 2*(d^2*e - c*d*f)*log(-1/2*cos(f*x + e) + 1/2*I*sin(f*x + e) +
 1/2)*sin(f*x + e) + 2*(d^2*e - c*d*f)*log(-1/2*cos(f*x + e) - 1/2*I*sin(f
*x + e) + 1/2)*sin(f*x + e) - 2*(d^2*f*x + d^2*e)*log(-cos(f*x + e) + I*si
n(f*x + e) + 1)*sin(f*x + e) - 2*(d^2*f*x + d^2*e)*log(-cos(f*x + e) - I*s
in(f*x + e) + 1)*sin(f*x + e) + (d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*cos(
f*x + e))/(a*f^3*sin(f*x + e))
```

3.139.6 Sympy [F]

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = -\frac{\int \frac{c^2}{\cos(e+fx)-1}\,dx + \int \frac{d^2x^2}{\cos(e+fx)-1}\,dx + \int \frac{2cdx}{\cos(e+fx)-1}\,dx}{a}$$

input
```
integrate((d*x+c)**2/(a-a*cos(f*x+e)),x)
```

output
```
-(Integral(c**2/(cos(e + f*x) - 1), x) + Integral(d**2*x**2/(cos(e + f*x)
- 1), x) + Integral(2*c*d*x/(cos(e + f*x) - 1), x))/a
```

3.139.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(89) = 178.

Time = 0.39 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.04

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx = -\frac{2\left(c^2f^2 - 2\left(cdf\cos(fx+e) + i\,cdf\sin(fx+e) - cdf\right)\arctan\left(\sin(fx+e), \cos(fx+e)-1\right) + 2\left(d^2f\right.\right.}{}$$

input
```
integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
-2*(c^2*f^2 - 2*(c*d*f*cos(f*x + e) + I*c*d*f*sin(f*x + e) - c*d*f)*arctan
2(sin(f*x + e), cos(f*x + e) - 1) + 2*(d^2*f*x*cos(f*x + e) + I*d^2*f*x*si
n(f*x + e) - d^2*f*x)*arctan2(sin(f*x + e), -cos(f*x + e) + 1) + (d^2*f^2*
x^2 + 2*c*d*f^2*x)*cos(f*x + e) + 2*(d^2*cos(f*x + e) + I*d^2*sin(f*x + e)
 - d^2)*dilog(e^(I*f*x + I*e)) + (-I*d^2*f*x - I*c*d*f + (I*d^2*f*x + I*c*
d*f)*cos(f*x + e) - (d^2*f*x + c*d*f)*sin(f*x + e))*log(cos(f*x + e)^2 + s
in(f*x + e)^2 - 2*cos(f*x + e) + 1) + (I*d^2*f^2*x^2 + 2*I*c*d*f^2*x)*sin(
f*x + e))/(-I*a*f^3*cos(f*x + e) + a*f^3*sin(f*x + e) + I*a*f^3)
```

3.139.8 Giac [F]

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx=\int -\frac{(dx+c)^2}{a\cos(fx+e)-a}\,dx$$

input
```
integrate((d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-(d*x + c)^2/(a*cos(f*x + e) - a), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a-a\cos(e+fx)}\,dx=\int \frac{(c+d\,x)^2}{a-a\,\cos(e+f\,x)}\,dx$$

input
```
int((c + d*x)^2/(a - a*cos(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a - a*cos(e + f*x)), x)
```

3.140 $\int \frac{c+dx}{a-a\cos(e+fx)} \, dx$

3.140.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{c+dx}{a-a\cos(e+fx)} \, dx = -\frac{(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{2d\log\left(\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

output
```
-(d*x+c)*cot(1/2*f*x+1/2*e)/a/f+2*d*ln(sin(1/2*f*x+1/2*e))/a/f^2
```

3.140.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{c+dx}{a-a\cos(e+fx)} \, dx = \frac{-4d\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)+f(c+dx)\sin(e+fx)}{af^2(-1+\cos(e+fx))}$$

input
```
Integrate[(c + d*x)/(a - a*Cos[e + f*x]),x]
```

output
```
(-4*d*Log[Sin[(e + f*x)/2]]*Sin[(e + f*x)/2]^2 + f*(c + d*x)*Sin[e + f*x])
/(a*f^2*(-1 + Cos[e + f*x]))
```

3.140.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3799, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a-a\cos(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a-a\sin\left(e+fx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)\csc^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{e}{2}+\frac{fx}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2d\int \cot\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2d\int -\tan\left(\frac{e}{2}+\frac{fx}{2}+\frac{\pi}{2}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{2d\int \tan\left(\frac{e+\pi}{2}+\frac{fx}{2}\right)dx}{f}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{4d\log\left(-\sin\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}-\frac{2(c+dx)\cot\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)/(a - a*Cos[e + f*x]),x]
```

output
```
((-2*(c + d*x)*Cot[e/2 + (f*x)/2])/f + (4*d*Log[-Sin[e/2 + (f*x)/2]])/f^2)
/(2*a)
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.140.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-d\ln\left(\sec^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)+2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)-\cot\left(\frac{fx}{2}+\frac{e}{2}\right)f(dx+c)}{f^{2}a}$	54
risch	$-\frac{2idx}{af}-\frac{2ide}{a\,f^{2}}-\frac{2i(dx+c)}{fa\left(e^{i(fx+e)}-1\right)}+\frac{2d\ln\left(e^{i(fx+e)}-1\right)}{a\,f^{2}}$	72
norman	$\frac{-\frac{c}{af}-\frac{dx}{af}}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}+\frac{2d\ln\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a\,f^{2}}-\frac{d\ln\left(1+\tan^{2}\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a\,f^{2}}$	76

input
```
int((d*x+c)/(a-cos(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
(-d*ln(sec(1/2*f*x+1/2*e)^2)+2*d*ln(tan(1/2*f*x+1/2*e))-cot(1/2*f*x+1/2*e)
*f*(d*x+c))/f^2/a
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = -\frac{dfx-d\log\left(-\frac{1}{2}\,\cos\left(fx+e\right)+\frac{1}{2}\right)\sin\left(fx+e\right)+cf+\left(dfx+cf\right)\cos\left(fx+e\right)}{af^2\sin\left(fx+e\right)}$$

input
```
integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
-(d*f*x - d*log(-1/2*cos(f*x + e) + 1/2)*sin(f*x + e) + c*f + (d*f*x + c*f
)*cos(f*x + e))/(a*f^2*sin(f*x + e))
```

3.140.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = \begin{cases} -\frac{c}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}-\frac{dx}{af\tan\left(\frac{e}{2}+\frac{fx}{2}\right)}-\frac{d\log\left(\tan^2\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2}+\frac{2d\log\left(\tan\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} & \text{for } f\neq 0 \\ \frac{cx+\frac{dx^2}{2}}{-a\cos\left(e\right)+a} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)/(a-a*cos(f*x+e)),x)
```

output
```
Piecewise((-c/(a*f*tan(e/2 + f*x/2)) - d*x/(a*f*tan(e/2 + f*x/2)) - d*log(
tan(e/2 + f*x/2)**2 + 1)/(a*f**2) + 2*d*log(tan(e/2 + f*x/2))/(a*f**2), Ne
(f, 0)), ((c*x + d*x**2/2)/(-a*cos(e) + a), True))
```

3.140.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.20

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = \frac{\frac{\left(\left(\cos(fx+e)^2+\sin(fx+e)^2-2\cos(fx+e)+1\right)\log\left(\cos(fx+e)^2+\sin(fx+e)^2-2\cos(fx+e)+1\right)-2(fx+e)\sin(fx+e)\right)d}{af\cos(fx+e)^2+af\sin(fx+e)^2-2af\cos(fx+e)+af} - \frac{c(\cos(fx+e)+1)}{a\sin(fx+e)} +}{f}$$

input
```
integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
(((cos(f*x + e)^2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1)*log(cos(f*x + e)^
2 + sin(f*x + e)^2 - 2*cos(f*x + e) + 1) - 2*(f*x + e)*sin(f*x + e))*d/(a*
f*cos(f*x + e)^2 + a*f*sin(f*x + e)^2 - 2*a*f*cos(f*x + e) + a*f) - c*(cos
(f*x + e) + 1)/(a*sin(f*x + e)) + d*e*(cos(f*x + e) + 1)/(a*f*sin(f*x + e)
))/f
```

3.140.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(42) = 84$.

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.78

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = \frac{dfx\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+cf\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)-dfx+d\log\left(\frac{4\left(\tan\left(\frac{1}{2}fx\right)^2+2\tan\left(\frac{1}{2}fx\right)\tan\left(\frac{1}{2}e\right)+\tan\left(\frac{1}{2}e\right)^2\right)}{\tan\left(\frac{1}{2}fx\right)^2\tan\left(\frac{1}{2}e\right)^2+\tan\left(\frac{1}{2}fx\right)^2+\tan\left(\frac{1}{2}e\right)^2+1}\right)}{af^2\tan\left(\frac{1}{2}fx\right)+af^2\tan\left(\frac{1}{2}e\right)}$$

input
```
integrate((d*x+c)/(a-a*cos(f*x+e)),x, algorithm="giac")
```

output
```
(d*f*x*tan(1/2*f*x)*tan(1/2*e) + c*f*tan(1/2*f*x)*tan(1/2*e) - d*f*x + d*l
og(4*(tan(1/2*f*x)^2 + 2*tan(1/2*f*x)*tan(1/2*e) + tan(1/2*e)^2)/(tan(1/2*
f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*f*x) + d
*log(4*(tan(1/2*f*x)^2 + 2*tan(1/2*f*x)*tan(1/2*e) + tan(1/2*e)^2)/(tan(1/
2*f*x)^2*tan(1/2*e)^2 + tan(1/2*f*x)^2 + tan(1/2*e)^2 + 1))*tan(1/2*e) - c
*f)/(a*f^2*tan(1/2*f*x) + a*f^2*tan(1/2*e))
```

3.140.9 Mupad [B] (verification not implemented)

Time = 13.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{c+dx}{a-a\cos(e+fx)}\,dx = \frac{2\,d\,\ln\left(\mathrm{e}^{e\,1\mathrm{i}}\,\mathrm{e}^{f\,x\,1\mathrm{i}}-1\right)}{a\,f^2} - \frac{(c+d\,x)\;2\mathrm{i}}{a\,f\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}-1\right)} - \frac{d\,x\,2\mathrm{i}}{a\,f}$$

input
```
int((c + d*x)/(a - a*cos(e + f*x)),x)
```

output
```
(2*d*log(exp(e*1i)*exp(f*x*1i) - 1))/(a*f^2) - ((c + d*x)*2i)/(a*f*(exp(e*
1i + f*x*1i) - 1)) - (d*x*2i)/(a*f)
```

3.141 $\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$

3.141.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a-a\cos(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

3.141.2 Mathematica [N/A]

Not integrable

Time = 3.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a - a*Cos[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a - a*Cos[e + f*x])), x]
```

3.141.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(c+dx)\left(a-a\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 3807$$

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a - a*Cos[e + f*x])),x]
```

output
```
$Aborted
```

3.141.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.141.4 Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a-\cos\,(fx+e)\,a)}dx$$

input
```
int(1/(d*x+c)/(a-cos(f*x+e)*a),x)
```

output
```
int(1/(d*x+c)/(a-cos(f*x+e)*a),x)
```

3.141.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\cos\,(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c - (a*d*x + a*c)*cos(f*x + e)), x)
```

3.141.6 Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = -\frac{\int \frac{1}{c\cos\,(e+fx)-c+dx\cos\,(e+fx)-dx}\,dx}{a}$$

input
```
integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x)
```

output
```
-Integral(1/(c*cos(e + f*x) - c + d*x*cos(e + f*x) - d*x), x)/a
```

3.141.7 Maxima [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 285, normalized size of antiderivative = 13.57

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\cos(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
-2*((a*d^2*f*x + a*c*d*f + (a*d^2*f*x + a*c*d*f)*cos(f*x + e)^2 + (a*d^2*f
*x + a*c*d*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x + a*c*d*f)*cos(f*x + e))*integ
rate(sin(f*x + e)/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2 + 2*
a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f
)*sin(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)),
x) + sin(f*x + e))/(a*d*f*x + a*c*f + (a*d*f*x + a*c*f)*cos(f*x + e)^2 + (
a*d*f*x + a*c*f)*sin(f*x + e)^2 - 2*(a*d*f*x + a*c*f)*cos(f*x + e))
```

3.141.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)(a\cos(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a-a*cos(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-1/((d*x + c)*(a*cos(f*x + e) - a)), x)
```

3.141.9 Mupad [N/A]

Not integrable

Time = 13.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a-a\cos(e+fx))}\,dx = \int \frac{1}{(a-a\,\cos\left(e+f\,x\right))\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a - a*cos(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a - a*cos(e + f*x))*(c + d*x)), x)
```

3.142 $\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$

3.142.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a-a\cos(e+fx))},x\right)$$

output
```
Unintegrable(1/(d*x+c)^2/(a-a*cos(f*x+e)),x)
```

3.142.2 Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a - a*Cos[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a - a*Cos[e + f*x])), x]
```

3.142.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2\left(a-a\sin\left(e+fx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a - a*Cos[e + f*x])),x]
```

output
```
$Aborted
```

3.142.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.142.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(dx+c\right)^{2} \left(a-\cos \left(fx+e\right) a\right)}dx$$

input
```
int(1/(d*x+c)^2/(a-cos(f*x+e)*a),x)
```

output
```
int(1/(d*x+c)^2/(a-cos(f*x+e)*a),x)
```

3.142.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.48

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int -\frac{1}{\left(dx+c\right)^{2}\left(a\cos\left(fx+e\right)-a\right)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 - (a*d^2*x^2 + 2*a*c*d*x + a*c^2
)*cos(f*x + e)), x)
```

3.142.6 Sympy [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.86

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = -\frac{\int \frac{1}{c^2\cos{(e+fx)}-c^2+2cdx\cos{(e+fx)}-2cdx+d^2x^2\cos{(e+fx)}-d^2x^2}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**2/(a-a*cos(f*x+e)),x)
```

output
```
-Integral(1/(c**2*cos(e + f*x) - c**2 + 2*c*d*x*cos(e + f*x) - 2*c*d*x + d
**2*x**2*cos(e + f*x) - d**2*x**2), x)/a
```

3.142.7 Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 442, normalized size of antiderivative = 21.05

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)^2(a\cos(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="maxima")
```

output
```
-2*(2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f + (a*d^3*f*x^2 + 2*a*c*d^2*
f*x + a*c^2*d*f)*cos(f*x + e)^2 + (a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f
)*sin(f*x + e)^2 - 2*(a*d^3*f*x^2 + 2*a*c*d^2*f*x + a*c^2*d*f)*cos(f*x + e
))*integrate(sin(f*x + e)/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x +
 a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*cos(f
*x + e)^2 + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*sin(
f*x + e)^2 - 2*(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*c
os(f*x + e)), x) + sin(f*x + e))/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a
*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*cos(f*x + e)^2 + (a*d^2*f*x^2 + 2*a*c*
d*f*x + a*c^2*f)*sin(f*x + e)^2 - 2*(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*
cos(f*x + e))
```

3.142.8 Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int -\frac{1}{(dx+c)^2(a\cos(fx+e)-a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a-a*cos(f*x+e)),x, algorithm="giac")
```

output
```
integrate(-1/((d*x + c)^2*(a*cos(f*x + e) - a)), x)
```

3.142.9 Mupad [N/A]

Not integrable

Time = 13.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a-a\cos(e+fx))}\,dx = \int \frac{1}{(a-a\,\cos\left(e+f\,x\right))\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a - a*cos(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a - a*cos(e + f*x))*(c + d*x)^2), x)
```

3.143 $\int x^3\sqrt{a+a\cos(c+dx)}\,dx$

3.143.1 Optimal result

Integrand size = 18, antiderivative size = 110

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx=-\frac{96\sqrt{a+a\cos(c+dx)}}{d^4}+\frac{12x^2\sqrt{a+a\cos(c+dx)}}{d^2}-\frac{48x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3}+\frac{2x^3\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

output
```
-96*(a+a*cos(d*x+c))^(1/2)/d^4+12*x^2*(a+a*cos(d*x+c))^(1/2)/d^2-48*x*(a+a
*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)/d^3+2*x^3*(a+a*cos(d*x+c))^(1/2)*tan
(1/2*d*x+1/2*c)/d
```

3.143.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.48

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx=\frac{2\sqrt{a(1+\cos(c+dx))}\left(6(-8+d^2x^2)+dx(-24+d^2x^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d^4}$$

input
```
Integrate[x^3*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(6*(-8 + d^2*x^2) + d*x*(-24 + d^2*x^2)*Tan[
(c + d*x)/2]))/d^4
```

3.143.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3800, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt{a \cos(c+dx)+a} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3 \sqrt{a \sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int x^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int x^3\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{6\int -x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}+\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\int x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\int x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\left(\frac{4\int x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\left(\frac{4\int x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)dx}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\left(\frac{4\left(\frac{2\int -\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}+\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\left(\frac{4\left(\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\int \sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\left(\frac{4\left(\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\int \sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{3118}$$

3.143. $\int x^3\sqrt{a+a\cos(c+dx)}\,dx$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\left(\frac{4\left(\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2}+\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}-\frac{2x^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

input
```
Int[x^3*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*((2*x^3*Sin[c/2 + (d*x)/2])/d
- (6*((-2*x^2*Cos[c/2 + (d*x)/2])/d + (4*((4*Cos[c/2 + (d*x)/2])/d^2 + (2*
x*Sin[c/2 + (d*x)/2])/d))/d))/d)
```

3.143.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.143.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{i(dx+c)}+1\right)^{2}\mathrm{e}^{-i(dx+c)}}\,\left(d^{3}x^{3}\mathrm{e}^{i(dx+c)}+6id^{2}x^{2}\mathrm{e}^{i(dx+c)}-d^{3}x^{3}+6id^{2}x^{2}-24dx\,\mathrm{e}^{i(dx+c)}-48i\mathrm{e}^{i(dx+c)}+24dx-48i\right)}{\left(\mathrm{e}^{i(dx+c)}+1\right)d^{4}}$	132

input
```
int(x^3*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+
1)*(d^3*x^3*exp(I*(d*x+c))+6*I*d^2*x^2*exp(I*(d*x+c))-d^3*x^3+6*I*d^2*x^2-
24*d*x*exp(I*(d*x+c))-48*I*exp(I*(d*x+c))+24*d*x-48*I)/d^4
```

3.143.5 Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.143.6 Sympy [F]

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx = \int x^3\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

input
```
integrate(x**3*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*(cos(c + d*x) + 1)), x)
```

3.143.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(94) = 188.

Time = 0.50 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx =$$

$$-\frac{2\left(\sqrt{2}\sqrt{a}c^3\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\left(\sqrt{2}(dx+c)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}c^2+3\left(\sqrt{2}(dx\right.}{}$$

input
```
integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*(sqrt(2)*sqrt(a)*c^3*sin(1/2*d*x + 1/2*c) - 3*(sqrt(2)*(d*x + c)*sin(1/
2*d*x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a)*c^2 + 3*(sqrt(2)*
(d*x + c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*(d*x + c)*cos(1/2*d*x + 1/2*c
) - 8*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a)*c - (sqrt(2)*(d*x + c)^3*sin(1
/2*d*x + 1/2*c) + 6*sqrt(2)*(d*x + c)^2*cos(1/2*d*x + 1/2*c) - 24*sqrt(2)*
(d*x + c)*sin(1/2*d*x + 1/2*c) - 48*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a))
/d^4
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx$$

$$=2\,\sqrt{2}\sqrt{a}\left(\frac{6\left(d^2x^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-8\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\right)\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d^4}+\frac{\left(d^3x^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx\right.\right.\right.}{}\right.$$

input
```
integrate(x^3*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*(6*(d^2*x^2*sgn(cos(1/2*d*x + 1/2*c)) - 8*sgn(cos(1/2*d*
x + 1/2*c)))*cos(1/2*d*x + 1/2*c)/d^4 + (d^3*x^3*sgn(cos(1/2*d*x + 1/2*c))
 - 24*d*x*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/2*d*x + 1/2*c)/d^4)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 13.77 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int x^3\sqrt{a+a\cos(c+dx)}\,dx = -\frac{2\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(48\,\cos\left(c+d\,x\right)-6\,d^2\,x^2-6\,d^2\,x^2\,\cos\left(c+d\,x\right)-d^3\,x^3\,\sin\left(c+d\,x\right)+24\,d\,x\right.}{d^4\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input

```
int(x^3*(a + a*cos(c + d*x))^(1/2),x)
```

output

```
-(2*(a*(cos(c + d*x) + 1))^(1/2)*(48*cos(c + d*x) - 6*d^2*x^2 - 6*d^2*x^2*
cos(c + d*x) - d^3*x^3*sin(c + d*x) + 24*d*x*sin(c + d*x) + 48))/(d^4*(cos
(c + d*x) + 1))
```

3.144 $\int x^2 \sqrt{a + a\cos(c + dx)}\, dx$

3.144.1 Optimal result

Integrand size = 18, antiderivative size = 88

$$\int x^2 \sqrt{a + a\cos(c + dx)}\, dx = \frac{8x\sqrt{a + a\cos(c + dx)}}{d^2} - \frac{16\sqrt{a + a\cos(c + dx)}\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^3} + \frac{2x^2\sqrt{a + a\cos(c + dx)}\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

output
```
8*x*(a+a*cos(d*x+c))^(1/2)/d^2-16*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c
)/d^3+2*x^2*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)/d
```

3.144.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.50

$$\int x^2 \sqrt{a + a\cos(c + dx)}\, dx = \frac{2\sqrt{a(1 + \cos(c + dx))}\left(4dx + (-8 + d^2x^2)\tan\left(\frac{1}{2}(c + dx)\right)\right)}{d^3}$$

input
```
Integrate[x^2*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(4*d*x + (-8 + d^2*x^2)*Tan[(c + d*x)/2]))/d
^3
```

3.144.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3800, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int x^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int x^2\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{4\int -x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}+\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\int x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\int x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\left(\frac{2\int\cos\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\left(\frac{2\int\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)dx}{d}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{3117}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x^2\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\left(\frac{4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2}-\frac{2x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)}{d}\right)$$

input
```
Int[x^2*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*((2*x^2*Sin[c/2 + (d*x)/2])/d
- (4*((-2*x*Cos[c/2 + (d*x)/2])/d + (4*Sin[c/2 + (d*x)/2])/d^2))/d)
```

3.144.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.144.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{i(dx+c)}+1\right)^{2}\mathrm{e}^{-i(dx+c)}}\,\left(d^{2}x^{2}\mathrm{e}^{i(dx+c)}+4idx\,\mathrm{e}^{i(dx+c)}-x^{2}d^{2}+4idx-8\,\mathrm{e}^{i(dx+c)}+8\right)}{\left(\mathrm{e}^{i(dx+c)}+1\right)d^{3}}$	105

input
```
int(x^2*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+
1)*(d^2*x^2*exp(I*(d*x+c))+4*I*d*x*exp(I*(d*x+c))-x^2*d^2+4*I*d*x-8*exp(I*
(d*x+c))+8)/d^3
```

3.144.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+a\cos(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.144.6 Sympy [F]

$$\int x^2\sqrt{a+a\cos(c+dx)}\,dx = \int x^2\sqrt{a\left(\cos\left(c+dx\right)+1\right)}\,dx$$

input
```
integrate(x**2*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*(cos(c + d*x) + 1)), x)
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.39

$$\int x^2\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{2\left(\sqrt{2}\sqrt{a}c^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\left(\sqrt{2}(dx+c)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\sqrt{2}\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sqrt{a}c+\left(\sqrt{2}(dx+c)\right.}{d^3}$$

input
```
integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*sqrt(a)*c^2*sin(1/2*d*x + 1/2*c) - 2*(sqrt(2)*(d*x + c)*sin(1/2
*d*x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a)*c + (sqrt(2)*(d*x
+ c)^2*sin(1/2*d*x + 1/2*c) + 4*sqrt(2)*(d*x + c)*cos(1/2*d*x + 1/2*c) - 8
*sqrt(2)*sin(1/2*d*x + 1/2*c))*sqrt(a))/d^3
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int x^2\sqrt{a+a\cos(c+dx)}\,dx$$
$$=2\sqrt{2}\sqrt{a}\left(\frac{4\,x\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d^2}+\frac{\left(d^2x^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)-8\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{d^3}\right.$$

input
```
integrate(x^2*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*(4*x*cos(1/2*d*x + 1/2*c)*sgn(cos(1/2*d*x + 1/2*c))/d^2
+ (d^2*x^2*sgn(cos(1/2*d*x + 1/2*c)) - 8*sgn(cos(1/2*d*x + 1/2*c)))*sin(1/
2*d*x + 1/2*c)/d^3)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 13.78 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int x^2\sqrt{a+a\cos(c+dx)}\,dx$$
$$=\frac{2\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,\left(4\,d\,x-8\,\sin\left(c+d\,x\right)+d^2\,x^2\,\sin\left(c+d\,x\right)+4\,d\,x\,\cos\left(c+d\,x\right)\right)}{d^3\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int(x^2*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(2*(a*(cos(c + d*x) + 1))^(1/2)*(4*d*x - 8*sin(c + d*x) + d^2*x^2*sin(c +
d*x) + 4*d*x*cos(c + d*x)))/(d^3*(cos(c + d*x) + 1))
```

3.145 $\int x\sqrt{a+a\cos(c+dx)}\,dx$

3.145.1 Optimal result

Integrand size = 16, antiderivative size = 53

$$\int x\sqrt{a+a\cos(c+dx)}\,dx=\frac{4\sqrt{a+a\cos(c+dx)}}{d^2}+\frac{2x\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

output
```
4*(a+a*cos(d*x+c))^(1/2)/d^2+2*x*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2*c)
/d
```

3.145.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int x\sqrt{a+a\cos(c+dx)}\,dx=\frac{2\sqrt{a(1+\cos(c+dx))}\left(2+dx\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d^2}$$

input
```
Integrate[x*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*(2 + d*x*Tan[(c + d*x)/2]))/d^2
```

3.145.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3800, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+adx}$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int x\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2\int -\sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}+\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\int \sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\int \sin\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3118}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{4\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2}+\frac{2x\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right)$$

input
```
Int[x*Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*((4*Cos[c/2 + (d*x)/2])/d^2 +
(2*x*Sin[c/2 + (d*x)/2])/d)
```

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.145.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{i(dx+c)}+1\right)^{2} e^{-i(dx+c)}}\,\left(dx\,e^{i(dx+c)}+2i e^{i(dx+c)}-dx+2i\right)}{\left(e^{i(dx+c)}+1\right)d^{2}}$	80

input
```
int(x*(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(a*(exp(I*(d*x+c))+1)^2*exp(-I*(d*x+c)))^(1/2)/(exp(I*(d*x+c))+
1)*(d*x*exp(I*(d*x+c))+2*I*exp(I*(d*x+c))-d*x+2*I)/d^2
```

3.145.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+a\cos(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.145.6 Sympy [F]

$$\int x\sqrt{a+a\cos(c+dx)}\,dx = \int x\sqrt{a(\cos(c+dx)+1)}\,dx$$

input
```
integrate(x*(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(x*sqrt(a*(cos(c + d*x) + 1)), x)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int x\sqrt{a+a\cos(c+dx)}\,dx$$
$$= -\frac{2\left(\sqrt{2}\sqrt{a}c\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\left(\sqrt{2}(dx+c)\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)\sqrt{a}\right)}{d^2}$$

input
```
integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*(sqrt(2)*sqrt(a)*c*sin(1/2*d*x + 1/2*c) - (sqrt(2)*(d*x + c)*sin(1/2*d*
x + 1/2*c) + 2*sqrt(2)*cos(1/2*d*x + 1/2*c))*sqrt(a))/d^2
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int x\sqrt{a+a\cos(c+dx)}\,dx$$
$$=2\sqrt{2}\left(\frac{x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d}+\frac{2\,\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d^2}\right)\sqrt{a}$$

input
```
integrate(x*(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*(x*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d + 2*cos(1/2*
d*x + 1/2*c)*sgn(cos(1/2*d*x + 1/2*c))/d^2)*sqrt(a)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int x\sqrt{a+a\cos(c+dx)}\,dx=\frac{2\,\sqrt{a\,\left(\cos\left(c+d\,x\right)+1\right)}\left(2\,\cos\left(c+d\,x\right)+d\,x\,\sin\left(c+d\,x\right)+2\right)}{d^2\,\left(\cos\left(c+d\,x\right)+1\right)}$$

input
```
int(x*(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(2*(a*(cos(c + d*x) + 1))^(1/2)*(2*cos(c + d*x) + d*x*sin(c + d*x) + 2))/(
d^2*(cos(c + d*x) + 1))
```

3.146 $\int \sqrt{a+a\cos(c+dx)}\,dx$

3.146.1 Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2a\sin(c+dx)}{d\sqrt{a+a\cos(c+dx)}}$$

output
```
2*a*sin(d*x+c)/d/(a+a*cos(d*x+c))^(1/2)
```

3.146.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a+a\cos(c+dx)}\,dx=\frac{2\sqrt{a(1+\cos(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cos[c + d*x])]*Tan[(c + d*x)/2])/d
```

3.146.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3125}$$

$$\frac{2a\sin(c+dx)}{d\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*a*Sin[c + d*x])/(d*Sqrt[a + a*Cos[c + d*x]])
```

3.146.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.146.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{2a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,d}$	43
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{i(dx+c)}+1\right)^2 e^{-i(dx+c)}}\,\left(e^{i(dx+c)}-1\right)}{\left(e^{i(dx+c)}+1\right)d}$	60

input
```
int((a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)*2^(1/2)/(a*cos(1/2*d*x+1/2*c)^2)
^(1/2)/d
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \frac{2\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{d\cos(dx+c)+d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*cos(d*x + c) + a)*sin(d*x + c)/(d*cos(d*x + c) + d)
```

3.146.6 Sympy [F]

$$\int \sqrt{a+a\cos(c+dx)}\,dx = \int \sqrt{a\cos(c+dx)+a}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*cos(c + d*x) + a), x)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sqrt{a + a\cos(c + dx)}\, dx = \frac{2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(a)*sin(1/2*d*x + 1/2*c)/d
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \sqrt{a + a\cos(c + dx)}\, dx = \frac{2\sqrt{2}\sqrt{a}\operatorname{sgn}\left(\cos\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)\right)\sin\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)}{d}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))*sin(1/2*d*x + 1/2*c)/d
```

3.146.9 Mupad [B] (verification not implemented)

Time = 13.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \sqrt{a + a\cos(c + dx)}\, dx = \frac{2\,\sin\left(c + d\,x\right)\,\sqrt{a\,\left(\cos\left(c + d\,x\right) + 1\right)}}{d\,\left(\cos\left(c + d\,x\right) + 1\right)}$$

input
```
int((a + a*cos(c + d*x))^(1/2),x)
```

output
```
(2*sin(c + d*x)*(a*(cos(c + d*x) + 1))^(1/2))/(d*(cos(c + d*x) + 1))
```

3.147 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx$

3.147.1 Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right) - \sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)$$

output
```
Ci(1/2*d*x)*cos(1/2*c)*sec(1/2*d*x+1/2*c)*(a+a*cos(d*x+c))^(1/2)-sec(1/2*d
*x+1/2*c)*Si(1/2*d*x)*sin(1/2*c)*(a+a*cos(d*x+c))^(1/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \sqrt{a(1+\cos(c+dx))}\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]/x,x]
```

output
```
Sqrt[a*(1 + Cos[c + d*x])]*Sec[(c + d*x)/2]*(Cos[c/2]*CosIntegral[(d*x)/2]
 - Sin[c/2]*SinIntegral[(d*x)/2])
```

3.147.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 3800, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\cos\left(\frac{c}{2}\right)\int \frac{\cos\left(\frac{dx}{2}\right)}{x}dx-\sin\left(\frac{c}{2}\right)\int \frac{\sin\left(\frac{dx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\cos\left(\frac{c}{2}\right)\int \frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx-\sin\left(\frac{c}{2}\right)\int \frac{\sin\left(\frac{dx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{3780}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\cos\left(\frac{c}{2}\right)\int \frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)$$

$$\Big\downarrow \text{3783}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/x,x]
```

output
```
Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*(Cos[c/2]*CosIntegral[(d*x)/2]
 - Sin[c/2]*SinIntegral[(d*x)/2])
```

3.147.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.147.4 Maple [F]

$$\int \frac{\sqrt{a + \cos(dx + c)\, a}}{x} dx$$

input
```
int((a+cos(d*x+c)*a)^(1/2)/x,x)
```

output
```
int((a+cos(d*x+c)*a)^(1/2)/x,x)
```

3.147.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.147.6 Sympy [F]

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x}\, dx = \int \frac{\sqrt{a\,(\cos{(c + dx)} + 1)}}{x}\, dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/x,x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))/x, x)
```

3.147.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x}\, dx = -\frac{1}{2}\left(\left(\sqrt{2}E_1\left(\frac{1}{2}i\,dx\right) + \sqrt{2}E_1\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}\,c\right) - \left(i\,\sqrt{2}E_1\left(\frac{1}{2}i\,dx\right) - i\,\sqrt{2}E_1\left(-\frac{1}{2}i\,dx\right)\right)\sin\left(\frac{1}{2}\right.$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

output
```
-1/2*((sqrt(2)*exp_integral_e(1, 1/2*I*d*x) + sqrt(2)*exp_integral_e(1, -1
/2*I*d*x))*cos(1/2*c) - (I*sqrt(2)*exp_integral_e(1, 1/2*I*d*x) - I*sqrt(2
)*exp_integral_e(1, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)
```

3.147.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = -\frac{\sqrt{2}\left(\Re\left(\mathrm{Ci}\left(\frac{1}{2}\,dx\right)\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{4}\,c\right)^2+\Re\left(\mathrm{Ci}\left(-\frac{1}{2}\,dx\right)\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)\tan\left(\frac{1}{4}\,c\right)^2+\right.}{}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/4*c)^2 + real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*
tan(1/4*c)^2 + 2*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)
)*tan(1/4*c) - 2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c
))*tan(1/4*c) + 4*sgn(cos(1/2*d*x + 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*
c) - real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)) - real_par
t(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)))*sqrt(a)/(tan(1/4*c)^2
 + 1)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x}\,dx = \int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{x}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/x,x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/x, x)
```

3.148 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx$

3.148.1 Optimal result

Integrand size = 18, antiderivative size = 110

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = -\frac{\sqrt{a+a\cos(c+dx)}}{x} - \frac{1}{2}d\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right) - \frac{1}{2}d\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Si}\left(\frac{dx}{2}\right)$$

output
```
-(a+a*cos(d*x+c))^(1/2)/x-1/2*d*cos(1/2*c)*sec(1/2*d*x+1/2*c)*Si(1/2*d*x)*
(a+a*cos(d*x+c))^(1/2)-1/2*d*Ci(1/2*d*x)*sec(1/2*d*x+1/2*c)*sin(1/2*c)*(a+
a*cos(d*x+c))^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = -\frac{\sqrt{a(1+\cos(c+dx))}\left(2+dx\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}\right)+dx\cos\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\text{Si}\left(\frac{d}{2}\right.\right.}{2x}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]/x^2,x]
```

output
```
-1/2*(Sqrt[a*(1 + Cos[c + d*x])]*(2 + d*x*CosIntegral[(d*x)/2]*Sec[(c + d*
x)/2]*Sin[c/2] + d*x*Cos[c/2]*Sec[(c + d*x)/2]*SinIntegral[(d*x)/2]))/x
```

3.148.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3800, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{x^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{x^2}dx$$

$$\Big\downarrow \ 3800$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}dx$$

$$\Big\downarrow \ 3042$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \ 3778$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{1}{2}d\int -\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)$$

$$\Big\downarrow \ 25$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{2}d\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)$$

$$\Big\downarrow \ 3042$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{2}d\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{2}d\left(\sin\left(\frac{c}{2}\right)\int\frac{\cos\left(\frac{dx}{2}\right)}{x}dx+\cos\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{dx}{2}\right)}{x}dx\right)-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{2}d\left(\sin\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx+\cos\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{dx}{2}\right)}{x}dx\right)-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)$$

$$\Big\downarrow \text{3780}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{2}d\left(\sin\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx+\cos\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)$$

$$\Big\downarrow \text{3783}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{2}d\left(\sin\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)+\cos\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)$$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/x^2,x]
```

output
```
Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*(-(Cos[c/2 + (d*x)/2]/x) - (d*
(CosIntegral[(d*x)/2]*Sin[c/2] + Cos[c/2]*SinIntegral[(d*x)/2]))/2)
```

3.148.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

3.148. $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx$

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.148.4 Maple [F]

$$\int \frac{\sqrt{a+\cos(dx+c)\,a}}{x^2}\,dx$$

input
```
int((a+cos(d*x+c)*a)^(1/2)/x^2,x)
```

output
```
int((a+cos(d*x+c)*a)^(1/2)/x^2,x)
```

3.148.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

3.148.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{x^2}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))/x**2, x)
```

3.148.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = -\frac{\left(\left(E_2\left(\frac{1}{2}i\,dx\right)+E_2\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)^3+\left(E_2\left(\frac{1}{2}i\,dx\right)+E_2\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)\sin\left(\frac{1}{2}c\right)^2+\left(-i\,E_2\left(\frac{1}{2}i\,dx\right.\right.\right.}{2\left(\left(\sqrt{2}\cos\right.\right.}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/2*((exp_integral_e(2, 1/2*I*d*x) + exp_integral_e(2, -1/2*I*d*x))*cos(1
/2*c)^3 + (exp_integral_e(2, 1/2*I*d*x) + exp_integral_e(2, -1/2*I*d*x))*c
os(1/2*c)*sin(1/2*c)^2 + (-I*exp_integral_e(2, 1/2*I*d*x) + I*exp_integral
_e(2, -1/2*I*d*x))*sin(1/2*c)^3 + (exp_integral_e(2, 1/2*I*d*x) + exp_inte
gral_e(2, -1/2*I*d*x))*cos(1/2*c) + ((-I*exp_integral_e(2, 1/2*I*d*x) + I*
exp_integral_e(2, -1/2*I*d*x))*cos(1/2*c)^2 - I*exp_integral_e(2, 1/2*I*d*
x) + I*exp_integral_e(2, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)*d/((sqrt(2)*cos(
1/2*c)^2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c) - (sqrt(2)*cos(1/2*c)^2 + sqrt(
2)*sin(1/2*c)^2)*c)
```

3.148.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 560, normalized size of antiderivative = 5.09

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c)
)*tan(1/4*d*x)^2*tan(1/4*c)^2 - d*x*imag_part(cos_integral(-1/2*d*x))*sgn(
cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d*x*sgn(cos(1/2*d*x
+ 1/2*c))*sin_integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c)^2 - 2*d*x*real_p
art(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/
4*c) - 2*d*x*real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/4*d*x)^2*tan(1/4*c) - d*x*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/
2*d*x + 1/2*c))*tan(1/4*d*x)^2 + d*x*imag_part(cos_integral(-1/2*d*x))*sgn
(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - 2*d*x*sgn(cos(1/2*d*x + 1/2*c))*si
n_integral(1/2*d*x)*tan(1/4*d*x)^2 + d*x*imag_part(cos_integral(1/2*d*x))*
sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 - d*x*imag_part(cos_integral(-1/2*d
*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 + 2*d*x*sgn(cos(1/2*d*x + 1/2*
c))*sin_integral(1/2*d*x)*tan(1/4*c)^2 - 2*d*x*real_part(cos_integral(1/2*
d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c) - 2*d*x*real_part(cos_integral(
-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c) - 4*sgn(cos(1/2*d*x + 1/2*
c))*tan(1/4*d*x)^2*tan(1/4*c)^2 - d*x*imag_part(cos_integral(1/2*d*x))*sgn
(cos(1/2*d*x + 1/2*c)) + d*x*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2
*d*x + 1/2*c)) - 2*d*x*sgn(cos(1/2*d*x + 1/2*c))*sin_integral(1/2*d*x) + 4
*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 + 16*sgn(cos(1/2*d*x + 1/2*c))*t
an(1/4*d*x)*tan(1/4*c) + 4*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 - 4*s...
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^2}\,dx = \int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{x^2}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/x^2,x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/x^2, x)
```

3.149 $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx$

3.149.1 Optimal result

Integrand size = 18, antiderivative size = 151

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = -\frac{\sqrt{a+a\cos(c+dx)}}{2x^2} - \frac{1}{8}d^2\cos\left(\frac{c}{2}\right)\sqrt{a+a\cos(c+dx)}\,\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{1}{8}d^2\sqrt{a+a\cos(c+dx)}\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right) + \frac{d\sqrt{a+a\cos(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x}$$

output
```
-1/2*(a+a*cos(d*x+c))^(1/2)/x^2-1/8*d^2*Ci(1/2*d*x)*cos(1/2*c)*sec(1/2*d*x
+1/2*c)*(a+a*cos(d*x+c))^(1/2)+1/8*d^2*sec(1/2*d*x+1/2*c)*Si(1/2*d*x)*sin(
1/2*c)*(a+a*cos(d*x+c))^(1/2)+1/4*d*(a+a*cos(d*x+c))^(1/2)*tan(1/2*d*x+1/2
*c)/x
```

3.149.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx$$
$$= \frac{\sqrt{a(1+\cos(c+dx))}\left(-4-d^2x^2\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)+d^2x^2\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{c}{2}\right)\text{S}\right.}{8x^2}$$

input
```
Integrate[Sqrt[a + a*Cos[c + d*x]]/x^3,x]
```

output
```
(Sqrt[a*(1 + Cos[c + d*x])]*(-4 - d^2*x^2*Cos[c/2]*CosIntegral[(d*x)/2]*Se
c[(c + d*x)/2] + d^2*x^2*Sec[(c + d*x)/2]*Sin[c/2]*SinIntegral[(d*x)/2] +
2*d*x*Tan[(c + d*x)/2]))/(8*x^2)
```

3.149.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 3778, 25, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(c+dx)+a}}{x^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{x^3}dx$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(\frac{1}{4}d\int -\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}\right)$$

$\downarrow$ 25

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{4}d\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}\right)$$

$\downarrow$ 3042

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{4}d\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}dx-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}\right)$$

$\downarrow$ 3778

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{4}d\left(\frac{1}{2}d\int \frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}\right)$$

$\downarrow$ 3042

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\left(-\frac{1}{4}d\left(\frac{1}{2}d\int \frac{\sin\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)-\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}\right)$$

$\downarrow$ 3784

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\Bigg(-\frac{1}{4}d\Bigg(\frac{1}{2}d\left(\cos\left(\frac{c}{2}\right)\int \frac{\cos\left(\frac{dx}{2}\right)}{x}dx-\sin\left(\frac{c}{2}\right)\int \frac{\sin\left(\frac{dx}{2}\right)}{x}dx\right)-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\Bigg)$$

$\downarrow$ 3042

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\Bigg(-\frac{1}{4}d\Bigg(\frac{1}{2}d\left(\cos\left(\frac{c}{2}\right)\int \frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx-\sin\left(\frac{c}{2}\right)\int \frac{\sin\left(\frac{dx}{2}\right)}{x}dx\right)-\frac{\sin\left(\frac{c}{2}+\right.}{x}$$

$\downarrow$ 3780

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\Bigg(-\frac{1}{4}d\Bigg(\frac{1}{2}d\left(\cos\left(\frac{c}{2}\right)\int \frac{\sin\left(\frac{dx}{2}+\frac{\pi}{2}\right)}{x}dx-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\Bigg)$$

$\downarrow$ 3783

$$\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cos(c+dx)+a}\Bigg(-\frac{1}{4}d\Bigg(\frac{1}{2}d\left(\cos\left(\frac{c}{2}\right)\text{CosIntegral}\left(\frac{dx}{2}\right)-\sin\left(\frac{c}{2}\right)\text{Si}\left(\frac{dx}{2}\right)\right)-\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\Bigg)$$

3.149. $\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx$

input
```
Int[Sqrt[a + a*Cos[c + d*x]]/x^3,x]
```

output
```
Sqrt[a + a*Cos[c + d*x]]*Sec[c/2 + (d*x)/2]*(-1/2*Cos[c/2 + (d*x)/2]/x^2 -
 (d*(-(Sin[c/2 + (d*x)/2]/x) + (d*(Cos[c/2]*CosIntegral[(d*x)/2] - Sin[c/2
]*SinIntegral[(d*x)/2]))/2))/4)
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.149.4 Maple [F]

$$\int \frac{\sqrt{a+\cos\left(dx+c\right)a}}{x^3}dx$$

input
```
int((a+cos(d*x+c)*a)^(1/2)/x^3,x)
```

output
```
int((a+cos(d*x+c)*a)^(1/2)/x^3,x)
```

3.149.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.149.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = \int \frac{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}{x^3}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*(cos(c + d*x) + 1))/x**3, x)
```

3.149.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x^3}\,dx =$$

$$-\frac{\left(\left(E_3\left(\frac{1}{2}i\,dx\right) + E_3\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)^3 + \left(E_3\left(\frac{1}{2}i\,dx\right) + E_3\left(-\frac{1}{2}i\,dx\right)\right)\cos\left(\frac{1}{2}c\right)\sin\left(\frac{1}{2}c\right)^2 + \left(-i\,E_3\left(\frac{1}{2}i\,dx\right.\right.}{2\left(\left(\sqrt{2}\cos\left(\frac{1}{2}c\right)^2 + \sqrt{2}\sin\left(\frac{1}{2}c\right)^2\right)\right.}$$

input

```
integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

output

```
-1/2*((exp_integral_e(3, 1/2*I*d*x) + exp_integral_e(3, -1/2*I*d*x))*cos(1
/2*c)^3 + (exp_integral_e(3, 1/2*I*d*x) + exp_integral_e(3, -1/2*I*d*x))*c
os(1/2*c)*sin(1/2*c)^2 + (-I*exp_integral_e(3, 1/2*I*d*x) + I*exp_integral
_e(3, -1/2*I*d*x))*sin(1/2*c)^3 + (exp_integral_e(3, 1/2*I*d*x) + exp_inte
gral_e(3, -1/2*I*d*x))*cos(1/2*c) + ((-I*exp_integral_e(3, 1/2*I*d*x) + I*
exp_integral_e(3, -1/2*I*d*x))*cos(1/2*c)^2 - I*exp_integral_e(3, 1/2*I*d*
x) + I*exp_integral_e(3, -1/2*I*d*x))*sin(1/2*c))*sqrt(a)*d^2/((sqrt(2)*co
s(1/2*c)^2 + sqrt(2)*sin(1/2*c)^2)*(d*x + c)^2 - 2*(sqrt(2)*cos(1/2*c)^2 +
 sqrt(2)*sin(1/2*c)^2)*(d*x + c)*c + (sqrt(2)*cos(1/2*c)^2 + sqrt(2)*sin(1
/2*c)^2)*c^2)
```

3.149.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 662, normalized size of antiderivative = 4.38

$$\int \frac{\sqrt{a + a\cos(c + dx)}}{x^3}\,dx = \text{Too large to display}$$

input

```
integrate((a+a*cos(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1
/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + d^2*x^2*real_part(cos_integral(-1/2*d
*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 2*d^2*x^2*ima
g_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan
(1/4*c) - 2*d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/
2*c))*tan(1/4*d*x)^2*tan(1/4*c) + 4*d^2*x^2*sgn(cos(1/2*d*x + 1/2*c))*sin_
integral(1/2*d*x)*tan(1/4*d*x)^2*tan(1/4*c) - d^2*x^2*real_part(cos_integr
al(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 - d^2*x^2*real_part(
cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2 + d^2*x^2
*real_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c)^2 +
 d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d*x + 1/2*c))*tan(1
/4*c)^2 + 2*d^2*x^2*imag_part(cos_integral(1/2*d*x))*sgn(cos(1/2*d*x + 1/2
*c))*tan(1/4*c) - 2*d^2*x^2*imag_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*
d*x + 1/2*c))*tan(1/4*c) + 4*d^2*x^2*sgn(cos(1/2*d*x + 1/2*c))*sin_integra
l(1/2*d*x)*tan(1/4*c) - d^2*x^2*real_part(cos_integral(1/2*d*x))*sgn(cos(1
/2*d*x + 1/2*c)) - d^2*x^2*real_part(cos_integral(-1/2*d*x))*sgn(cos(1/2*d
*x + 1/2*c)) - 8*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c) -
 8*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*d*x)*tan(1/4*c)^2 - 8*sgn(cos(1/2
*d*x + 1/2*c))*tan(1/4*d*x)^2*tan(1/4*c)^2 + 8*d*x*sgn(cos(1/2*d*x + 1/2*c
))*tan(1/4*d*x) + 8*d*x*sgn(cos(1/2*d*x + 1/2*c))*tan(1/4*c) + 8*sgn(co...
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(c+dx)}}{x^3}\,dx = \int \frac{\sqrt{a+a\,\cos\left(c+d\,x\right)}}{x^3}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/2)/x^3,x)
```

output
```
int((a + a*cos(c + d*x))^(1/2)/x^3, x)
```

3.150 $\int x^3 \sqrt{a + a\cos(x)}\, dx$

3.150.1 Optimal result

Integrand size = 14, antiderivative size = 68

$$\int x^3 \sqrt{a + a\cos(x)}\, dx = -96\sqrt{a + a\cos(x)} + 12x^2\sqrt{a + a\cos(x)} - 48x\sqrt{a + a\cos(x)}\tan\left(\frac{x}{2}\right) + 2x^3\sqrt{a + a\cos(x)}\tan\left(\frac{x}{2}\right)$$

output
```
-96*(a+a*cos(x))^(1/2)+12*x^2*(a+a*cos(x))^(1/2)-48*x*(a+a*cos(x))^(1/2)*t
an(1/2*x)+2*x^3*(a+a*cos(x))^(1/2)*tan(1/2*x)
```

3.150.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int x^3 \sqrt{a + a\cos(x)}\, dx = 2\sqrt{a(1 + \cos(x))}\left(6(-8 + x^2) + x(-24 + x^2)\tan\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x^3*Sqrt[a + a*Cos[x]],x]
```

output
```
2*Sqrt[a*(1 + Cos[x])]*(6*(-8 + x^2) + x*(-24 + x^2)*Tan[x/2])
```

3.150.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 3800, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a\cos(x)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int x^3\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \ 3800$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x^3\cos\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \ 3042$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x^3\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \ 3777$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(6\int -x^2\sin\left(\frac{x}{2}\right)dx+2x^3\sin\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \ 25$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^3\sin\left(\frac{x}{2}\right)-6\int x^2\sin\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \ 3042$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^3\sin\left(\frac{x}{2}\right)-6\int x^2\sin\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \ 3777$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\int x\cos\left(\frac{x}{2}\right)dx-2x^2\cos\left(\frac{x}{2}\right)\right)\right)$$

$$\Big\downarrow \ 3042$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2x^2\cos\left(\frac{x}{2}\right)\right)\right)$$

$$\Big\downarrow \ 3777$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\left(2\int-\sin\left(\frac{x}{2}\right)dx+2x\sin\left(\frac{x}{2}\right)\right)-2x^2\cos\left(\frac{x}{2}\right)\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)-2x^2\cos\left(\frac{x}{2}\right)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)-2x^2\cos\left(\frac{x}{2}\right)\right)\right)$$

$$\Big\downarrow \text{3118}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\left(2x\sin\left(\frac{x}{2}\right)+4\cos\left(\frac{x}{2}\right)\right)-2x^2\cos\left(\frac{x}{2}\right)\right)\right)$$

input
```
Int[x^3*Sqrt[a + a*Cos[x]],x]
```

output
```
Sqrt[a + a*Cos[x]]*Sec[x/2]*(2*x^3*Sin[x/2] - 6*(-2*x^2*Cos[x/2] + 4*(4*Co
s[x/2] + 2*x*Sin[x/2])))
```

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.150.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{ix}+1\right)^{2} e^{-ix}}\,\left(6ix^{2} e^{ix}+x^{3} e^{ix}+6ix^{2}-x^{3}-48i e^{ix}-24x\,e^{ix}-48i+24x\right)}{e^{ix}+1}$	87

input
```
int(x^3*(a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(6*I*x^2*exp(I*
x)+x^3*exp(I*x)+6*I*x^2-x^3-48*I*exp(I*x)-24*x*exp(I*x)-48*I+24*x)
```

3.150.5 Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.150.6 Sympy [F]

$$\int x^3\sqrt{a+a\cos(x)}\,dx=\int x^3\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

input
```
integrate(x**3*(a+a*cos(x))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*(cos(x) + 1)), x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{a+a\cos(x)}\,dx$$
$$=2\left(\sqrt{2}x^3\sin\left(\frac{1}{2}\,x\right)+6\,\sqrt{2}x^2\cos\left(\frac{1}{2}\,x\right)-24\,\sqrt{2}x\sin\left(\frac{1}{2}\,x\right)-48\,\sqrt{2}\cos\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

input
```
integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*x^3*sin(1/2*x) + 6*sqrt(2)*x^2*cos(1/2*x) - 24*sqrt(2)*x*sin(1/
2*x) - 48*sqrt(2)*cos(1/2*x))*sqrt(a)
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int x^3\sqrt{a+a\cos(x)}\,dx$$
$$=2\,\sqrt{2}\left(6\left(x^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-8\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\cos\left(\frac{1}{2}\,x\right)+\left(x^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-24\,x\mathrm{sgn}\left(\cos\right.\right.\right.$$

input
```
integrate(x^3*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*(6*(x^2*sgn(cos(1/2*x)) - 8*sgn(cos(1/2*x)))*cos(1/2*x) + (x^3*s
gn(cos(1/2*x)) - 24*x*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

3.150.9 Mupad [B] (verification not implemented)

Time = 14.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.34

$$\int x^3\sqrt{a+a\cos(x)}\,dx$$
$$= \frac{2\sqrt{a}\,\sqrt{\cos(x)+1}\,(24\,x-\cos(x)\ 48\mathrm{i}+48\ \sin(x)+x^2\ \cos(x)\ 6\mathrm{i}+x^3\ \cos(x)-6\,x^2\ \sin(x)+x^3\ \sin(x)}{\cos(x)\ 1\mathrm{i}-\sin(x)+1\mathrm{i}}$$

input
```
int(x^3*(a + a*cos(x))^(1/2),x)
```

output
```
(2*a^(1/2)*(cos(x) + 1)^(1/2)*(24*x - cos(x)*48i + 48*sin(x) + x^2*cos(x)*
6i + x^3*cos(x) - 6*x^2*sin(x) + x^3*sin(x)*1i - 24*x*cos(x) - x*sin(x)*24
i + x^2*6i - x^3 - 48i))/(cos(x)*1i - sin(x) + 1i)
```

3.151 $\int x^2\sqrt{a+a\cos(x)}\,dx$

3.151.1 Optimal result

Integrand size = 14, antiderivative size = 53

$$\int x^2\sqrt{a+a\cos(x)}\,dx=8x\sqrt{a+a\cos(x)}-16\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+2x^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

output
```
8*x*(a+a*cos(x))^(1/2)-16*(a+a*cos(x))^(1/2)*tan(1/2*x)+2*x^2*(a+a*cos(x))
^(1/2)*tan(1/2*x)
```

3.151.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int x^2\sqrt{a+a\cos(x)}\,dx=8\sqrt{a(1+\cos(x))}\left(x+\frac{1}{4}(-8+x^2)\tan\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x^2*Sqrt[a + a*Cos[x]],x]
```

output
```
8*Sqrt[a*(1 + Cos[x])]*(x + ((-8 + x^2)*Tan[x/2])/4)
```

3.151.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 3800, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a\cos(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x^2\cos\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x^2\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(4\int -x\sin\left(\frac{x}{2}\right)dx+2x^2\sin\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^2\sin\left(\frac{x}{2}\right)-4\int x\sin\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^2\sin\left(\frac{x}{2}\right)-4\int x\sin\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(2\int\cos\left(\frac{x}{2}\right)dx-2x\cos\left(\frac{x}{2}\right)\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(2\int\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2x\cos\left(\frac{x}{2}\right)\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(4\sin\left(\frac{x}{2}\right)-2x\cos\left(\frac{x}{2}\right)\right)\right)$$

input
```
Int[x^2*Sqrt[a + a*Cos[x]],x]
```

output
```
Sqrt[a + a*Cos[x]]*Sec[x/2]*(2*x^2*Sin[x/2] - 4*(-2*x*Cos[x/2] + 4*Sin[x/2
]))
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.151.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{ix}+1\right)^{2} e^{-ix}}\,\left(4ix\,e^{ix}+x^{2} e^{ix}+4ix-x^{2}-8\,e^{ix}+8\right)}{e^{ix}+1}$	70

input
```
int(x^2*(a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(4*I*x*exp(I*x)
+x^2*exp(I*x)+4*I*x-x^2-8*exp(I*x)+8)
```

3.151.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.151.6 Sympy [F]

$$\int x^2\sqrt{a+a\cos(x)}\,dx = \int x^2\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

input
```
integrate(x**2*(a+a*cos(x))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*(cos(x) + 1)), x)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{a+a\cos(x)}\,dx = 2\left(\sqrt{2}x^2\sin\left(\frac{1}{2}\,x\right)+4\,\sqrt{2}x\cos\left(\frac{1}{2}\,x\right)-8\,\sqrt{2}\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

input
```
integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*x^2*sin(1/2*x) + 4*sqrt(2)*x*cos(1/2*x) - 8*sqrt(2)*sin(1/2*x))
*sqrt(a)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int x^2\sqrt{a+a\cos(x)}\,dx$$
$$=2\,\sqrt{2}\left(4\,x\cos\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+\left(x^2\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-8\operatorname{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

input
```
integrate(x^2*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*(4*x*cos(1/2*x)*sgn(cos(1/2*x)) + (x^2*sgn(cos(1/2*x)) - 8*sgn(c
os(1/2*x)))*sin(1/2*x))*sqrt(a)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 13.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int x^2\sqrt{a+a\cos(x)}\,dx$$
$$=\frac{2\,\sqrt{a}\,\sqrt{\cos\left(x\right)+1}\,\left(x^2\,\cos\left(x\right)-8\,\cos\left(x\right)-4\,x\,\sin\left(x\right)-x^2+8+x\,4\mathrm{i}-\sin\left(x\right)\,8\mathrm{i}+x^2\,\sin\left(x\right)\,1\mathrm{i}+x\,\cos\right.}{\cos\left(x\right)\,1\mathrm{i}-\sin\left(x\right)+1\mathrm{i}}$$

input
```
int(x^2*(a + a*cos(x))^(1/2),x)
```

output
```
(2*a^(1/2)*(cos(x) + 1)^(1/2)*(x*4i - 8*cos(x) - sin(x)*8i + x^2*cos(x) +
x^2*sin(x)*1i + x*cos(x)*4i - 4*x*sin(x) - x^2 + 8))/(cos(x)*1i - sin(x) +
 1i)
```

3.152 $\int x\sqrt{a+a\cos(x)}\,dx$

3.152.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int x\sqrt{a+a\cos(x)}\,dx=4\sqrt{a+a\cos(x)}+2x\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

output
```
4*(a+a*cos(x))^(1/2)+2*x*(a+a*cos(x))^(1/2)*tan(1/2*x)
```

3.152.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int x\sqrt{a+a\cos(x)}\,dx=2\sqrt{a(1+\cos(x))}\left(2+x\tan\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x*Sqrt[a + a*Cos[x]],x]
```

output
```
2*Sqrt[a*(1 + Cos[x])]*(2 + x*Tan[x/2])
```

3.152.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3800, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a\cos(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x\cos\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2\int -\sin\left(\frac{x}{2}\right)dx+2x\sin\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x\sin\left(\frac{x}{2}\right)-2\int \sin\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x\sin\left(\frac{x}{2}\right)-2\int \sin\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{3118}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(2x\sin\left(\frac{x}{2}\right)+4\cos\left(\frac{x}{2}\right)\right)$$

input
```
Int[x*Sqrt[a + a*Cos[x]],x]
```

output
```
Sqrt[a + a*Cos[x]]*Sec[x/2]*(4*Cos[x/2] + 2*x*Sin[x/2])
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.152.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(e^{i x}+1\right)^{2} e^{-i x}}\,\left(2 i e^{i x}+x\,e^{i x}+2 i-x\right)}{e^{i x}+1}$	55

input
```
int(x*(a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(a*(exp(I*x)+1)^2*exp(-I*x))^(1/2)/(exp(I*x)+1)*(2*I*exp(I*x)+x
*exp(I*x)+2*I-x)
```

3.152.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.152.6 Sympy [F]

$$\int x\sqrt{a+a\cos(x)}\,dx = \int x\sqrt{a\left(\cos\left(x\right)+1\right)}\,dx$$

input
```
integrate(x*(a+a*cos(x))**(1/2),x)
```

output
```
Integral(x*sqrt(a*(cos(x) + 1)), x)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int x\sqrt{a+a\cos(x)}\,dx = 2\left(\sqrt{2}x\sin\left(\frac{1}{2}\,x\right)+2\,\sqrt{2}\cos\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

input
```
integrate(x*(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*x*sin(1/2*x) + 2*sqrt(2)*cos(1/2*x))*sqrt(a)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int x\sqrt{a+a\cos(x)}\,dx = 2\sqrt{2}\left(x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)+2\,\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}$$

input
```
integrate(x*(a+a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*(x*sgn(cos(1/2*x))*sin(1/2*x) + 2*cos(1/2*x)*sgn(cos(1/2*x)))*sq
rt(a)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 14.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

$$\int x\sqrt{a+a\cos(x)}\,dx = \frac{2\sqrt{a}\,\sqrt{\cos(x)+1}\,(x\,\cos(x)+\cos(x)\,2\mathrm{i}-2\,\sin(x)-x+x\,\sin(x)\,1\mathrm{i}+2\mathrm{i})}{\cos(x)\,1\mathrm{i}-\sin(x)+1\mathrm{i}}$$

input
```
int(x*(a + a*cos(x))^(1/2),x)
```

output
```
(2*a^(1/2)*(cos(x) + 1)^(1/2)*(cos(x)*2i - x - 2*sin(x) + x*cos(x) + x*sin
(x)*1i + 2i))/(cos(x)*1i - sin(x) + 1i)
```

3.153 $\int \sqrt{a+a\cos(x)}\,dx$

3.153.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \sqrt{a+a\cos(x)}\,dx = \frac{2a\sin(x)}{\sqrt{a+a\cos(x)}}$$

output
```
2*a*sin(x)/(a+a*cos(x))^(1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \sqrt{a+a\cos(x)}\,dx = 2\sqrt{a(1+\cos(x))}\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a + a*Cos[x]],x]
```

output
```
2*Sqrt[a*(1 + Cos[x])]*Tan[x/2]
```

3.153.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3125}$$

$$\frac{2a\sin(x)}{\sqrt{a\cos(x)+a}}$$

input
```
Int[Sqrt[a + a*Cos[x]],x]
```

output
```
(2*a*Sin[x])/Sqrt[a + a*Cos[x]]
```

3.153.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.153.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{2a\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{a\left(\cos^2\left(\frac{x}{2}\right)\right)}}$	25
risch	$-\frac{i\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{ix}+1\right)^2\mathrm{e}^{-ix}}\,\left(\mathrm{e}^{ix}-1\right)}{\mathrm{e}^{ix}+1}$	41

input
```
int((a+cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(a*cos(1/2*x)^2)^(1/2)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \sqrt{a+a\cos(x)}\,dx = \frac{2\sqrt{a\cos(x)+a}\sin(x)}{\cos(x)+1}$$

input
```
integrate((a+a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a*cos(x) + a)*sin(x)/(cos(x) + 1)
```

3.153.6 Sympy [F]

$$\int \sqrt{a+a\cos(x)}\,dx = \int \sqrt{a\cos(x)+a}\,dx$$

input
```
integrate((a+a*cos(x))**(1/2),x)
```

output
```
Integral(sqrt(a*cos(x) + a), x)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \sqrt{a + a\cos(x)}\, dx = 2\sqrt{2}\sqrt{a}\sin\left(\frac{1}{2}x\right)$$

input
```
integrate((a+a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*sqrt(a)*sin(1/2*x)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \sqrt{a + a\cos(x)}\, dx = 2\sqrt{2}\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)$$

input
```
integrate((a+a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*sqrt(a)*sgn(cos(1/2*x))*sin(1/2*x)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.27

$$\int \sqrt{a + a\cos(x)}\, dx = \frac{2\sqrt{a}\sqrt{\cos(x) + 1}\,(\cos(x) - 1 + \sin(x)\ 1\mathrm{i})}{\cos(x)\ 1\mathrm{i} - \sin(x) + 1\mathrm{i}}$$

input
```
int((a + a*cos(x))^(1/2),x)
```

output
```
(2*a^(1/2)*(cos(x) + 1)^(1/2)*(cos(x) + sin(x)*1i - 1))/(cos(x)*1i - sin(x
) + 1i)
```

3.154 $\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx$

3.154.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)$$

output
```
Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \sqrt{a(1+\cos(x))}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a + a*Cos[x]]/x,x]
```

output
```
Sqrt[a*(1 + Cos[x])]*CosIntegral[x/2]*Sec[x/2]
```

3.154.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3800, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(x)+a}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\cos\left(\frac{x}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3783}$$

$$\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

input
```
Int[Sqrt[a + a*Cos[x]]/x,x]
```

output
```
Sqrt[a + a*Cos[x]]*CosIntegral[x/2]*Sec[x/2]
```

3.154.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.154.4 Maple [F]

$$\int \frac{\sqrt{a+\cos(x)\,a}}{x}dx$$

input
```
int((a+cos(x)*a)^(1/2)/x,x)
```

output
```
int((a+cos(x)*a)^(1/2)/x,x)
```

3.154.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(x))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.154.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \int \frac{\sqrt{a(\cos(x)+1)}}{x}\,dx$$

input
```
integrate((a+a*cos(x))**(1/2)/x,x)
```

output
```
Integral(sqrt(a*(cos(x) + 1))/x, x)
```

3.154.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \frac{1}{2}\sqrt{2}\sqrt{a}\left(\mathrm{Ei}\left(\frac{1}{2}i\,x\right)+\mathrm{Ei}\left(-\frac{1}{2}i\,x\right)\right)$$

input
```
integrate((a+a*cos(x))^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*sqrt(a)*(Ei(1/2*I*x) + Ei(-1/2*I*x))
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \sqrt{2}\sqrt{a}\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)$$

input
```
integrate((a+a*cos(x))^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(2)*sqrt(a)*cos_integral(1/2*x)*sgn(cos(1/2*x))
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(x)}}{x}\,dx = \int \frac{\sqrt{a+a\,\cos(x)}}{x}\,dx$$

input
```
int((a + a*cos(x))^(1/2)/x,x)
```

output
```
int((a + a*cos(x))^(1/2)/x, x)
```

3.155 $\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx$

3.155.1 Optimal result

Integrand size = 14, antiderivative size = 42

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx=-\frac{\sqrt{a+a\cos(x)}}{x}-\frac{1}{2}\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)$$

output
```
-(a+a*cos(x))^(1/2)/x-1/2*sec(1/2*x)*Si(1/2*x)*(a+a*cos(x))^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx=-\frac{\sqrt{a(1+\cos(x))}\left(2+x\sec\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)\right)}{2x}$$

input
```
Integrate[Sqrt[a + a*Cos[x]]/x^2,x]
```

output
```
-1/2*(Sqrt[a*(1 + Cos[x])]*(2 + x*Sec[x/2]*SinIntegral[x/2]))/x
```

3.155.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3800, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(x)+a}}{x^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}}{x^2}dx$$

$$\Big\downarrow \ 3800$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\cos\left(\frac{x}{2}\right)}{x^2}dx$$

$$\Big\downarrow \ 3042$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \ 3778$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{1}{2}\int -\frac{\sin\left(\frac{x}{2}\right)}{x}dx-\frac{\cos\left(\frac{x}{2}\right)}{x}\right)$$

$$\Big\downarrow \ 25$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{1}{2}\int \frac{\sin\left(\frac{x}{2}\right)}{x}dx-\frac{\cos\left(\frac{x}{2}\right)}{x}\right)$$

$$\Big\downarrow \ 3042$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{1}{2}\int \frac{\sin\left(\frac{x}{2}\right)}{x}dx-\frac{\cos\left(\frac{x}{2}\right)}{x}\right)$$

$$\Big\downarrow \ 3780$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{\text{Si}\left(\frac{x}{2}\right)}{2}-\frac{\cos\left(\frac{x}{2}\right)}{x}\right)$$

input
```
Int[Sqrt[a + a*Cos[x]]/x^2,x]
```

output
```
Sqrt[a + a*Cos[x]]*Sec[x/2]*(-(Cos[x/2]/x) - SinIntegral[x/2]/2)
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.155.4 Maple [F]

$$\int \frac{\sqrt{a + \cos(x) a}}{x^2} dx$$

input
```
int((a+cos(x)*a)^(1/2)/x^2,x)
```

output
```
int((a+cos(x)*a)^(1/2)/x^2,x)
```

3.155.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.155.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{a(\cos(x)+1)}}{x^2}\,dx$$

input
```
integrate((a+a*cos(x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*(cos(x) + 1))/x**2, x)
```

3.155.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = -\frac{1}{4}\sqrt{2}\sqrt{a}\left(i\,\Gamma\left(-1,\frac{1}{2}i\,x\right)-i\,\Gamma\left(-1,-\frac{1}{2}i\,x\right)\right)$$

input
```
integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-1/4*sqrt(2)*sqrt(a)*(I*gamma(-1, 1/2*I*x) - I*gamma(-1, -1/2*I*x))
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = -\frac{\sqrt{2}\left(x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{1}{2}\,x\right)+2\,\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}}{2\,x}$$

input
```
integrate((a+a*cos(x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(x*sgn(cos(1/2*x))*sin_integral(1/2*x) + 2*cos(1/2*x)*sgn(cos
(1/2*x)))*sqrt(a)/x
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{a+a\,\cos(x)}}{x^2}\,dx$$

input
```
int((a + a*cos(x))^(1/2)/x^2,x)
```

output
```
int((a + a*cos(x))^(1/2)/x^2, x)
```

3.156 $\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx$

3.156.1 Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx=-\frac{\sqrt{a+a\cos(x)}}{2x^2}-\frac{1}{8}\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)+\frac{\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)}{4x}$$

output
```
-1/2*(a+a*cos(x))^(1/2)/x^2-1/8*Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+1/
4*(a+a*cos(x))^(1/2)*tan(1/2*x)/x
```

3.156.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx=-\frac{\sqrt{a(1+\cos(x))}\left(4+x^2\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-2x\tan\left(\frac{x}{2}\right)\right)}{8x^2}$$

input
```
Integrate[Sqrt[a + a*Cos[x]]/x^3,x]
```

output
```
-1/8*(Sqrt[a*(1 + Cos[x])]*(4 + x^2*CosIntegral[x/2]*Sec[x/2] - 2*x*Tan[x/
2]))/x^2
```

3.156.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 3800, 3042, 3778, 25, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cos(x)+a}}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}}{x^3}dx$$

$$\Big\downarrow \text{3800}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\cos\left(\frac{x}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{1}{4}\int -\frac{\sin\left(\frac{x}{2}\right)}{x^2}dx-\frac{\cos\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{1}{4}\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}dx-\frac{\cos\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{1}{4}\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}dx-\frac{\cos\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{1}{4}\left(\frac{\sin\left(\frac{x}{2}\right)}{x}-\frac{1}{2}\int \frac{\cos\left(\frac{x}{2}\right)}{x}dx\right)-\frac{\cos\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{1}{4}\left(\frac{\sin\left(\frac{x}{2}\right)}{x}-\frac{1}{2}\int\frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)}{x}dx\right)-\frac{\cos\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{ 3783}$$

$$\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{1}{4}\left(\frac{\sin\left(\frac{x}{2}\right)}{x}-\frac{\text{CosIntegral}\left(\frac{x}{2}\right)}{2}\right)-\frac{\cos\left(\frac{x}{2}\right)}{2x^2}\right)$$

input
```
Int[Sqrt[a + a*Cos[x]]/x^3,x]
```

output
```
Sqrt[a + a*Cos[x]]*Sec[x/2]*(-1/2*Cos[x/2]/x^2 + (-1/2*CosIntegral[x/2] +
Sin[x/2]/x)/4)
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.156.4 Maple [F]

$$\int \frac{\sqrt{a+\cos\left(x\right) a}}{x^{3}}dx$$

input
```
int((a+cos(x)*a)^(1/2)/x^3,x)
```

output
```
int((a+cos(x)*a)^(1/2)/x^3,x)
```

3.156.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.156.6 Sympy [F]

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{a\left(\cos\left(x\right)+1\right)}}{x^{3}}\,dx$$

input
```
integrate((a+a*cos(x))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*(cos(x) + 1))/x**3, x)
```

3.156.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \frac{1}{8}\sqrt{2}\sqrt{a}\left(\Gamma\left(-2,\frac{1}{2}i\,x\right)+\Gamma\left(-2,-\frac{1}{2}i\,x\right)\right)$$

input
```
integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*sqrt(a)*(gamma(-2, 1/2*I*x) + gamma(-2, -1/2*I*x))
```

3.156.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{2}\left(x^2\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-2\,x\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)+4\,\cos\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}}{8\,x^2}$$

input
```
integrate((a+a*cos(x))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(x^2*cos_integral(1/2*x)*sgn(cos(1/2*x)) - 2*x*sgn(cos(1/2*x)
)*sin(1/2*x) + 4*cos(1/2*x)*sgn(cos(1/2*x)))*sqrt(a)/x^2
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{a+a\,\cos(x)}}{x^3}\,dx$$

input
```
int((a + a*cos(x))^(1/2)/x^3,x)
```

output
```
int((a + a*cos(x))^(1/2)/x^3, x)
```

3.157 $\int x^3 \sqrt{a-a\cos(x)}\,dx$

3.157.1 Optimal result

Integrand size = 15, antiderivative size = 72

$$\int x^3 \sqrt{a-a\cos(x)}\,dx = -96\sqrt{a-a\cos(x)}+12x^2\sqrt{a-a\cos(x)} + 48x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)-2x^3\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

output
```
-96*(a-a*cos(x))^(1/2)+12*x^2*(a-a*cos(x))^(1/2)+48*x*cot(1/2*x)*(a-a*cos(
x))^(1/2)-2*x^3*cot(1/2*x)*(a-a*cos(x))^(1/2)
```

3.157.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.47

$$\int x^3 \sqrt{a-a\cos(x)}\,dx = -2\sqrt{a-a\cos(x)}\left(-6\left(-8+x^2\right)+x\left(-24+x^2\right)\cot\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x^3*Sqrt[a - a*Cos[x]],x]
```

output
```
-2*Sqrt[a - a*Cos[x]]*(-6*(-8 + x^2) + x*(-24 + x^2)*Cot[x/2])
```

3.157.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3042, 3800, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a-a\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int x^3\sin\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int x^3\sin\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(6\int x^2\cos\left(\frac{x}{2}\right)dx-2x^3\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(6\int x^2\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2x^3\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(6\left(4\int -x\sin\left(\frac{x}{2}\right)dx+2x^2\sin\left(\frac{x}{2}\right)\right)-2x^3\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(6\left(2x^2\sin\left(\frac{x}{2}\right)-4\int x\sin\left(\frac{x}{2}\right)dx\right)-2x^3\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(6\left(2x^2\sin\left(\frac{x}{2}\right)-4\int x\sin\left(\frac{x}{2}\right)dx\right)-2x^3\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(6\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(2\int\cos\left(\frac{x}{2}\right)dx-2x\cos\left(\frac{x}{2}\right)\right)\right)-2x^3\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(6\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(2\int\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2x\cos\left(\frac{x}{2}\right)\right)\right)-2x^3\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(6\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(4\sin\left(\frac{x}{2}\right)-2x\cos\left(\frac{x}{2}\right)\right)\right)-2x^3\cos\left(\frac{x}{2}\right)\right)$$

input
```
Int[x^3*Sqrt[a - a*Cos[x]],x]
```

output
```
Sqrt[a - a*Cos[x]]*Csc[x/2]*(-2*x^3*Cos[x/2] + 6*(2*x^2*Sin[x/2] - 4*(-2*x
*Cos[x/2] + 4*Sin[x/2])))
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.157.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a\left(e^{ix}-1\right)^{2} e^{-ix}}\,\left(6 i x^{2} e^{ix}+x^{3} e^{ix}-6 i x^{2}+x^{3}-48 i e^{ix}-24 x\,e^{ix}+48 i-24 x\right)}{e^{ix}-1}$	86

input
```
int(x^3*(a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(6*I*x^2*exp(I
*x)+x^3*exp(I*x)-6*I*x^2+x^3-48*I*exp(I*x)-24*x*exp(I*x)+48*I-24*x)
```

3.157.5 Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a-a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.157.6 Sympy [F]

$$\int x^3\sqrt{a-a\cos(x)}\,dx = \int x^{3}\sqrt{-a\left(\cos\left(x\right)-1\right)}\,dx$$

input
```
integrate(x**3*(a-a*cos(x))**(1/2),x)
```

output
```
Integral(x**3*sqrt(-a*(cos(x) - 1)), x)
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(60) = 120.

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.79

$$\int x^3\sqrt{a-a\cos(x)}\,dx =$$
$$-\left(\left(6\sqrt{2}x^2-6\left(\sqrt{2}x^2-8\sqrt{2}\right)\cos(x)-\left(\sqrt{2}x^3-24\sqrt{2}x\right)\sin(x)-48\sqrt{2}\right)\cos\left(\frac{1}{2}\pi+\frac{1}{2}\arctan(\sin\right.\right.$$

input
```
integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
-((6*sqrt(2)*x^2 - 6*(sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - (sqrt(2)*x^3 - 24*
sqrt(2)*x)*sin(x) - 48*sqrt(2))*cos(1/2*pi + 1/2*arctan2(sin(x), cos(x)))
+ (sqrt(2)*x^3 + (sqrt(2)*x^3 - 24*sqrt(2)*x)*cos(x) - 6*(sqrt(2)*x^2 - 8*
sqrt(2))*sin(x) - 24*sqrt(2)*x)*sin(1/2*pi + 1/2*arctan2(sin(x), cos(x))))
*sqrt(a)
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int x^3\sqrt{a-a\cos(x)}\,dx =$$
$$-2\sqrt{2}\left(\left(x^3\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-24\,x\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\right)\cos\left(\frac{1}{2}x\right)-6\left(x^2\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-8\,\mathrm{sgn}\left(\sin\right.\right.\right.$$

input
```
integrate(x^3*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*((x^3*sgn(sin(1/2*x)) - 24*x*sgn(sin(1/2*x)))*cos(1/2*x) - 6*(x
^2*sgn(sin(1/2*x)) - 8*sgn(sin(1/2*x)))*sin(1/2*x))*sqrt(a)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 14.84 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int x^3\sqrt{a-a\cos(x)}\,dx$$
$$=\frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,(24\,x+\cos(x)\ 48\mathrm{i}-48\,\sin(x)-x^2\,\cos(x)\ 6\mathrm{i}-x^3\,\cos(x)+6\,x^2\,\sin(x)-x^3\,\sin(x)}{\sin(x)-\cos(x)\ 1\mathrm{i}+1\mathrm{i}}$$

input
```
int(x^3*(a - a*cos(x))^(1/2),x)
```

output
```
(2*a^(1/2)*(1 - cos(x))^(1/2)*(24*x + cos(x)*48i - 48*sin(x) - x^2*cos(x)*
6i - x^3*cos(x) + 6*x^2*sin(x) - x^3*sin(x)*1i + 24*x*cos(x) + x*sin(x)*24
i + x^2*6i - x^3 - 48i))/(sin(x) - cos(x)*1i + 1i)
```

3.158 $\int x^2 \sqrt{a-a\cos(x)}\,dx$

3.158.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int x^2 \sqrt{a-a\cos(x)}\,dx = 8x\sqrt{a-a\cos(x)}+16\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right) - 2x^2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

output
```
8*x*(a-a*cos(x))^(1/2)+16*cot(1/2*x)*(a-a*cos(x))^(1/2)-2*x^2*cot(1/2*x)*(
a-a*cos(x))^(1/2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.54

$$\int x^2 \sqrt{a-a\cos(x)}\,dx = 8\sqrt{a-a\cos(x)}\left(x-\frac{1}{4}\left(-8+x^2\right)\cot\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x^2*Sqrt[a - a*Cos[x]],x]
```

output
```
8*Sqrt[a - a*Cos[x]]*(x - ((-8 + x^2)*Cot[x/2])/4)
```

3.158.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3800, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-a\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int x^2\sin\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int x^2\sin\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(4\int x\cos\left(\frac{x}{2}\right)dx-2x^2\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(4\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2x^2\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(4\left(2\int -\sin\left(\frac{x}{2}\right)dx+2x\sin\left(\frac{x}{2}\right)\right)-2x^2\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(4\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)-2x^2\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(4\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)-2x^2\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3118}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(4\left(2x\sin\left(\frac{x}{2}\right)+4\cos\left(\frac{x}{2}\right)\right)-2x^2\cos\left(\frac{x}{2}\right)\right)$$

input
```
Int[x^2*Sqrt[a - a*Cos[x]],x]
```

output
```
Sqrt[a - a*Cos[x]]*Csc[x/2]*(-2*x^2*Cos[x/2] + 4*(4*Cos[x/2] + 2*x*Sin[x/2
]))
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.158.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a\left(\mathrm{e}^{ix}-1\right)^{2}\mathrm{e}^{-ix}}\,\left(4ix\,\mathrm{e}^{ix}+x^{2}\mathrm{e}^{ix}-4ix+x^{2}-8\,\mathrm{e}^{ix}-8\right)}{\mathrm{e}^{ix}-1}$	69

input
```
int(x^2*(a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(4*I*x*exp(I*x
)+x^2*exp(I*x)-4*I*x+x^2-8*exp(I*x)-8)
```

3.158.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a-a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.158.6 Sympy [F]

$$\int x^2\sqrt{a-a\cos(x)}\,dx = \int x^2\sqrt{-a\left(\cos\left(x\right)-1\right)}\,dx$$

input
```
integrate(x**2*(a-a*cos(x))**(1/2),x)
```

output
```
Integral(x**2*sqrt(-a*(cos(x) - 1)), x)
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(46) = 92$.

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.79

$$\int x^2\sqrt{a-a\cos(x)}\,dx$$
$$= \left(\left(4\sqrt{2}x\cos(x) + \left(\sqrt{2}x^2 - 8\sqrt{2}\right)\sin(x) - 4\sqrt{2}x\right)\cos\left(\frac{1}{2}\pi + \frac{1}{2}\arctan(\sin(x),\cos(x))\right) - \left(\sqrt{2}x^2 - \right.\right.$$

input
```
integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
((4*sqrt(2)*x*cos(x) + (sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 4*sqrt(2)*x)*cos
(1/2*pi + 1/2*arctan2(sin(x), cos(x))) - (sqrt(2)*x^2 - 4*sqrt(2)*x*sin(x)
 + (sqrt(2)*x^2 - 8*sqrt(2))*cos(x) - 8*sqrt(2))*sin(1/2*pi + 1/2*arctan2(
sin(x), cos(x))))*sqrt(a)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{a-a\cos(x)}\,dx$$
$$= 2\sqrt{2}\left(4\,x\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right) - \left(x^2\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right) - 8\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\right)\cos\left(\frac{1}{2}\,x\right) - 8\,\mathrm{sgn}\left(\right.\right.$$

input
```
integrate(x^2*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*(4*x*sgn(sin(1/2*x))*sin(1/2*x) - (x^2*sgn(sin(1/2*x)) - 8*sgn(s
in(1/2*x)))*cos(1/2*x) - 8*sgn(sin(1/2*x)))*sqrt(a)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27

$$\int x^2\sqrt{a-a\cos(x)}\,dx$$
$$=\frac{2\sqrt{a}\,\sqrt{1-\cos(x)}\,(8\cos(x)-x^2\cos(x)+4x\sin(x)-x^2+8+x\,4\mathrm{i}+\sin(x)\ 8\mathrm{i}-x^2\sin(x)\ 1\mathrm{i}-x\ \mathrm{co}}{\sin(x)-\cos(x)\ 1\mathrm{i}+1\mathrm{i}}$$

input
```
int(x^2*(a - a*cos(x))^(1/2),x)
```

output
```
(2*a^(1/2)*(1 - cos(x))^(1/2)*(x*4i + 8*cos(x) + sin(x)*8i - x^2*cos(x) -
x^2*sin(x)*1i - x*cos(x)*4i + 4*x*sin(x) - x^2 + 8))/(sin(x) - cos(x)*1i +
 1i)
```

3.159 $\int x\sqrt{a-a\cos(x)}\,dx$

3.159.1 Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x\sqrt{a-a\cos(x)}\,dx=4\sqrt{a-a\cos(x)}-2x\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

output
```
4*(a-a*cos(x))^(1/2)-2*x*cot(1/2*x)*(a-a*cos(x))^(1/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int x\sqrt{a-a\cos(x)}\,dx=-2\sqrt{a-a\cos(x)}\left(-2+x\cot\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x*Sqrt[a - a*Cos[x]],x]
```

output
```
-2*Sqrt[a - a*Cos[x]]*(-2 + x*Cot[x/2])
```

3.159.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3800, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a-a\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int x\sin\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int x\sin\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(2\int\cos\left(\frac{x}{2}\right)dx-2x\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(2\int\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2x\cos\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(4\sin\left(\frac{x}{2}\right)-2x\cos\left(\frac{x}{2}\right)\right)$$

input
```
Int[x*Sqrt[a - a*Cos[x]],x]
```

output
```
Sqrt[a - a*Cos[x]]*Csc[x/2]*(-2*x*Cos[x/2] + 4*Sin[x/2])
```

3.159.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.159.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

method	result	size
risch	$-\frac{i\sqrt{2}\,\sqrt{-a\left(e^{ix}-1\right)^{2} e^{-ix}}\,\left(2i e^{ix}+x\,e^{ix}-2i+x\right)}{e^{ix}-1}$	54

input
```
int(x*(a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(-a*(exp(I*x)-1)^2*exp(-I*x))^(1/2)/(exp(I*x)-1)*(2*I*exp(I*x)+
x*exp(I*x)-2*I+x)
```

3.159.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a-a\cos(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.159.6 Sympy [F]

$$\int x\sqrt{a-a\cos(x)}\,dx = \int x\sqrt{-a\left(\cos\left(x\right)-1\right)}\,dx$$

input
```
integrate(x*(a-a*cos(x))**(1/2),x)
```

output
```
Integral(x*sqrt(-a*(cos(x) - 1)), x)
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(28) = 56$.

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.12

$$\int x\sqrt{a-a\cos(x)}\,dx$$
$$= \left(\left(\sqrt{2}x\sin\left(x\right)+2\sqrt{2}\cos\left(x\right)-2\sqrt{2}\right)\cos\left(\frac{1}{2}\pi+\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right)\right)\right)-\left(\sqrt{2}x\cos\left(x\right)+\sqrt{2}x-\right.$$

input
```
integrate(x*(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
((sqrt(2)*x*sin(x) + 2*sqrt(2)*cos(x) - 2*sqrt(2))*cos(1/2*pi + 1/2*arctan
2(sin(x), cos(x))) - (sqrt(2)*x*cos(x) + sqrt(2)*x - 2*sqrt(2)*sin(x))*sin
(1/2*pi + 1/2*arctan2(sin(x), cos(x))))*sqrt(a)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int x\sqrt{a-a\cos(x)}\,dx$$
$$= -2\sqrt{2}\left(x\cos\left(\frac{1}{2}x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)-2\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}$$

input
```
integrate(x*(a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*(x*cos(1/2*x)*sgn(sin(1/2*x)) - 2*sgn(sin(1/2*x))*sin(1/2*x))*s
qrt(a)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 14.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int x\sqrt{a-a\cos(x)}\,dx$$
$$= -\frac{2\sqrt{a}\sqrt{1-\cos(x)}\,(x+\cos(x)\ 2\mathrm{i}-2\sin(x)+x\cos(x)+x\sin(x)\ 1\mathrm{i}-2\mathrm{i})}{\sin(x)-\cos(x)\ 1\mathrm{i}+1\mathrm{i}}$$

input
```
int(x*(a - a*cos(x))^(1/2),x)
```

output
```
-(2*a^(1/2)*(1 - cos(x))^(1/2)*(x + cos(x)*2i - 2*sin(x) + x*cos(x) + x*si
n(x)*1i - 2i))/(sin(x) - cos(x)*1i + 1i)
```

3.160 $\int \sqrt{a-a\cos(x)}\,dx$

3.160.1 Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \sqrt{a-a\cos(x)}\,dx=-\frac{2a\sin(x)}{\sqrt{a-a\cos(x)}}$$

output
```
-2*a*sin(x)/(a-a*cos(x))^(1/2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \sqrt{a-a\cos(x)}\,dx=-2\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a - a*Cos[x]],x]
```

output
```
-2*Sqrt[a - a*Cos[x]]*Cot[x/2]
```

3.160.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\cos(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3125}$$

$$-\frac{2a\sin(x)}{\sqrt{a-a\cos(x)}}$$

input
```
Int[Sqrt[a - a*Cos[x]],x]
```

output
```
(-2*a*Sin[x])/Sqrt[a - a*Cos[x]]
```

3.160.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

3.160.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

method	result	size
default	$-\frac{2\sin\left(\frac{x}{2}\right)a\cos\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{a\left(\sin^2\left(\frac{x}{2}\right)\right)}}$	25
risch	$-\frac{i\sqrt{2}\,\sqrt{-a(\mathrm{e}^{ix}-1)^2\mathrm{e}^{-ix}}\,(\mathrm{e}^{ix}+1)}{\mathrm{e}^{ix}-1}$	42

input
```
int((a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*sin(1/2*x)*a*cos(1/2*x)*2^(1/2)/(a*sin(1/2*x)^2)^(1/2)
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \sqrt{a-a\cos(x)}\,dx = -\frac{2\sqrt{-a\cos(x)+a}(\cos(x)+1)}{\sin(x)}$$

input
```
integrate((a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a*cos(x) + a)*(cos(x) + 1)/sin(x)
```

3.160.6 Sympy [F]

$$\int \sqrt{a-a\cos(x)}\,dx = \int \sqrt{-a\cos(x)+a}\,dx$$

input
```
integrate((a-a*cos(x))**(1/2),x)
```

output
```
Integral(sqrt(-a*cos(x) + a), x)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \sqrt{a - a\cos(x)}\, dx = -\frac{2\sqrt{2}\sqrt{a}}{\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1}}$$

input
```
integrate((a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(2)*sqrt(a)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \sqrt{a - a\cos(x)}\, dx = -2\sqrt{2}\left(\cos\left(\frac{1}{2}x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right) - \operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\right)\sqrt{a}$$

input
```
integrate((a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*(cos(1/2*x)*sgn(sin(1/2*x)) - sgn(sin(1/2*x)))*sqrt(a)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 14.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \sqrt{a - a\cos(x)}\, dx = -\frac{2\sqrt{a}\sqrt{1-\cos(x)}\,(\cos(x) + 1 + \sin(x)\ 1\mathrm{i})}{\sin(x) - \cos(x)\ 1\mathrm{i} + 1\mathrm{i}}$$

input
```
int((a - a*cos(x))^(1/2),x)
```

output
```
-(2*a^(1/2)*(1 - cos(x))^(1/2)*(cos(x) + sin(x)*1i + 1))/(sin(x) - cos(x)*
1i + 1i)
```

3.161 $\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx$

3.161.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)$$

output
```
csc(1/2*x)*Si(1/2*x)*(a-a*cos(x))^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a - a*Cos[x]]/x,x]
```

output
```
Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

3.161.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3800, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\cos(x)}}{x}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}}{x}dx$$
$$\Big\downarrow \text{3800}$$
$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int \frac{\sin\left(\frac{x}{2}\right)}{x}dx$$
$$\Big\downarrow \text{3042}$$
$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int \frac{\sin\left(\frac{x}{2}\right)}{x}dx$$
$$\Big\downarrow \text{3780}$$
$$\text{Si}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}$$

input
```
Int[Sqrt[a - a*Cos[x]]/x,x]
```

output
```
Sqrt[a - a*Cos[x]]*Csc[x/2]*SinIntegral[x/2]
```

3.161.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.161.4 Maple [F]

$$\int \frac{\sqrt{a-\cos \left(x \right) a}}{x}dx$$

input
```
int((a-cos(x)*a)^(1/2)/x,x)
```

output
```
int((a-cos(x)*a)^(1/2)/x,x)
```

3.161.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-a*cos(x))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.161.6 Sympy [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \int \frac{\sqrt{-a\left(\cos\left(x\right)-1\right)}}{x}\,dx$$

input
```
integrate((a-a*cos(x))**(1/2)/x,x)
```

output
```
Integral(sqrt(-a*(cos(x) - 1))/x, x)
```

3.161.7 Maxima [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \int \frac{\sqrt{-a\cos(x)+a}}{x}\,dx$$

input
```
integrate((a-a*cos(x))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*cos(x) + a)/x, x)
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \sqrt{2}\sqrt{a}\operatorname{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\operatorname{Si}\left(\frac{1}{2}x\right)$$

input
```
integrate((a-a*cos(x))^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(2)*sqrt(a)*sgn(sin(1/2*x))*sin_integral(1/2*x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\cos(x)}}{x}\,dx = \int \frac{\sqrt{a-a\cos(x)}}{x}\,dx$$

input
```
int((a - a*cos(x))^(1/2)/x,x)
```

output
```
int((a - a*cos(x))^(1/2)/x, x)
```

3.162 $\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx$

3.162.1 Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx=-\frac{\sqrt{a-a\cos(x)}}{x}+\frac{1}{2}\sqrt{a-a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

output
```
-(a-a*cos(x))^(1/2)/x+1/2*Ci(1/2*x)*csc(1/2*x)*(a-a*cos(x))^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx=\frac{\sqrt{a-a\cos(x)}\left(-2+x\,\text{CosIntegral}\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\right)}{2x}$$

input
```
Integrate[Sqrt[a - a*Cos[x]]/x^2,x]
```

output
```
(Sqrt[a - a*Cos[x]]*(-2 + x*CosIntegral[x/2]*Csc[x/2]))/(2*x)
```

3.162.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3800, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}}{x^2}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int \frac{\sin\left(\frac{x}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{1}{2}\int \frac{\cos\left(\frac{x}{2}\right)}{x}dx-\frac{\sin\left(\frac{x}{2}\right)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{1}{2}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)}{x}dx-\frac{\sin\left(\frac{x}{2}\right)}{x}\right)$$

$$\Big\downarrow \text{3783}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{\text{CosIntegral}\left(\frac{x}{2}\right)}{2}-\frac{\sin\left(\frac{x}{2}\right)}{x}\right)$$

input
```
Int[Sqrt[a - a*Cos[x]]/x^2,x]
```

output
```
Sqrt[a - a*Cos[x]]*Csc[x/2]*(CosIntegral[x/2]/2 - Sin[x/2]/x)
```

3.162.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.162.4 Maple [F]

$$\int \frac{\sqrt{a-\cos(x)\,a}}{x^2}dx$$

input
```
int((a-cos(x)*a)^(1/2)/x^2,x)
```

output
```
int((a-cos(x)*a)^(1/2)/x^2,x)
```

3.162.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.162.6 Sympy [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{-a\left(\cos\left(x\right)-1\right)}}{x^2}\,dx$$

input
```
integrate((a-a*cos(x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(-a*(cos(x) - 1))/x**2, x)
```

3.162.7 Maxima [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{-a\cos\left(x\right)+a}}{x^2}\,dx$$

input
```
integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*cos(x) + a)/x^2, x)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a - a\cos(x)}}{x^2}\,dx = \frac{\sqrt{2}\left(x\operatorname{Ci}\left(\frac{1}{2}\,x\right)\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right) - 2\operatorname{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}}{2\,x}$$

input
```
integrate((a-a*cos(x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(x*cos_integral(1/2*x)*sgn(sin(1/2*x)) - 2*sgn(sin(1/2*x))*sin
(1/2*x))*sqrt(a)/x
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a - a\cos(x)}}{x^2}\,dx = \int \frac{\sqrt{a - a\,\cos\left(x\right)}}{x^2}\,dx$$

input
```
int((a - a*cos(x))^(1/2)/x^2,x)
```

output
```
int((a - a*cos(x))^(1/2)/x^2, x)
```

3.163 $\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx$

3.163.1 Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{a-a\cos(x)}}{2x^2} - \frac{\sqrt{a-a\cos(x)}\cot\left(\frac{x}{2}\right)}{4x} - \frac{1}{8}\sqrt{a-a\cos(x)}\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)$$

output
```
-1/2*(a-a*cos(x))^(1/2)/x^2-1/4*cot(1/2*x)*(a-a*cos(x))^(1/2)/x-1/8*csc(1/
2*x)*Si(1/2*x)*(a-a*cos(x))^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{a-a\cos(x)}\left(4+2x\cot\left(\frac{x}{2}\right)+x^2\csc\left(\frac{x}{2}\right)\text{Si}\left(\frac{x}{2}\right)\right)}{8x^2}$$

input
```
Integrate[Sqrt[a - a*Cos[x]]/x^3,x]
```

output
```
-1/8*(Sqrt[a - a*Cos[x]]*(4 + 2*x*Cot[x/2] + x^2*Csc[x/2]*SinIntegral[x/2]
))/x^2
```

3.163.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3800, 3042, 3778, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}}{x^3}dx$$

$$\Big\downarrow \text{3800}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int \frac{\sin\left(\frac{x}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\int \frac{\sin\left(\frac{x}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{1}{4}\int \frac{\cos\left(\frac{x}{2}\right)}{x^2}dx-\frac{\sin\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{1}{4}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)}{x^2}dx-\frac{\sin\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{1}{4}\left(\frac{1}{2}\int -\frac{\sin\left(\frac{x}{2}\right)}{x}dx-\frac{\cos\left(\frac{x}{2}\right)}{x}\right)-\frac{\sin\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{1}{4}\left(-\frac{1}{2}\int \frac{\sin\left(\frac{x}{2}\right)}{x}dx-\frac{\cos\left(\frac{x}{2}\right)}{x}\right)-\frac{\sin\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{1}{4}\left(-\frac{1}{2}\int\frac{\sin\left(\frac{x}{2}\right)}{x}dx-\frac{\cos\left(\frac{x}{2}\right)}{x}\right)-\frac{\sin\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3780}$$

$$\csc\left(\frac{x}{2}\right)\sqrt{a-a\cos(x)}\left(\frac{1}{4}\left(-\frac{\text{Si}\left(\frac{x}{2}\right)}{2}-\frac{\cos\left(\frac{x}{2}\right)}{x}\right)-\frac{\sin\left(\frac{x}{2}\right)}{2x^2}\right)$$

input
```
Int[Sqrt[a - a*Cos[x]]/x^3,x]
```

output
```
Sqrt[a - a*Cos[x]]*Csc[x/2]*(-1/2*Sin[x/2]/x^2 + (-(Cos[x/2]/x) - SinInteg
ral[x/2]/2)/4)
```

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.163.4 Maple [F]

$$\int \frac{\sqrt{a-\cos\left(x\right)a}}{x^{3}}dx$$

input
```
int((a-cos(x)*a)^(1/2)/x^3,x)
```

output
```
int((a-cos(x)*a)^(1/2)/x^3,x)
```

3.163.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.163.6 Sympy [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{-a\left(\cos\left(x\right)-1\right)}}{x^{3}}\,dx$$

input
```
integrate((a-a*cos(x))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(-a*(cos(x) - 1))/x**3, x)
```

3.163.7 Maxima [F]

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{-a\cos(x)+a}}{x^3}\,dx$$

input
```
integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*cos(x) + a)/x^3, x)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = -\frac{\sqrt{2}\left(x^2\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\mathrm{Si}\left(\frac{1}{2}x\right)+2x\cos\left(\frac{1}{2}x\right)\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)+4\,\mathrm{sgn}\left(\sin\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)\right)\sqrt{a}}{8\,x^2}$$

input
```
integrate((a-a*cos(x))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*(x^2*sgn(sin(1/2*x))*sin_integral(1/2*x) + 2*x*cos(1/2*x)*sgn
(sin(1/2*x)) + 4*sgn(sin(1/2*x))*sin(1/2*x))*sqrt(a)/x^2
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx = \int \frac{\sqrt{a-a\cos(x)}}{x^3}\,dx$$

input
```
int((a - a*cos(x))^(1/2)/x^3,x)
```

output
```
int((a - a*cos(x))^(1/2)/x^3, x)
```

3.164 $\int x^3(a+a\cos(x))^{3/2}\,dx$

3.164.1 Optimal result

Integrand size = 14, antiderivative size = 185

$$\begin{aligned}\int x^3(a+a\cos(x))^{3/2}\,dx = & -\frac{1280}{9}a\sqrt{a+a\cos(x)}+16ax^2\sqrt{a+a\cos(x)} \\ & -\frac{64}{27}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{8}{3}ax^2\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)} \\ & -\frac{32}{9}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)+\frac{4}{3}ax^3\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right) \\ & -\frac{640}{9}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+\frac{8}{3}ax^3\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)\end{aligned}$$

output
```
-1280/9*a*(a+a*cos(x))^(1/2)+16*a*x^2*(a+a*cos(x))^(1/2)-64/27*a*cos(1/2*x
)^2*(a+a*cos(x))^(1/2)+8/3*a*x^2*cos(1/2*x)^2*(a+a*cos(x))^(1/2)-32/9*a*x*
cos(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)+4/3*a*x^3*cos(1/2*x)*sin(1/2*x)*(
a+a*cos(x))^(1/2)-640/9*a*x*(a+a*cos(x))^(1/2)*tan(1/2*x)+8/3*a*x^3*(a+a*c
os(x))^(1/2)*tan(1/2*x)
```

3.164.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.36

$$\begin{aligned}\int x^3(a+a\cos(x))^{3/2}\,dx = & \frac{2}{27}a\sqrt{a(1+\cos(x))}\Big(-1936+234x^2 \\ & +3x(-328+15x^2)\tan\left(\frac{x}{2}\right)+\cos(x)\left(2(-8+9x^2)+3x(-8+3x^2)\tan\left(\frac{x}{2}\right)\right)\Big)\end{aligned}$$

input
```
Integrate[x^3*(a + a*Cos[x])^(3/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cos[x])]*(-1936 + 234*x^2 + 3*x*(-328 + 15*x^2)*Tan[x/2]
+ Cos[x]*(2*(-8 + 9*x^2) + 3*x*(-8 + 3*x^2)*Tan[x/2])))/27
```

3.164.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.89, number of steps used = 20, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {3042, 3800, 3042, 3792, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a\cos(x)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3800}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x^3\cos^3\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x^3\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{ 3792}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\int x^3\cos\left(\frac{x}{2}\right)dx-\frac{8}{3}\int x\cos^3\left(\frac{x}{2}\right)dx+\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{4}{3}x^2\cos^3\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\int x^3\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-\frac{8}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{4}{3}x^2\cos^3\right.$$

$$\Big\downarrow \text{ 3777}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(6\int -x^2\sin\left(\frac{x}{2}\right)dx+2x^3\sin\left(\frac{x}{2}\right)\right)-\frac{8}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos^2\right.$$

$\downarrow$ 25

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-6\int x^2\sin\left(\frac{x}{2}\right)dx\right)-\frac{8}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos^2\left(\right.\right.$$

$\downarrow$ 3042

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-6\int x^2\sin\left(\frac{x}{2}\right)dx\right)-\frac{8}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos^2\left(\right.\right.$$

$\downarrow$ 3777

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\int x\cos\left(\frac{x}{2}\right)dx-2x^2\cos\left(\frac{x}{2}\right)\right)\right)-\frac{8}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx+\right.$$

$\downarrow$ 3042

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2x^2\cos\left(\frac{x}{2}\right)\right)\right)-\frac{8}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3d\right.$$

$\downarrow$ 3777

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\left(2\int -\sin\left(\frac{x}{2}\right)dx+2x\sin\left(\frac{x}{2}\right)\right)-2x^2\cos\left(\frac{x}{2}\right)\right)\right)-\frac{8}{3}\int x\,\mathrm{si}\right.$$

$\downarrow$ 25

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)-2x^2\cos\left(\frac{x}{2}\right)\right)\right)-\frac{8}{3}\int x\sin\right.$$

$\downarrow$ 3042

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)-2x^2\cos\left(\frac{x}{2}\right)\right)\right)-\frac{8}{3}\int x\sin\right.$$

$\downarrow$ 3118

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{8}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{4}{3}x^2\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-\right.\right.$$

$\downarrow$ 3791

3.164. $\int x^3(a+a\cos(x))^{3/2}\,dx$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{8}{3}\left(\frac{2}{3}\int x\cos\left(\frac{x}{2}\right)dx+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{8}{3}\left(\frac{2}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos\right.$$

$$\Big\downarrow \text{3777}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{8}{3}\left(\frac{2}{3}\left(2\int -\sin\left(\frac{x}{2}\right)dx+2x\sin\left(\frac{x}{2}\right)\right)+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)+\frac{2}{3}\right.$$

$$\Big\downarrow \text{25}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{8}{3}\left(\frac{2}{3}\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^3\right.$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{8}{3}\left(\frac{2}{3}\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^3\right.$$

$$\Big\downarrow \text{3118}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}x^3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{4}{3}x^2\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}\left(2x^3\sin\left(\frac{x}{2}\right)-6\left(4\left(2x\sin\left(\frac{x}{2}\right)+4\cos\left(\frac{x}{2}\right.\right.\right.\right.\right.$$

input
```
Int[x^3*(a + a*Cos[x])^(3/2),x]
```

output
```
2*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*((4*x^2*Cos[x/2]^3)/3 + (2*x^3*Cos[x/2]^2*
Sin[x/2])/3 - (8*((4*Cos[x/2]^3)/9 + (2*x*Cos[x/2]^2*Sin[x/2])/3 + (2*(4*C
os[x/2] + 2*x*Sin[x/2]))/3))/3 + (2*(2*x^3*Sin[x/2] - 6*(-2*x^2*Cos[x/2] +
 4*(4*Cos[x/2] + 2*x*Sin[x/2]))))/3)
```

3.164. $\int x^3(a+a\cos(x))^{3/2}\,dx$

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.164.4 Maple [F]

$$\int x^3(a+\cos(x)\,a)^{\frac{3}{2}}\,dx$$

input
```
int(x^3*(a+cos(x)*a)^(3/2),x)
```

output
```
int(x^3*(a+cos(x)*a)^(3/2),x)
```

3.164.5 Fricas [F(-2)]

Exception generated.

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.164.6 Sympy [F]

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \int x^3(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(a+a*cos(x))**(3/2),x)
```

output
```
Integral(x**3*(a*(cos(x) + 1))**(3/2), x)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.53

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \frac{1}{27}\left(81\sqrt{2}ax^3\sin\left(\frac{1}{2}x\right)+486\sqrt{2}ax^2\cos\left(\frac{1}{2}x\right)-1944\sqrt{2}ax\sin\left(\frac{1}{2}x\right)-3888\sqrt{2}a\cos\right.$$

input
```
integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

output
```
1/27*(81*sqrt(2)*a*x^3*sin(1/2*x) + 486*sqrt(2)*a*x^2*cos(1/2*x) - 1944*sq
rt(2)*a*x*sin(1/2*x) - 3888*sqrt(2)*a*cos(1/2*x) + 2*(9*sqrt(2)*a*x^2 - 8*
sqrt(2)*a)*cos(3/2*x) + 3*(3*sqrt(2)*a*x^3 - 8*sqrt(2)*a*x)*sin(3/2*x))*sq
rt(a)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61

$$\int x^3(a+a\cos(x))^{3/2}\,dx = \frac{1}{27}\sqrt{2}\left(2\left(9\,ax^2\text{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)-8\,a\text{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\right)\cos\left(\frac{3}{2}x\right)+486\left(ax^2\text{sgn}\left(c\right.\right.\right.$$

input
```
integrate(x^3*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

output
```
1/27*sqrt(2)*(2*(9*a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*cos(3/2*x)
 + 486*(a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*cos(1/2*x) + 3*(3*a*x
^3*sgn(cos(1/2*x)) - 8*a*x*sgn(cos(1/2*x)))*sin(3/2*x) + 81*(a*x^3*sgn(cos
(1/2*x)) - 24*a*x*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a+a\cos(x))^{3/2}\,dx=\int x^3\,(a+a\,\cos(x))^{3/2}\,dx$$

input
```
int(x^3*(a + a*cos(x))^(3/2),x)
```

output
```
int(x^3*(a + a*cos(x))^(3/2), x)
```

3.165 $\int x^2(a+a\cos(x))^{3/2}\,dx$

3.165.1 Optimal result

Integrand size = 14, antiderivative size = 145

$$\int x^2(a+a\cos(x))^{3/2}\,dx=\frac{32}{3}ax\sqrt{a+a\cos(x)}+\frac{16}{9}ax\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax^2\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)-\frac{224}{9}a\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+\frac{8}{3}ax^2\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)+\frac{32}{27}a\sqrt{a+a\cos(x)}\sin^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$$

output
```
32/3*a*x*(a+a*cos(x))^(1/2)+16/9*a*x*cos(1/2*x)^2*(a+a*cos(x))^(1/2)+4/3*a
*x^2*cos(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)-224/9*a*(a+a*cos(x))^(1/2)*t
an(1/2*x)+8/3*a*x^2*(a+a*cos(x))^(1/2)*tan(1/2*x)+32/27*a*sin(1/2*x)^2*(a+
a*cos(x))^(1/2)*tan(1/2*x)
```

3.165.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int x^2(a+a\cos(x))^{3/2}\,dx=\frac{2}{27}a\sqrt{a(1+\cos(x))}\Big(156x+\left(-328+45x^2\right)\tan\left(\frac{x}{2}\right)+\cos(x)\left(12x+\left(-8+9x^2\right)\tan\left(\frac{x}{2}\right)\right)\Big)$$

input
```
Integrate[x^2*(a + a*Cos[x])^(3/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cos[x])]*(156*x + (-328 + 45*x^2)*Tan[x/2] + Cos[x]*(12*x
 + (-8 + 9*x^2)*Tan[x/2])))/27
```

3.165.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 3800, 3042, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a\cos(x)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x^2\cos^3\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x^2\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3792}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\int x^2\cos\left(\frac{x}{2}\right)dx-\frac{8}{9}\int\cos^3\left(\frac{x}{2}\right)dx+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{8}{9}x\cos^3\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\int x^2\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-\frac{8}{9}\int\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{8}{9}x\cos^3\left(\frac{x}{2}\right.\right.$$

$$\Big\downarrow \text{3113}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\int x^2\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+\frac{16}{9}\int\left(1-\sin^2\left(\frac{x}{2}\right)\right)d\left(-\sin\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\int x^2\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{16}{9}\left(\frac{1}{3}\sin^3\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{8}{9}x\,c$$

$$\Big\downarrow \; 3777$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(4\int -x\sin\left(\frac{x}{2}\right)dx+2x^2\sin\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{16}{9}\left(\frac{1}{3}\sin^3\left(\frac{x}{2}\right)-\mathrm{si}\right.\right.$$

$$\Big\downarrow \; 25$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^2\sin\left(\frac{x}{2}\right)-4\int x\sin\left(\frac{x}{2}\right)dx\right)+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{16}{9}\left(\frac{1}{3}\sin^3\left(\frac{x}{2}\right)-\sin\right.\right.$$

$$\Big\downarrow \; 3042$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^2\sin\left(\frac{x}{2}\right)-4\int x\sin\left(\frac{x}{2}\right)dx\right)+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{16}{9}\left(\frac{1}{3}\sin^3\left(\frac{x}{2}\right)-\sin\right.\right.$$

$$\Big\downarrow \; 3777$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(2\int\cos\left(\frac{x}{2}\right)dx-2x\cos\left(\frac{x}{2}\right)\right)\right)+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{16}{9}\left(\right.\right.$$

$$\Big\downarrow \; 3042$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(2\int\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2x\cos\left(\frac{x}{2}\right)\right)\right)+\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\right.$$

$$\Big\downarrow \; 3117$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}x^2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{2}{3}\left(2x^2\sin\left(\frac{x}{2}\right)-4\left(4\sin\left(\frac{x}{2}\right)-2x\cos\left(\frac{x}{2}\right)\right)\right)+\frac{16}{9}\left(\frac{1}{3}\sin^3\right.\right.$$

input
```
Int[x^2*(a + a*Cos[x])^(3/2),x]
```

output
```
2*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*((8*x*Cos[x/2]^3)/9 + (2*x^2*Cos[x/2]^2*Si
n[x/2])/3 + (16*(-Sin[x/2] + Sin[x/2]^3/3))/9 + (2*(2*x^2*Sin[x/2] - 4*(-2
*x*Cos[x/2] + 4*Sin[x/2])))/3)
```

3.165. $\int x^2(a+a\cos(x))^{3/2}\,dx$

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.165. $\int x^2(a+a\cos(x))^{3/2}\,dx$

3.165.4 Maple [F]

$$\int x^2(a+\cos(x)a)^{\frac{3}{2}}dx$$

input
```
int(x^2*(a+cos(x)*a)^(3/2),x)
```

output
```
int(x^2*(a+cos(x)*a)^(3/2),x)
```

3.165.5 Fricas [F(-2)]

Exception generated.

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.165.6 Sympy [F]

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \int x^2(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+a*cos(x))**(3/2),x)
```

output
```
Integral(x**2*(a*(cos(x) + 1))**(3/2), x)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \frac{1}{27}\left(81\sqrt{2}ax^2\sin\left(\frac{1}{2}x\right)+12\sqrt{2}ax\cos\left(\frac{3}{2}x\right)+324\sqrt{2}ax\cos\left(\frac{1}{2}x\right)-648\sqrt{2}a\sin\left(\frac{1}{2}\right.\right.$$

input
```
integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

output
```
1/27*(81*sqrt(2)*a*x^2*sin(1/2*x) + 12*sqrt(2)*a*x*cos(3/2*x) + 324*sqrt(2
)*a*x*cos(1/2*x) - 648*sqrt(2)*a*sin(1/2*x) + (9*sqrt(2)*a*x^2 - 8*sqrt(2)
*a)*sin(3/2*x))*sqrt(a)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \frac{1}{27}\sqrt{2}\left(12\,ax\cos\left(\frac{3}{2}x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)+324\,ax\cos\left(\frac{1}{2}x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)+\left(9\,ax\right.\right.$$

input
```
integrate(x^2*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

output
```
1/27*sqrt(2)*(12*a*x*cos(3/2*x)*sgn(cos(1/2*x)) + 324*a*x*cos(1/2*x)*sgn(c
os(1/2*x)) + (9*a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*sin(3/2*x) +
81*(a*x^2*sgn(cos(1/2*x)) - 8*a*sgn(cos(1/2*x)))*sin(1/2*x))*sqrt(a)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+a\cos(x))^{3/2}\,dx = \int x^2\left(a+a\,\cos\left(x\right)\right)^{3/2}\,dx$$

input
```
int(x^2*(a + a*cos(x))^(3/2),x)
```

output
```
int(x^2*(a + a*cos(x))^(3/2), x)
```

3.166 $\int x(a+a\cos(x))^{3/2}\,dx$

3.166.1 Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x(a+a\cos(x))^{3/2}\,dx=\frac{16}{3}a\sqrt{a+a\cos(x)}+\frac{8}{9}a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}+\frac{4}{3}ax\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)+\frac{8}{3}ax\sqrt{a+a\cos(x)}\tan\left(\frac{x}{2}\right)$$

output
```
16/3*a*(a+a*cos(x))^(1/2)+8/9*a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)+4/3*a*x*co
s(1/2*x)*sin(1/2*x)*(a+a*cos(x))^(1/2)+8/3*a*x*(a+a*cos(x))^(1/2)*tan(1/2*
x)
```

3.166.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int x(a+a\cos(x))^{3/2}\,dx=\frac{1}{9}a\sqrt{a(1+\cos(x))}\left(52+4\cos(x)+3x\sec\left(\frac{x}{2}\right)\sin\left(\frac{3x}{2}\right)+27x\tan\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x*(a + a*Cos[x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cos[x])]*(52 + 4*Cos[x] + 3*x*Sec[x/2]*Sin[(3*x)/2] + 27*x*
Tan[x/2]))/9
```

3.166.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3800, 3042, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a\cos(x)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x\cos^3\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3791}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\int x\cos\left(\frac{x}{2}\right)dx+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\int x\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2\int-\sin\left(\frac{x}{2}\right)dx+2x\sin\left(\frac{x}{2}\right)\right)+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{2}{3}\left(2x\sin\left(\frac{x}{2}\right)-2\int\sin\left(\frac{x}{2}\right)dx\right)+\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3118}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{4}{9}\cos^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)+\frac{2}{3}\left(2x\sin\left(\frac{x}{2}\right)+4\cos\left(\frac{x}{2}\right)\right)\right)$$

input
```
Int[x*(a + a*Cos[x])^(3/2),x]
```

output
```
2*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*((4*Cos[x/2]^3)/9 + (2*x*Cos[x/2]^2*Sin[x/
2])/3 + (2*(4*Cos[x/2] + 2*x*Sin[x/2]))/3)
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.166.4 Maple [F]

$$\int x(a+\cos(x)\,a)^{\frac{3}{2}}\,dx$$

input
```
int(x*(a+cos(x)*a)^(3/2),x)
```

output
```
int(x*(a+cos(x)*a)^(3/2),x)
```

3.166.5 Fricas [F(-2)]

Exception generated.

$$\int x(a+a\cos(x))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.166.6 Sympy [F]

$$\int x(a+a\cos(x))^{3/2}\,dx = \int x(a(\cos(x)+1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+a*cos(x))**(3/2),x)
```

output
```
Integral(x*(a*(cos(x) + 1))**(3/2), x)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

$$\int x(a+a\cos(x))^{3/2}\,dx=\frac{1}{9}\left(3\sqrt{2}ax\sin\left(\frac{3}{2}x\right)+27\sqrt{2}ax\sin\left(\frac{1}{2}x\right)+2\sqrt{2}a\cos\left(\frac{3}{2}x\right)+54\sqrt{2}a\cos\left(\frac{1}{2}x\right)\right)\sqrt{}$$

input
```
integrate(x*(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

output
```
1/9*(3*sqrt(2)*a*x*sin(3/2*x) + 27*sqrt(2)*a*x*sin(1/2*x) + 2*sqrt(2)*a*co
s(3/2*x) + 54*sqrt(2)*a*cos(1/2*x))*sqrt(a)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int x(a+a\cos(x))^{3/2}\,dx=\frac{1}{9}\sqrt{2}\left(3\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{3}{2}x\right)+27\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)+2\,a\cos\left(\frac{3}{2}\right.\right.$$

input
```
integrate(x*(a+a*cos(x))^(3/2),x, algorithm="giac")
```

output
```
1/9*sqrt(2)*(3*a*x*sgn(cos(1/2*x))*sin(3/2*x) + 27*a*x*sgn(cos(1/2*x))*sin
(1/2*x) + 2*a*cos(3/2*x)*sgn(cos(1/2*x)) + 54*a*cos(1/2*x)*sgn(cos(1/2*x))
)*sqrt(a)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int x(a+a\cos(x))^{3/2}\,dx=\int x\,(a+a\,\cos(x))^{3/2}\,dx$$

input
```
int(x*(a + a*cos(x))^(3/2),x)
```

output
```
int(x*(a + a*cos(x))^(3/2), x)
```

3.167 $\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx$

3.167.1 Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx=\frac{3}{2}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)+\frac{1}{2}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right)$$

output
```
3/2*a*Ci(1/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+1/2*a*Ci(3/2*x)*sec(1/2*x)*(
a+a*cos(x))^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx=\frac{1}{2}a\sqrt{a(1+\cos(x))}\left(3\,\text{CosIntegral}\left(\frac{x}{2}\right)+\text{CosIntegral}\left(\frac{3x}{2}\right)\right)\sec\left(\frac{x}{2}\right)$$

input
```
Integrate[(a + a*Cos[x])^(3/2)/x,x]
```

output
```
(a*Sqrt[a*(1 + Cos[x])]*(3*CosIntegral[x/2] + CosIntegral[(3*x)/2])*Sec[x/
2])/2
```

3.167.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3800, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(x)+a)^{3/2}}{x}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}{x}dx$$
$$\Big\downarrow \text{3800}$$
$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\cos^3\left(\frac{x}{2}\right)}{x}dx$$
$$\Big\downarrow \text{3042}$$
$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3}{x}dx$$
$$\Big\downarrow \text{3793}$$
$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \left(\frac{3\cos\left(\frac{x}{2}\right)}{4x}+\frac{\cos\left(\frac{3x}{2}\right)}{4x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2a\left(\frac{3\,\text{CosIntegral}\left(\frac{x}{2}\right)}{4}+\frac{\text{CosIntegral}\left(\frac{3x}{2}\right)}{4}\right)\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}$$

input
```
Int[(a + a*Cos[x])^(3/2)/x,x]
```

output
```
2*a*Sqrt[a + a*Cos[x]]*((3*CosIntegral[x/2])/4 + CosIntegral[(3*x)/2]/4)*S
ec[x/2]
```

3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.167.4 Maple [F]

$$\int \frac{(a+\cos(x)\,a)^{\frac{3}{2}}}{x}dx$$

input
```
int((a+cos(x)*a)^(3/2)/x,x)
```

output
```
int((a+cos(x)*a)^(3/2)/x,x)
```

3.167.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(x))^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.167.6 Sympy [F]

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx=\int \frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+a*cos(x))**(3/2)/x,x)
```

output
```
Integral((a*(cos(x) + 1))**(3/2)/x, x)
```

3.167.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx=\frac{1}{4}\sqrt{2}a^{\frac{3}{2}}\left(\mathrm{Ei}\left(\frac{3}{2}i\,x\right)+3\,\mathrm{Ei}\left(\frac{1}{2}i\,x\right)+3\,\mathrm{Ei}\left(-\frac{1}{2}i\,x\right)+\mathrm{Ei}\left(-\frac{3}{2}i\,x\right)\right)$$

input
```
integrate((a+a*cos(x))^(3/2)/x,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*a^(3/2)*(Ei(3/2*I*x) + 3*Ei(1/2*I*x) + 3*Ei(-1/2*I*x) + Ei(-3/
2*I*x))
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.58

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx=\frac{1}{2}\sqrt{2}\left(a\,\mathrm{Ci}\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+3\,a\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\right)\sqrt{a}$$

input
```
integrate((a+a*cos(x))^(3/2)/x,x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(a*cos_integral(3/2*x)*sgn(cos(1/2*x)) + 3*a*cos_integral(1/2*
x)*sgn(cos(1/2*x)))*sqrt(a)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(x))^{3/2}}{x}\,dx=\int \frac{(a+a\,\cos(x))^{3/2}}{x}\,dx$$

input
```
int((a + a*cos(x))^(3/2)/x,x)
```

output
```
int((a + a*cos(x))^(3/2)/x, x)
```

3.168 $\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx$

3.168.1 Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = -\frac{2a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x} - \frac{3}{4}a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\mathrm{Si}\left(\frac{x}{2}\right) - \frac{3}{4}a\sqrt{a+a\cos(x)}\sec\left(\frac{x}{2}\right)\mathrm{Si}\left(\frac{3x}{2}\right)$$

output
```
-2*a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)/x-3/4*a*sec(1/2*x)*Si(1/2*x)*(a+a*cos
(x))^(1/2)-3/4*a*sec(1/2*x)*Si(3/2*x)*(a+a*cos(x))^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.67

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a(1+\cos(x))}\sec\left(\frac{x}{2}\right)\left(8\cos^3\left(\frac{x}{2}\right)+3x\mathrm{Si}\left(\frac{x}{2}\right)+3x\mathrm{Si}\left(\frac{3x}{2}\right)\right)}{4x}$$

input
```
Integrate[(a + a*Cos[x])^(3/2)/x^2,x]
```

output
```
-1/4*(a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(8*Cos[x/2]^3 + 3*x*SinIntegral[x/2]
 + 3*x*SinIntegral[(3*x)/2]))/x
```

3.168.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3800, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(x)+a)^{3/2}}{x^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}{x^2}dx$$
$$\Big\downarrow \text{3800}$$
$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\cos^3\left(\frac{x}{2}\right)}{x^2}dx$$
$$\Big\downarrow \text{3042}$$
$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3}{x^2}dx$$
$$\Big\downarrow \text{3794}$$
$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{3}{2}\int \left(-\frac{\sin\left(\frac{x}{2}\right)}{4x}-\frac{\sin\left(\frac{3x}{2}\right)}{4x}\right)dx-\frac{\cos^3\left(\frac{x}{2}\right)}{x}\right)$$
$$\Big\downarrow \text{2009}$$
$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{3}{2}\left(-\frac{\text{Si}\left(\frac{x}{2}\right)}{4}-\frac{\text{Si}\left(\frac{3x}{2}\right)}{4}\right)-\frac{\cos^3\left(\frac{x}{2}\right)}{x}\right)$$

input
```
Int[(a + a*Cos[x])^(3/2)/x^2,x]
```

output
```
2*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*(-(Cos[x/2]^3/x) + (3*(-1/4*SinIntegral[x/
2] - SinIntegral[(3*x)/2]/4))/2)
```

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.168.4 Maple [F]

$$\int \frac{(a+\cos(x)\,a)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a+cos(x)*a)^(3/2)/x^2,x)
```

output
```
int((a+cos(x)*a)^(3/2)/x^2,x)
```

3.168.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

3.168.6 Sympy [F]

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = \int \frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+a*cos(x))**(3/2)/x**2,x)
```

output
```
Integral((a*(cos(x) + 1))**(3/2)/x**2, x)
```

3.168.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = \frac{3}{8}\sqrt{2}a^{\frac{3}{2}}\left(-i\,\Gamma\left(-1,\frac{3}{2}i\,x\right)-i\,\Gamma\left(-1,\frac{1}{2}i\,x\right)+i\,\Gamma\left(-1,-\frac{1}{2}i\,x\right)+i\,\Gamma\left(-1,-\frac{3}{2}i\,x\right)\right)$$

input
```
integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
3/8*sqrt(2)*a^(3/2)*(-I*gamma(-1, 3/2*I*x) - I*gamma(-1, 1/2*I*x) + I*gamm
a(-1, -1/2*I*x) + I*gamma(-1, -3/2*I*x))
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\cos(x))^{3/2}}{x^2}\,dx = -\frac{\sqrt{2}\left(3\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{3}{2}\,x\right)+3\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\mathrm{Si}\left(\frac{1}{2}\,x\right)+2\,a\cos\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+6\,a\cos\left(\frac{1}{2}\,x\right)\right.}{4\,x}$$

input
```
integrate((a+a*cos(x))^(3/2)/x^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(3*a*x*sgn(cos(1/2*x))*sin_integral(3/2*x) + 3*a*x*sgn(cos(1/
2*x))*sin_integral(1/2*x) + 2*a*cos(3/2*x)*sgn(cos(1/2*x)) + 6*a*cos(1/2*x
)*sgn(cos(1/2*x)))*sqrt(a)/x
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\cos(x))^{3/2}}{x^2}\,dx = \int \frac{(a + a\ \cos(x))^{3/2}}{x^2}\,dx$$

input
```
int((a + a*cos(x))^(3/2)/x^2,x)
```

output
```
int((a + a*cos(x))^(3/2)/x^2, x)
```

3.169 $\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx$

3.169.1 Optimal result

Integrand size = 14, antiderivative size = 109

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = -\frac{a\cos^2\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}}{x^2} - \frac{3}{16}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right) - \frac{9}{16}a\sqrt{a+a\cos(x)}\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec\left(\frac{x}{2}\right) + \frac{3a\cos\left(\frac{x}{2}\right)\sqrt{a+a\cos(x)}\sin\left(\frac{x}{2}\right)}{2x}$$

output
```
-a*cos(1/2*x)^2*(a+a*cos(x))^(1/2)/x^2-3/16*a*Ci(1/2*x)*sec(1/2*x)*(a+a*co
s(x))^(1/2)-9/16*a*Ci(3/2*x)*sec(1/2*x)*(a+a*cos(x))^(1/2)+3/2*a*cos(1/2*x
)*sin(1/2*x)*(a+a*cos(x))^(1/2)/x
```

3.169.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = -\frac{(a(1+\cos(x)))^{3/2}\left(16+3x^2\,\text{CosIntegral}\left(\frac{x}{2}\right)\sec^3\left(\frac{x}{2}\right)+9x^2\,\text{CosIntegral}\left(\frac{3x}{2}\right)\sec^3\left(\frac{x}{2}\right)-24x\tan\left(\frac{x}{2}\right)\right)}{32x^2}$$

input
```
Integrate[(a + a*Cos[x])^(3/2)/x^3,x]
```

output
```
-1/32*((a*(1 + Cos[x]))^(3/2)*(16 + 3*x^2*CosIntegral[x/2]*Sec[x/2]^3 + 9*
x^2*CosIntegral[(3*x)/2]*Sec[x/2]^3 - 24*x*Tan[x/2]))/x^2
```

3.169.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3800, 3042, 3795, 3042, 3783, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cos(x)+a)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\cos^3\left(\frac{x}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{9}{8}\int \frac{\cos^3\left(\frac{x}{2}\right)}{x}dx+\frac{3}{4}\int \frac{\cos\left(\frac{x}{2}\right)}{x}dx-\frac{\cos^3\left(\frac{x}{2}\right)}{2x^2}+\frac{3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)}{4x}\right)$$

$$\Big\downarrow \text{3042}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{3}{4}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)}{x}dx-\frac{9}{8}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3}{x}dx-\frac{\cos^3\left(\frac{x}{2}\right)}{2x^2}+\frac{3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)}{4x}\right)$$

$$\Big\downarrow \text{3783}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{9}{8}\int \frac{\sin\left(\frac{x}{2}+\frac{\pi}{2}\right)^3}{x}dx+\frac{3\,\text{CosIntegral}\left(\frac{x}{2}\right)}{4}-\frac{\cos^3\left(\frac{x}{2}\right)}{2x^2}+\frac{3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)}{4x}\right)$$

$$\downarrow \text{3793}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(-\frac{9}{8}\int\left(\frac{3\cos\left(\frac{x}{2}\right)}{4x}+\frac{\cos\left(\frac{3x}{2}\right)}{4x}\right)dx+\frac{3\,\text{CosIntegral}\left(\frac{x}{2}\right)}{4}-\frac{\cos^3\left(\frac{x}{2}\right)}{2x^2}+\frac{3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)}{4x}\right)$$

$$\downarrow \text{2009}$$

$$2a\sec\left(\frac{x}{2}\right)\sqrt{a\cos(x)+a}\left(\frac{3\,\text{CosIntegral}\left(\frac{x}{2}\right)}{4}-\frac{9}{8}\left(\frac{3\,\text{CosIntegral}\left(\frac{x}{2}\right)}{4}+\frac{\text{CosIntegral}\left(\frac{3x}{2}\right)}{4}\right)-\frac{\cos^3\left(\frac{x}{2}\right)}{2x^2}+\frac{3\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)}{4x}\right)$$

input
```
Int[(a + a*Cos[x])^(3/2)/x^3,x]
```

output
```
2*a*Sqrt[a + a*Cos[x]]*Sec[x/2]*(-1/2*Cos[x/2]^3/x^2 + (3*CosIntegral[x/2]
)/4 - (9*((3*CosIntegral[x/2])/4 + CosIntegral[(3*x)/2]/4))/8 + (3*Cos[x/2
]^2*Sin[x/2])/(4*x))
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

3.169. $\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx$

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

3.169.4 Maple [F]

$$\int \frac{(a+\cos(x)\,a)^{\frac{3}{2}}}{x^3}dx$$

input
```
int((a+cos(x)*a)^(3/2)/x^3,x)
```

output
```
int((a+cos(x)*a)^(3/2)/x^3,x)
```

3.169.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.169.6 Sympy [F]

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \int \frac{(a(\cos(x)+1))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+a*cos(x))**(3/2)/x**3,x)
```

output
```
Integral((a*(cos(x) + 1))**(3/2)/x**3, x)
```

3.169.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.30

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \frac{3}{16}\sqrt{2}a^{\frac{3}{2}}\left(3\,\Gamma\left(-2,\frac{3}{2}i\,x\right)+\Gamma\left(-2,\frac{1}{2}i\,x\right)+\Gamma\left(-2,-\frac{1}{2}i\,x\right)+3\,\Gamma\left(-2,-\frac{3}{2}i\,x\right)\right)$$

input
```
integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/16*sqrt(2)*a^(3/2)*(3*gamma(-2, 3/2*I*x) + gamma(-2, 1/2*I*x) + gamma(-2
, -1/2*I*x) + 3*gamma(-2, -3/2*I*x))
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = -\frac{\sqrt{2}\left(9\,ax^2\,\mathrm{Ci}\left(\frac{3}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)+3\,ax^2\,\mathrm{Ci}\left(\frac{1}{2}\,x\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-6\,ax\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{3}{2}\,x\right)-6\,ax\mathrm{sgn}}{16\,x^2}$$

input
```
integrate((a+a*cos(x))^(3/2)/x^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(9*a*x^2*cos_integral(3/2*x)*sgn(cos(1/2*x)) + 3*a*x^2*cos_i
ntegral(1/2*x)*sgn(cos(1/2*x)) - 6*a*x*sgn(cos(1/2*x))*sin(3/2*x) - 6*a*x*
sgn(cos(1/2*x))*sin(1/2*x) + 4*a*cos(3/2*x)*sgn(cos(1/2*x)) + 12*a*cos(1/2
*x)*sgn(cos(1/2*x)))*sqrt(a)/x^2
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cos(x))^{3/2}}{x^3}\,dx = \int \frac{\left(a+a\,\cos\left(x\right)\right)^{3/2}}{x^3}\,dx$$

input
```
int((a + a*cos(x))^(3/2)/x^3,x)
```

output
```
int((a + a*cos(x))^(3/2)/x^3, x)
```

3.170 $\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.170.1 Optimal result

Integrand size = 18, antiderivative size = 374

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx =& -\frac{4ix^3\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\ &- \frac{12ix^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} \\ &- \frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{48x\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}} \\ &- \frac{96i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(4,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^4\sqrt{a+a\cos(c+dx)}} \\ &+ \frac{96i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(4,ie^{\frac{1}{2}i(c+dx)}\right)}{d^4\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-4*I*x^3*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^
(1/2)+12*I*x^2*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+
a*cos(d*x+c))^(1/2)-12*I*x^2*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x
+c)))/d^2/(a+a*cos(d*x+c))^(1/2)-48*x*cos(1/2*d*x+1/2*c)*polylog(3,-I*exp(
1/2*I*(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)+48*x*cos(1/2*d*x+1/2*c)*polylog
(3,I*exp(1/2*I*(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)-96*I*cos(1/2*d*x+1/2*c
)*polylog(4,-I*exp(1/2*I*(d*x+c)))/d^4/(a+a*cos(d*x+c))^(1/2)+96*I*cos(1/2
*d*x+1/2*c)*polylog(4,I*exp(1/2*I*(d*x+c)))/d^4/(a+a*cos(d*x+c))^(1/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.53

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx =$$
$$-\frac{4i\cos\left(\frac{1}{2}(c+dx)\right)\left(d^3x^3\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)-3d^2x^2\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)+3d^2x^2\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(\cdots}\right.\right.}{}$$

input
```
Integrate[x^3/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((-4*I)*Cos[(c + d*x)/2]*(d^3*x^3*ArcTan[E^((I/2)*(c + d*x))] - 3*d^2*x^2*
PolyLog[2, (-I)*E^((I/2)*(c + d*x))] + 3*d^2*x^2*PolyLog[2, I*E^((I/2)*(c
+ d*x))] - (12*I)*d*x*PolyLog[3, (-I)*E^((I/2)*(c + d*x))] + (12*I)*d*x*Po
lyLog[3, I*E^((I/2)*(c + d*x))] + 24*PolyLog[4, (-I)*E^((I/2)*(c + d*x))]
- 24*PolyLog[4, I*E^((I/2)*(c + d*x))]))/(d^4*Sqrt[a*(1 + Cos[c + d*x])])
```

3.170.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.64, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3800, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a\cos(c+dx)+a}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x^3}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \; 3800$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\int x^3\sec\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\int x^3\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)dx}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 4669$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{6\int x^2\log\left(1-ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}+\frac{6\int x^2\log\left(1+ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}-\frac{4ix^3\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)}{d}\right)}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3011$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{6\left(\frac{2ix^2\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{4i\int x\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}\right)}{d}-\frac{6\left(\frac{2ix^2\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{4i\int x\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(\cdots}{d}\cdots\right.}{d}\cdots\right.}{\sqrt{a\cos(c+dx)+a}\cdots}$$

$$\Big\downarrow \; 7163$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{6\left(\frac{2ix^2\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{4i\left(\frac{2i\int\operatorname{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}-\frac{2ix\operatorname{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\frac{6\left(\frac{2ix^2\operatorname{PolyLog}\left(2,ie\cdots\right.}{d}\cdots\right.}{\cdots}\cdots\right.}{\cdots\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 2720$$

3.170. $\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx$

$$\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{6\left(\frac{2ix^2\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{4i\left(\frac{4\int e^{-\frac{1}{2}i(c+dx)}\,\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)de^{\frac{1}{2}i(c+dx)}}{d^2}-\frac{2ix\,\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}\right)}{d}\right)}{d}-\right.$$

$$\overline{\sqrt{a\cos(c}}$$

$\Big\downarrow$ 7143

$$\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{4ix^3\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)}{d}+\frac{6\left(\frac{2ix^2\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{4i\left(\frac{4\,\text{PolyLog}\left(4,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2}-\frac{2ix\,\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}\right)}{d}\right.}{d}\right.$$

$$\overline{\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[x^3/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Cos[c/2 + (d*x)/2]*(((-4*I)*x^3*ArcTan[E^((I/2)*(c + d*x))])/d + (6*(((2*
I)*x^2*PolyLog[2, (-I)*E^((I/2)*(c + d*x))])/d - ((4*I)*(((-2*I)*x*PolyLog
[3, (-I)*E^((I/2)*(c + d*x))])/d + (4*PolyLog[4, (-I)*E^((I/2)*(c + d*x))]
)/d^2))/d))/d - (6*(((2*I)*x^2*PolyLog[2, I*E^((I/2)*(c + d*x))])/d - ((4*
I)*(((-2*I)*x*PolyLog[3, I*E^((I/2)*(c + d*x))])/d + (4*PolyLog[4, I*E^((I
/2)*(c + d*x))])/d^2))/d))/d))/Sqrt[a + a*Cos[c + d*x]]
```

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3.170.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.170. $\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.170.4 Maple [F]

$$\int \frac{x^3}{\sqrt{a+\cos\left(dx+c\right)a}}dx$$

input
```
int(x^3/(a+cos(d*x+c)*a)^(1/2),x)
```

output
```
int(x^3/(a+cos(d*x+c)*a)^(1/2),x)
```

3.170.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3/sqrt(a*cos(d*x + c) + a), x)
```

3.170.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(x**3/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(x**3/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.170.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*(6*sqrt(2)*d^2*x^2*cos(1/2*d*x + 1/2*c) + 24*(sqrt(2)*cos(d*x + c)^2 + s
qrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(cos(1/2*
d*x + 1/2*c), sin(1/2*d*x + 1/2*c) + 1) + 24*(sqrt(2)*cos(d*x + c)^2 + sqr
t(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(cos(1/2*d*
x + 1/2*c), -sin(1/2*d*x + 1/2*c) + 1) + (6*sqrt(2)*d^2*x^2*cos(1/2*d*x +
1/2*c) - (sqrt(2)*d^3*x^3 - 24*sqrt(2)*d*x)*sin(1/2*d*x + 1/2*c))*cos(d*x
+ c) + (sqrt(2)*a*d^7*cos(d*x + c)^2 + sqrt(2)*a*d^7*sin(d*x + c)^2 + 2*sq
rt(2)*a*d^7*cos(d*x + c) + sqrt(2)*a*d^7)*integrate((x^3*cos(2*d*x + 2*c)*
cos(1/2*d*x + 1/2*c) + 2*x^3*cos(d*x + c)*cos(1/2*d*x + 1/2*c) + x^3*sin(2
*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + 2*x^3*sin(d*x + c)*sin(1/2*d*x + 1/2*c)
 + x^3*cos(1/2*d*x + 1/2*c))/(a*d^3*cos(2*d*x + 2*c)^2 + 4*a*d^3*cos(d*x +
 c)^2 + a*d^3*sin(2*d*x + 2*c)^2 + 4*a*d^3*sin(2*d*x + 2*c)*sin(d*x + c) +
 4*a*d^3*sin(d*x + c)^2 + 4*a*d^3*cos(d*x + c) + a*d^3 + 2*(2*a*d^3*cos(d*
x + c) + a*d^3)*cos(2*d*x + 2*c)), x) - 6*(sqrt(2)*a*d^6*cos(d*x + c)^2 +
sqrt(2)*a*d^6*sin(d*x + c)^2 + 2*sqrt(2)*a*d^6*cos(d*x + c) + sqrt(2)*a*d^
6)*integrate((x^2*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + 2*x^2*cos(1/2*d*
x + 1/2*c)*sin(d*x + c) - x^2*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) - 2*x^
2*cos(d*x + c)*sin(1/2*d*x + 1/2*c) - x^2*sin(1/2*d*x + 1/2*c))/(a*d^3*cos
(2*d*x + 2*c)^2 + 4*a*d^3*cos(d*x + c)^2 + a*d^3*sin(2*d*x + 2*c)^2 + 4*a*
d^3*sin(2*d*x + 2*c)*sin(d*x + c) + 4*a*d^3*sin(d*x + c)^2 + 4*a*d^3*co...
```

3.170.8 Giac [F]

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate(x^3/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(a*cos(d*x + c) + a), x)
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^3}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(x^3/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(x^3/(a + a*cos(c + d*x))^(1/2), x)
```

3.171 $\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.171.1 Optimal result

Integrand size = 18, antiderivative size = 262

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = &-\frac{4ix^2\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}}\\ &+\frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}\\ &-\frac{8ix\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}\\ &-\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}\\ &+\frac{16\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(3,ie^{\frac{1}{2}i(c+dx)}\right)}{d^3\sqrt{a+a\cos(c+dx)}}\end{aligned}$$

output
```
-4*I*x^2*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^
(1/2)+8*I*x*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+a*c
os(d*x+c))^(1/2)-8*I*x*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x+c)))/
d^2/(a+a*cos(d*x+c))^(1/2)-16*cos(1/2*d*x+1/2*c)*polylog(3,-I*exp(1/2*I*(d
*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)+16*cos(1/2*d*x+1/2*c)*polylog(3,I*exp(1
/2*I*(d*x+c)))/d^3/(a+a*cos(d*x+c))^(1/2)
```

3.171.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx$$
$$= \frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(-id^2x^2\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)+2idx\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)-2idx\,\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+d}\right.\right.}{d^3\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[x^2/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(4*Cos[(c + d*x)/2]*((-I)*d^2*x^2*ArcTan[E^((I/2)*(c + d*x))] + (2*I)*d*x*
PolyLog[2, (-I)*E^((I/2)*(c + d*x))] - (2*I)*d*x*PolyLog[2, I*E^((I/2)*(c
+ d*x))] - 4*PolyLog[3, (-I)*E^((I/2)*(c + d*x))] + 4*PolyLog[3, I*E^((I/2
)*(c + d*x))]))/(d^3*Sqrt[a*(1 + Cos[c + d*x])])
```

3.171.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.64, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3042, 3800, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a\cos(c+dx)+a}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{x^2}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \text{ 3800}$$
$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\int x^2\sec\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{\sqrt{a\cos(c+dx)+a}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\int x^2\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)dx}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 4669$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{4\int x\log\left(1-ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}+\frac{4\int x\log\left(1+ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}-\frac{4ix^2\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)}{d}\right)}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3011$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{2i\int \text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}\right)}{d}-\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{2i\int \text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right.}{d}\right.}{d}\right.}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 2720$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{4\int e^{-\frac{1}{2}i(c+dx)}\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)de^{\frac{1}{2}i(c+dx)}}{d^2}\right)}{d}-\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{4\int e^{-}}{}\right.}{}\right.}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 7143$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{4ix^2\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)}{d}+\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d}-\frac{4\,\text{PolyLog}\left(3,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2}\right)}{d}-\frac{4\left(\frac{2ix\,\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d}\right.}{d}\right.}{\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[x^2/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Cos[c/2 + (d*x)/2]*(((-4*I)*x^2*ArcTan[E^((I/2)*(c + d*x))])/d + (4*(((2*
I)*x*PolyLog[2, (-I)*E^((I/2)*(c + d*x))])/d - (4*PolyLog[3, (-I)*E^((I/2)
*(c + d*x))])/d^2))/d - (4*(((2*I)*x*PolyLog[2, I*E^((I/2)*(c + d*x))])/d
- (4*PolyLog[3, I*E^((I/2)*(c + d*x))])/d^2))/d))/Sqrt[a + a*Cos[c + d*x]]
```

3.171. $\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.171.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.171.4 Maple [F]

$$\int \frac{x^2}{\sqrt{a+\cos\left(dx+c\right)a}}dx$$

input
```
int(x^2/(a+cos(d*x+c)*a)^(1/2),x)
```

output
```
int(x^2/(a+cos(d*x+c)*a)^(1/2),x)
```

3.171.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/sqrt(a*cos(d*x + c) + a), x)
```

3.171.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(x**2/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(x**2/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.171.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*(sqrt(2)*d^2*x^2*sin(1/2*d*x + 1/2*c) - 4*sqrt(2)*d*x*cos(1/2*d*x + 1/2
*c) + (sqrt(2)*d^2*x^2*sin(1/2*d*x + 1/2*c) - 4*sqrt(2)*d*x*cos(1/2*d*x +
1/2*c))*cos(d*x + c) - (sqrt(2)*a*d^5*cos(d*x + c)^2 + sqrt(2)*a*d^5*sin(d
*x + c)^2 + 2*sqrt(2)*a*d^5*cos(d*x + c) + sqrt(2)*a*d^5)*integrate((x^2*c
os(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + 2*x^2*cos(d*x + c)*cos(1/2*d*x + 1/
2*c) + x^2*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + 2*x^2*sin(d*x + c)*sin(
1/2*d*x + 1/2*c) + x^2*cos(1/2*d*x + 1/2*c))/(a*d^2*cos(2*d*x + 2*c)^2 + 4
*a*d^2*cos(d*x + c)^2 + a*d^2*sin(2*d*x + 2*c)^2 + 4*a*d^2*sin(2*d*x + 2*c
)*sin(d*x + c) + 4*a*d^2*sin(d*x + c)^2 + 4*a*d^2*cos(d*x + c) + a*d^2 + 2
*(2*a*d^2*cos(d*x + c) + a*d^2)*cos(2*d*x + 2*c)), x) + 4*(sqrt(2)*a*d^4*c
os(d*x + c)^2 + sqrt(2)*a*d^4*sin(d*x + c)^2 + 2*sqrt(2)*a*d^4*cos(d*x + c
) + sqrt(2)*a*d^4)*integrate((x*cos(1/2*d*x + 1/2*c)*sin(2*d*x + 2*c) + 2*
x*cos(1/2*d*x + 1/2*c)*sin(d*x + c) - x*cos(2*d*x + 2*c)*sin(1/2*d*x + 1/2
*c) - 2*x*cos(d*x + c)*sin(1/2*d*x + 1/2*c) - x*sin(1/2*d*x + 1/2*c))/(a*d
^2*cos(2*d*x + 2*c)^2 + 4*a*d^2*cos(d*x + c)^2 + a*d^2*sin(2*d*x + 2*c)^2
+ 4*a*d^2*sin(2*d*x + 2*c)*sin(d*x + c) + 4*a*d^2*sin(d*x + c)^2 + 4*a*d^2
*cos(d*x + c) + a*d^2 + 2*(2*a*d^2*cos(d*x + c) + a*d^2)*cos(2*d*x + 2*c))
, x) + 2*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(
d*x + c) + sqrt(2))*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - 2*(sqrt(2)*cos(d*x + c)^2 + sqrt(2)*sin(d...
```

3.171.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate(x^2/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(a*cos(d*x + c) + a), x)
```

3.171.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{x^2}{\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(x^2/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(x^2/(a + a*cos(c + d*x))^(1/2), x)
```

3.172 $\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.172.1 Optimal result

Integrand size = 16, antiderivative size = 156

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{4ix\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cos(c+dx)}} + \frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}} - \frac{4i\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2\sqrt{a+a\cos(c+dx)}}$$

output
```
-4*I*x*arctan(exp(1/2*I*(d*x+c)))*cos(1/2*d*x+1/2*c)/d/(a+a*cos(d*x+c))^(1
/2)+4*I*cos(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*I*(d*x+c)))/d^2/(a+a*cos(d
*x+c))^(1/2)-4*I*cos(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*I*(d*x+c)))/d^2/(a
+a*cos(d*x+c))^(1/2)
```

3.172.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.57

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = -\frac{4i\cos\left(\frac{1}{2}(c+dx)\right)\left(dx\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)-\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)+\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)\right)}{d^2\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[x/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
((-4*I)*Cos[(c + d*x)/2]*(d*x*ArcTan[E^((I/2)*(c + d*x))] - PolyLog[2, (-I
)*E^((I/2)*(c + d*x))] + PolyLog[2, I*E^((I/2)*(c + d*x))]))/(d^2*Sqrt[a*(
1 + Cos[c + d*x])])
```

3.172.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3800, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \; 3800$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\int x\sec\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\int x\csc\left(\frac{c}{2}+\frac{dx}{2}+\frac{\pi}{2}\right)dx}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \; 4669$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{2\int \log\left(1-ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}+\frac{2\int \log\left(1+ie^{\frac{1}{2}i(c+dx)}\right)dx}{d}-\frac{4ix\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)}{d}\right)}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{4i\int e^{-\frac{1}{2}i(c+dx)}\log\left(1-ie^{\frac{1}{2}i(c+dx)}\right)de^{\frac{1}{2}i(c+dx)}}{d^2}-\frac{4i\int e^{-\frac{1}{2}i(c+dx)}\log\left(1+ie^{\frac{1}{2}i(c+dx)}\right)de^{\frac{1}{2}i(c+dx)}}{d^2}-\frac{4ix\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)}{d}\right)}{\sqrt{a\cos(c+dx)+a}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{4ix\arctan\left(e^{\frac{1}{2}i(c+dx)}\right)}{d}+\frac{4i\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}i(c+dx)}\right)}{d^2}-\frac{4i\,\text{PolyLog}\left(2,ie^{\frac{1}{2}i(c+dx)}\right)}{d^2}\right)}{\sqrt{a\cos(c+dx)+a}}$$

input
```
Int[x/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Cos[c/2 + (d*x)/2]*(((-4*I)*x*ArcTan[E^((I/2)*(c + d*x))])/d + ((4*I)*Pol
yLog[2, (-I)*E^((I/2)*(c + d*x))])/d^2 - ((4*I)*PolyLog[2, I*E^((I/2)*(c +
 d*x))])/d^2))/Sqrt[a + a*Cos[c + d*x]]
```

3.172.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.172.4 Maple [F]

$$\int \frac{x}{\sqrt{a+\cos \left(d x +c \right) a}}d x$$

input
```
int(x/(a+cos(d*x+c)*a)^(1/2),x)
```

output
```
int(x/(a+cos(d*x+c)*a)^(1/2),x)
```

3.172.5 Fricas [F]

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{x}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(a*cos(d*x + c) + a), x)
```

3.172.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{x}{\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(x/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(x/sqrt(a*(cos(c + d*x) + 1)), x)
```

3.172.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{x}{\sqrt{a\cos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*d*x*cos(1/2*d*x + 1/2*c)*sin(d*x + c) - sqrt(2)*d*x*cos(d*x + c
)*sin(1/2*d*x + 1/2*c) - sqrt(2)*d*x*sin(1/2*d*x + 1/2*c) - (sqrt(2)*cos(d
*x + c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arc
tan2(cos(1/2*d*x + 1/2*c), sin(1/2*d*x + 1/2*c) + 1) - (sqrt(2)*cos(d*x +
c)^2 + sqrt(2)*sin(d*x + c)^2 + 2*sqrt(2)*cos(d*x + c) + sqrt(2))*arctan2(
cos(1/2*d*x + 1/2*c), -sin(1/2*d*x + 1/2*c) + 1) + (sqrt(2)*a*d^3*cos(d*x
+ c)^2 + sqrt(2)*a*d^3*sin(d*x + c)^2 + 2*sqrt(2)*a*d^3*cos(d*x + c) + sqr
t(2)*a*d^3)*integrate((x*cos(2*d*x + 2*c)*cos(1/2*d*x + 1/2*c) + 2*x*cos(d
*x + c)*cos(1/2*d*x + 1/2*c) + x*sin(2*d*x + 2*c)*sin(1/2*d*x + 1/2*c) + 2
*x*sin(d*x + c)*sin(1/2*d*x + 1/2*c) + x*cos(1/2*d*x + 1/2*c))/(a*d*cos(2*
d*x + 2*c)^2 + 4*a*d*cos(d*x + c)^2 + a*d*sin(2*d*x + 2*c)^2 + 4*a*d*sin(2
*d*x + 2*c)*sin(d*x + c) + 4*a*d*sin(d*x + c)^2 + 4*a*d*cos(d*x + c) + a*d
 + 2*(2*a*d*cos(d*x + c) + a*d)*cos(2*d*x + 2*c)), x))/((d^2*cos(d*x + c)^
2 + d^2*sin(d*x + c)^2 + 2*d^2*cos(d*x + c) + d^2)*sqrt(a))
```

3.172.8 Giac [F]

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x}{\sqrt{a\cos(dx+c)+a}}\,dx$$

input
```
integrate(x/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(a*cos(d*x + c) + a), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{x}{\sqrt{a+a\,\cos(c+d\,x)}}\,dx$$

input
```
int(x/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
int(x/(a + a*cos(c + d*x))^(1/2), x)
```

3.173 $\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx$

3.173.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\cos(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
arctanh(1/2*sin(d*x+c)*a^(1/2)*2^(1/2)/(a+a*cos(d*x+c))^(1/2))*2^(1/2)/d/a
^(1/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx=\frac{2\text{arctanh}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cos(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(2*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2])/(d*Sqrt[a*(1 + Cos[c + d*x]
)])
```

3.173.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3128}$$

$$-\frac{2\int \frac{1}{2a-\frac{a^2\sin^2(c+dx)}{\cos(c+dx)a+a}}d\left(-\frac{a\sin(c+dx)}{\sqrt{\cos(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(c+dx)}{\sqrt{2}\sqrt{a\cos(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + a*Cos[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cos[c + d*x]])
])/(Sqrt[a]*d)
```

3.173.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.173.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{\sqrt{2}\,\operatorname{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle\vert 1\right)}{d\sec\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\operatorname{csgn}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	56

input
```
int(1/(a+cos(d*x+c)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*2^(1/2)/sec(1/2*d*x+1/2*c)/(a*cos(1/2*d*x+1/2*c)^2)^(1/2)/csgn(cos(1/2
*d*x+1/2*c))*InverseJacobiAM(1/2*d*x+1/2*c,1)
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{a\cos(dx+c)+a}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{2\sqrt{a}d},\ -\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{a\cos(dx+c)+a}\sqrt{-\frac{1}{a}}}{\sin(dx+c)}\right)}{d}\right]$$

input
```
integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a*cos(d*x + c) + a)*sin
(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1))/(sqrt(a)*d), -sqrt(2)*sqrt(-1/a)*arctan(sqrt(2)*sqrt(a*cos(d*x + c) +
 a)*sqrt(-1/a)/sin(d*x + c))/d]
```

3.173.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\cos\left(c+dx\right)+a}}\,dx$$

input
```
integrate(1/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*cos(c + d*x) + a), x)
```

3.173.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(37)=74$.

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\sin\right.}{2\sqrt{a}d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))/(sqrt(a)*d)
```

3.173.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(37) = 74.

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)} - \frac{\sqrt{2}\log\left(\left|\frac{1}{\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{4\,d}$$

input
```
integrate(1/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c) + sin(1/2*d*x + 1/2*c) + 2))/(
sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))) - sqrt(2)*log(abs(1/sin(1/2*d*x + 1/2*c
) + sin(1/2*d*x + 1/2*c) - 2))/(sqrt(a)*sgn(cos(1/2*d*x + 1/2*c))))/d
```

3.173.9 Mupad [B] (verification not implemented)

Time = 14.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{a+a\cos(c+dx)}}\,dx = \frac{\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|1\right)\,\sqrt{\frac{2\,(a+a\,\cos(c+d\,x))}{a}}}{d\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}$$

input
```
int(1/(a + a*cos(c + d*x))^(1/2),x)
```

output
```
(ellipticF(c/2 + (d*x)/2, 1)*((2*(a + a*cos(c + d*x)))/a)^(1/2))/(d*(a + a
*cos(c + d*x))^(1/2))
```

3.174 $\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$

3.174.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{a+a\cos(c+dx)}},x\right)$$

output
```
Unintegrable(1/x/(a+a*cos(d*x+c))^(1/2),x)
```

3.174.2 Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[a + a*Cos[c + d*x]]), x]
```

3.174.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a\cos(c+dx)+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x\sqrt{a\cos(c+dx)+a}}dx$$

input
```
Int[1/(x*Sqrt[a + a*Cos[c + d*x]]),x]
```

output
```
$Aborted
```

3.174.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.174.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a+\cos\left(dx+c\right)a}}dx$$

input
```
int(1/x/(a+cos(d*x+c)*a)^(1/2),x)
```

output
```
int(1/x/(a+cos(d*x+c)*a)^(1/2),x)
```

3.174.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\left(dx+c\right)+ax}}\,dx$$

input
```
integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cos(d*x + c) + a)/(a*x*cos(d*x + c) + a*x), x)
```

3.174.6 Sympy [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a\left(\cos\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(1/x/(a+a*cos(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*(cos(c + d*x) + 1))), x)
```

3.174.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}x}\,dx$$

input
```
integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*cos(d*x + c) + a)*x), x)
```

3.174.8 Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\left(dx+c\right)+a}x}\,dx$$

input
```
integrate(1/x/(a+a*cos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cos(d*x + c) + a)*x), x)
```

3.174.9 Mupad [N/A]

Not integrable

Time = 14.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cos(c+dx)}}\,dx = \int \frac{1}{x\,\sqrt{a+a\,\cos\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(x*(a + a*cos(c + d*x))^(1/2)),x)
```

output
```
int(1/(x*(a + a*cos(c + d*x))^(1/2)), x)
```

3.175 $\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx$

3.175.1 Optimal result

Integrand size = 15, antiderivative size = 235

$$\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx = -\frac{4x^3\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{12ix^2\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{12ix^2\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{48x\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{48x\operatorname{PolyLog}\left(3,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{96i\operatorname{PolyLog}\left(4,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{96i\operatorname{PolyLog}\left(4,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

output
```
-4*x^3*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+12*I*x^2*polylo
g(2,-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-12*I*x^2*polylog(2,exp(1/
2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-48*x*polylog(3,-exp(1/2*I*x))*sin(1/
2*x)/(a-a*cos(x))^(1/2)+48*x*polylog(3,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x
))^(1/2)-96*I*polylog(4,-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+96*I*
polylog(4,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)
```

3.175.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx =$$
$$-\frac{i\left(8\pi^4 - x^4 + 8ix^3\log\left(1 - e^{-\frac{ix}{2}}\right) - 8ix^3\log\left(1 + e^{\frac{ix}{2}}\right) - 48x^2\operatorname{PolyLog}\left(2, e^{-\frac{ix}{2}}\right) - 48x^2\operatorname{PolyLog}\left(2, -\right.\right.}{}$$

input
```
Integrate[x^3/Sqrt[a - a*Cos[x]],x]
```

output
```
((-1/4*I)*(8*Pi^4 - x^4 + (8*I)*x^3*Log[1 - E^((-1/2*I)*x)] - (8*I)*x^3*Lo
g[1 + E^((I/2)*x)] - 48*x^2*PolyLog[2, E^((-1/2*I)*x)] - 48*x^2*PolyLog[2,
 -E^((I/2)*x)] + (192*I)*x*PolyLog[3, E^((-1/2*I)*x)] - (192*I)*x*PolyLog[
3, -E^((I/2)*x)] + 384*PolyLog[4, E^((-1/2*I)*x)] + 384*PolyLog[4, -E^((I/
2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

3.175.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.64, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3800, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{x^3}{\sqrt{a - a\sin\left(x + \frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3800}$$
$$\frac{\sin\left(\frac{x}{2}\right)\int x^3\csc\left(\frac{x}{2}\right)dx}{\sqrt{a - a\cos(x)}}$$
$$\Big\downarrow \text{3042}$$

$$\frac{\sin\left(\frac{x}{2}\right)\int x^3\csc\left(\frac{x}{2}\right)dx}{\sqrt{a-a\cos(x)}}$$

$\Big\downarrow$ 4671

$$\frac{\sin\left(\frac{x}{2}\right)\left(-6\int x^2\log\left(1-e^{\frac{ix}{2}}\right)dx+6\int x^2\log\left(1+e^{\frac{ix}{2}}\right)dx-4x^3\text{arctanh}\left(e^{\frac{ix}{2}}\right)\right)}{\sqrt{a-a\cos(x)}}$$

$\Big\downarrow$ 3011

$$\frac{\sin\left(\frac{x}{2}\right)\left(6\left(2ix^2\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4i\int x\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)dx\right)-6\left(2ix^2\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)-4i\int x\operatorname{PolyLog}\left(\right.\right.\right.}{\sqrt{a-a\cos(x)}}$$

$\Big\downarrow$ 7163

$$\frac{\sin\left(\frac{x}{2}\right)\left(6\left(2ix^2\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4i\left(2i\int\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)dx-2ix\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\right)\right)-6\left(2ix^2\operatorname{PolyLo}\right.\right.}{\sqrt{a-a\cos(x)}}$$

$\Big\downarrow$ 2720

$$\frac{\sin\left(\frac{x}{2}\right)\left(6\left(2ix^2\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4i\left(4\int e^{-\frac{ix}{2}}\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)de^{\frac{ix}{2}}-2ix\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\right)\right)-6\left(2ix^2\operatorname{P}\right.\right.}{\sqrt{a-a\cos(x}}$$

$\Big\downarrow$ 7143

$$\frac{\sin\left(\frac{x}{2}\right)\left(-4x^3\text{arctanh}\left(e^{\frac{ix}{2}}\right)+6\left(2ix^2\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4i\left(4\operatorname{PolyLog}\left(4,-e^{\frac{ix}{2}}\right)-2ix\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\right)\right)-\right.}{\sqrt{a-a\cos(x)}}$$

input
```
Int[x^3/Sqrt[a - a*Cos[x]],x]
```

output
```
((-4*x^3*ArcTanh[E^((I/2)*x)] + 6*((2*I)*x^2*PolyLog[2, -E^((I/2)*x)] - (4
*I)*((-2*I)*x*PolyLog[3, -E^((I/2)*x)] + 4*PolyLog[4, -E^((I/2)*x)])) - 6*
((2*I)*x^2*PolyLog[2, E^((I/2)*x)] - (4*I)*((-2*I)*x*PolyLog[3, E^((I/2)*x
)] + 4*PolyLog[4, E^((I/2)*x)])))*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

3.175. $\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx$

3.175.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.175. $\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.175.4 Maple [F]

$$\int \frac{x^3}{\sqrt{a-\cos\left(x\right)a}}dx$$

input
```
int(x^3/(a-cos(x)*a)^(1/2),x)
```

output
```
int(x^3/(a-cos(x)*a)^(1/2),x)
```

3.175.5 Fricas [F]

$$\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{-a\cos\left(x\right)+a}}\,dx$$

input
```
integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*cos(x) + a)*x^3/(a*cos(x) - a), x)
```

3.175.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a-a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{-a\left(\cos\left(x\right)-1\right)}}\,dx$$

input
```
integrate(x**3/(a-a*cos(x))**(1/2),x)
```

output
```
Integral(x**3/sqrt(-a*(cos(x) - 1)), x)
```

3.175.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{-a\cos\left(x\right) + a}}\,dx$$

input
```
integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/sqrt(-a*cos(x) + a), x)
```

3.175.8 Giac [F]

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{-a\cos\left(x\right) + a}}\,dx$$

input
```
integrate(x^3/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(-a*cos(x) + a), x)
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^3}{\sqrt{a - a\,\cos\left(x\right)}}\,dx$$

input
```
int(x^3/(a - a*cos(x))^(1/2),x)
```

output
```
int(x^3/(a - a*cos(x))^(1/2), x)
```

3.176 $\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx$

3.176.1 Optimal result

Integrand size = 15, antiderivative size = 163

$$\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx = -\frac{4x^2\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{8ix\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{8ix\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} - \frac{16\operatorname{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}} + \frac{16\operatorname{PolyLog}\left(3,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

output
```
-4*x^2*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+8*I*x*polylog(2
,-exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-8*I*x*polylog(2,exp(1/2*I*x)
)*sin(1/2*x)/(a-a*cos(x))^(1/2)-16*polylog(3,-exp(1/2*I*x))*sin(1/2*x)/(a-
a*cos(x))^(1/2)+16*polylog(3,exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx$$
$$= \frac{2\left(x^2\log\left(1-e^{\frac{ix}{2}}\right)-x^2\log\left(1+e^{\frac{ix}{2}}\right)+4ix\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4ix\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)-8\operatorname{PolyLog}\left(3,-\right.\right.}{\sqrt{a-a\cos(x)}}$$

input
```
Integrate[x^2/Sqrt[a - a*Cos[x]],x]
```

output
```
(2*(x^2*Log[1 - E^((I/2)*x)] - x^2*Log[1 + E^((I/2)*x)] + (4*I)*x*PolyLog[
2, -E^((I/2)*x)] - (4*I)*x*PolyLog[2, E^((I/2)*x)] - 8*PolyLog[3, -E^((I/2
)*x)] + 8*PolyLog[3, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

3.176.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3800, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{x^2}{\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3800}$$
$$\frac{\sin\left(\frac{x}{2}\right)\int x^2\csc\left(\frac{x}{2}\right)dx}{\sqrt{a-a\cos(x)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sin\left(\frac{x}{2}\right)\int x^2\csc\left(\frac{x}{2}\right)dx}{\sqrt{a-a\cos(x)}}$$

$\big\downarrow$ 4671

$$\frac{\sin\left(\frac{x}{2}\right)\left(-4\int x\log\left(1-e^{\frac{ix}{2}}\right)dx+4\int x\log\left(1+e^{\frac{ix}{2}}\right)dx-4x^2\text{arctanh}\left(e^{\frac{ix}{2}}\right)\right)}{\sqrt{a-a\cos(x)}}$$

$\big\downarrow$ 3011

$$\frac{\sin\left(\frac{x}{2}\right)\left(4\left(2ix\,\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-2i\int \text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)dx\right)-4\left(2ix\,\text{PolyLog}\left(2,e^{\frac{ix}{2}}\right)-2i\int \text{PolyLog}\left(2,e^{\frac{ix}{2}}\right.\right.\right.}{\sqrt{a-a\cos(x)}}$$

$\big\downarrow$ 2720

$$\frac{\sin\left(\frac{x}{2}\right)\left(4\left(2ix\,\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4\int e^{-\frac{ix}{2}}\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)de^{\frac{ix}{2}}\right)-4\left(2ix\,\text{PolyLog}\left(2,e^{\frac{ix}{2}}\right)-4\int e^{-\frac{ix}{2}}\text{PolyL}\right.\right.}{\sqrt{a-a\cos(x)}}$$

$\big\downarrow$ 7143

$$\frac{\sin\left(\frac{x}{2}\right)\left(-4x^2\text{arctanh}\left(e^{\frac{ix}{2}}\right)+4\left(2ix\,\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4\,\text{PolyLog}\left(3,-e^{\frac{ix}{2}}\right)\right)-4\left(2ix\,\text{PolyLog}\left(2,e^{\frac{ix}{2}}\right)-4\,\text{Po}\right.\right.}{\sqrt{a-a\cos(x)}}$$

input
```
Int[x^2/Sqrt[a - a*Cos[x]],x]
```

output
```
((-4*x^2*ArcTanh[E^((I/2)*x)] + 4*((2*I)*x*PolyLog[2, -E^((I/2)*x)] - 4*Po
lyLog[3, -E^((I/2)*x)]) - 4*((2*I)*x*PolyLog[2, E^((I/2)*x)] - 4*PolyLog[3
, E^((I/2)*x)]))*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

3.176.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.176. $\int \frac{x^2}{\sqrt{a-a\cos(x)}}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.176.4 Maple [F]

$$\int \frac{x^2}{\sqrt{a-\cos(x)\,a}}dx$$

input
```
int(x^2/(a-cos(x)*a)^(1/2),x)
```

output
```
int(x^2/(a-cos(x)*a)^(1/2),x)
```

3.176.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{-a\cos(x) + a}}\,dx$$

input
```
integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*cos(x) + a)*x^2/(a*cos(x) - a), x)
```

3.176.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{-a(\cos(x) - 1)}}\,dx$$

input
```
integrate(x**2/(a-a*cos(x))**(1/2),x)
```

output
```
Integral(x**2/sqrt(-a*(cos(x) - 1)), x)
```

3.176.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{-a\cos(x) + a}}\,dx$$

input
```
integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-a*cos(x) + a), x)
```

3.176.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{-a\cos(x) + a}}\,dx$$

input
```
integrate(x^2/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-a*cos(x) + a), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a - a\cos(x)}}\,dx = \int \frac{x^2}{\sqrt{a - a\,\cos(x)}}\,dx$$

input
```
int(x^2/(a - a*cos(x))^(1/2),x)
```

output
```
int(x^2/(a - a*cos(x))^(1/2), x)
```

3.177 $\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx$

3.177.1 Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx = -\frac{4x\operatorname{arctanh}\left(e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}+\frac{4i\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}-\frac{4i\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

output
```
-4*x*arctanh(exp(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)+4*I*polylog(2,-ex
p(1/2*I*x))*sin(1/2*x)/(a-a*cos(x))^(1/2)-4*I*polylog(2,exp(1/2*I*x))*sin(
1/2*x)/(a-a*cos(x))^(1/2)
```

3.177.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx = \frac{2\left(x\left(\log\left(1-e^{\frac{ix}{2}}\right)-\log\left(1+e^{\frac{ix}{2}}\right)\right)+2i\operatorname{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-2i\operatorname{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

input
```
Integrate[x/Sqrt[a - a*Cos[x]],x]
```

output
```
(2*(x*(Log[1 - E^((I/2)*x)] - Log[1 + E^((I/2)*x)]) + (2*I)*PolyLog[2, -E^
((I/2)*x)] - (2*I)*PolyLog[2, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

3.177.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3800, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-a\cos(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x}{\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\sin\left(\frac{x}{2}\right)\int x\csc\left(\frac{x}{2}\right)dx}{\sqrt{a-a\cos(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin\left(\frac{x}{2}\right)\int x\csc\left(\frac{x}{2}\right)dx}{\sqrt{a-a\cos(x)}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{\sin\left(\frac{x}{2}\right)\left(-2\int \log\left(1-e^{\frac{ix}{2}}\right)dx+2\int \log\left(1+e^{\frac{ix}{2}}\right)dx-4x\text{arctanh}\left(e^{\frac{ix}{2}}\right)\right)}{\sqrt{a-a\cos(x)}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\sin\left(\frac{x}{2}\right)\left(4i\int e^{-\frac{ix}{2}}\log\left(1-e^{\frac{ix}{2}}\right)de^{\frac{ix}{2}}-4i\int e^{-\frac{ix}{2}}\log\left(1+e^{\frac{ix}{2}}\right)de^{\frac{ix}{2}}-4x\text{arctanh}\left(e^{\frac{ix}{2}}\right)\right)}{\sqrt{a-a\cos(x)}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\sin\left(\frac{x}{2}\right)\left(-4x\text{arctanh}\left(e^{\frac{ix}{2}}\right)+4i\,\text{PolyLog}\left(2,-e^{\frac{ix}{2}}\right)-4i\,\text{PolyLog}\left(2,e^{\frac{ix}{2}}\right)\right)}{\sqrt{a-a\cos(x)}}$$

input
```
Int[x/Sqrt[a - a*Cos[x]],x]
```

output
```
((-4*x*ArcTanh[E^((I/2)*x)] + (4*I)*PolyLog[2, -E^((I/2)*x)] - (4*I)*PolyL
og[2, E^((I/2)*x)])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

3.177.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.177.4 Maple [F]

$$\int \frac{x}{\sqrt{a-\cos \left(x \right) a}}d x$$

input
```
int(x/(a-cos(x)*a)^(1/2),x)
```

output
```
int(x/(a-cos(x)*a)^(1/2),x)
```

3.177.5 Fricas [F]

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx=\int \frac{x}{\sqrt{-a\cos\left(x\right)+a}}\,dx$$

input
```
integrate(x/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*cos(x) + a)*x/(a*cos(x) - a), x)
```

3.177.6 Sympy [F]

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx=\int \frac{x}{\sqrt{-a\left(\cos\left(x\right)-1\right)}}\,dx$$

input
```
integrate(x/(a-a*cos(x))**(1/2),x)
```

output
```
Integral(x/sqrt(-a*(cos(x) - 1)), x)
```

3.177.7 Maxima [F]

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx = \int \frac{x}{\sqrt{-a\cos(x)+a}}\,dx$$

input
```
integrate(x/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(-a*cos(x) + a), x)
```

3.177.8 Giac [F]

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx = \int \frac{x}{\sqrt{-a\cos(x)+a}}\,dx$$

input
```
integrate(x/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(-a*cos(x) + a), x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a-a\cos(x)}}\,dx = \int \frac{x}{\sqrt{a-a\cos(x)}}\,dx$$

input
```
int(x/(a - a*cos(x))^(1/2),x)
```

output
```
int(x/(a - a*cos(x))^(1/2), x)
```

3.178 $\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx$

3.178.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

output
```
-arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a-a*cos(x))^(1/2))*2^(1/2)/a^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx = \frac{2\left(-\log\left(\cos\left(\frac{x}{4}\right)\right)+\log\left(\sin\left(\frac{x}{4}\right)\right)\right)\sin\left(\frac{x}{2}\right)}{\sqrt{a-a\cos(x)}}$$

input
```
Integrate[1/Sqrt[a - a*Cos[x]],x]
```

output
```
(2*(-Log[Cos[x/4]] + Log[Sin[x/4]])*Sin[x/2])/Sqrt[a - a*Cos[x]]
```

3.178.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\cos(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$-2\int \frac{1}{2a-\frac{a^2\sin^2(x)}{a-a\cos(x)}}d\frac{a\sin(x)}{\sqrt{a-a\cos(x)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a-a\cos(x)}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[a - a*Cos[x]],x]
```

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a - a*Cos[x]])])/Sqrt[a]
)
```

3.178.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.178.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{\sin\left(\frac{x}{2}\right)\operatorname{arctanh}\left(\cos\left(\frac{x}{2}\right)\right)\sqrt{2}}{\sqrt{a\left(\sin^{2}\left(\frac{x}{2}\right)\right)}}$	25

input
```
int(1/(a-cos(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-sin(1/2*x)*arctanh(cos(1/2*x))*2^(1/2)/(a*sin(1/2*x)^2)^(1/2)
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx = \left[\frac{\sqrt{2}\log\left(-\frac{(\cos(x)+3)\sin(x)-\frac{2\sqrt{2}\sqrt{-a\cos(x)+a}(\cos(x)+1)}{\sqrt{a}}}{(\cos(x)-1)\sin(x)}\right)}{2\sqrt{a}}, \sqrt{2}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{-a\cos(x)+a}\sqrt{-\frac{1}{a}}}{\sin(x)}\right)\right]$$

input
```
integrate(1/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*log(-((cos(x) + 3)*sin(x) - 2*sqrt(2)*sqrt(-a*cos(x) + a)*(co
s(x) + 1)/sqrt(a))/((cos(x) - 1)*sin(x)))/sqrt(a), sqrt(2)*sqrt(-1/a)*arct
an(sqrt(2)*sqrt(-a*cos(x) + a)*sqrt(-1/a)/sin(x))]
```

3.178.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx=\int \frac{1}{\sqrt{-a\cos{(x)}+a}}\,dx$$

input
```
integrate(1/(a-a*cos(x))**(1/2),x)
```

output
```
Integral(1/sqrt(-a*cos(x) + a), x)
```

3.178.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(28) = 56.

Time = 0.49 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx=-\frac{\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin{(x)},\cos{(x)}\right)\right)^2+\sin\left(\frac{1}{2}\arctan\left(\sin{(x)},\cos{(x)}\right)\right)^2+2\cos\left(\frac{1}{2}\arctan\left(\sin{(x)}\right.\right.\right.}{}$$

input
```
integrate(1/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(2)*log(cos(1/2*arctan2(sin(x), cos(x)))^2 + sin(1/2*arctan2(sin
(x), cos(x)))^2 + 2*cos(1/2*arctan2(sin(x), cos(x))) + 1) - sqrt(2)*log(co
s(1/2*arctan2(sin(x), cos(x)))^2 + sin(1/2*arctan2(sin(x), cos(x)))^2 - 2*
cos(1/2*arctan2(sin(x), cos(x))) + 1))/sqrt(a)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.54

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx=\frac{\sqrt{2}\log\left(\left|\tan\left(\frac{1}{4}\,x\right)\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\sin\left(\frac{1}{2}\,x\right)\right)}$$

input
```
integrate(1/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*log(abs(tan(1/4*x)))/(sqrt(a)*sgn(sin(1/2*x)))
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{\sqrt{a-a\,\cos\left(x\right)}}\,dx$$

input
```
int(1/(a - a*cos(x))^(1/2),x)
```

output
```
int(1/(a - a*cos(x))^(1/2), x)
```

3.179 $\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$

3.179.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \operatorname{Int}\left(\frac{1}{x\sqrt{a-a\cos(x)}},x\right)$$

output
```
Unintegrable(1/x/(a-a*cos(x))^(1/2),x)
```

3.179.2 Mathematica [N/A]

Not integrable

Time = 4.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[a - a*Cos[x]]),x]
```

output
```
Integrate[1/(x*Sqrt[a - a*Cos[x]]), x]
```

3.179.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{x\sqrt{a-a\sin\left(x+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3807}$$

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}dx$$

input
```
Int[1/(x*Sqrt[a - a*Cos[x]]),x]
```

output
```
$Aborted
```

3.179.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.179.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\sqrt{a-\cos\left(x\right)a}}dx$$

input
```
int(1/x/(a-cos(x)*a)^(1/2),x)
```

output
```
int(1/x/(a-cos(x)*a)^(1/2),x)
```

3.179.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx=\int \frac{1}{\sqrt{-a\cos\left(x\right)+a}x}\,dx$$

input
```
integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-a*cos(x) + a)/(a*x*cos(x) - a*x), x)
```

3.179.6 Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx=\int \frac{1}{x\sqrt{-a\left(\cos\left(x\right)-1\right)}}\,dx$$

input
```
integrate(1/x/(a-a*cos(x))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-a*(cos(x) - 1))), x)
```

3.179.7 Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{\sqrt{-a\cos(x)+ax}}\,dx$$

input
```
integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a*cos(x) + a)*x), x)
```

3.179.8 Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{\sqrt{-a\cos(x)+ax}}\,dx$$

input
```
integrate(1/x/(a-a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a*cos(x) + a)*x), x)
```

3.179.9 Mupad [N/A]

Not integrable

Time = 13.98 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a-a\cos(x)}}\,dx = \int \frac{1}{x\,\sqrt{a-a\,\cos(x)}}\,dx$$

input
```
int(1/(x*(a - a*cos(x))^(1/2)),x)
```

output
```
int(1/(x*(a - a*cos(x))^(1/2)), x)
```

3.180 $\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx$

3.180.1 Optimal result

Integrand size = 14, antiderivative size = 423

$$\begin{aligned}\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = & -\frac{3x^2}{a\sqrt{a+a\cos(x)}}-\frac{24ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}\\ & -\frac{ix^3\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}\\ & +\frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}-\frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}\\ & -\frac{3ix^2\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}-\frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}\\ & +\frac{12x\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}-\frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(4,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}\\ & +\frac{24i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(4,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}+\frac{x^3\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}\end{aligned}$$

output
```
-3*x^2/a/(a+a*cos(x))^(1/2)-24*I*x*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*
cos(x))^(1/2)-I*x^3*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos(x))^(1/2)+2
4*I*cos(1/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+3*I*x^2*cos
(1/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-24*I*cos(1/2*x)*po
lylog(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-3*I*x^2*cos(1/2*x)*polylog(2,
I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-12*x*cos(1/2*x)*polylog(3,-I*exp(1/2*
I*x))/a/(a+a*cos(x))^(1/2)+12*x*cos(1/2*x)*polylog(3,I*exp(1/2*I*x))/a/(a+
a*cos(x))^(1/2)-24*I*cos(1/2*x)*polylog(4,-I*exp(1/2*I*x))/a/(a+a*cos(x))^
(1/2)+24*I*cos(1/2*x)*polylog(4,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+1/2*x
^3*tan(1/2*x)/a/(a+a*cos(x))^(1/2)
```

3.180.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx =$$

$$-\frac{i\cos\left(\frac{x}{2}\right)\left(-6ix^2\cos\left(\frac{x}{2}\right)+48x\arctan\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)+2x^3\arctan\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)-6(8+x^2)\cos^2\left(\frac{x}{2}\right)\mathrm{Pol}\right.}{}$$

input
```
Integrate[x^3/(a + a*Cos[x])^(3/2),x]
```

output
```
((-I)*Cos[x/2]*((-6*I)*x^2*Cos[x/2] + 48*x*ArcTan[E^((I/2)*x)]*Cos[x/2]^2
+ 2*x^3*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 - 6*(8 + x^2)*Cos[x/2]^2*PolyLog[2,
 (-I)*E^((I/2)*x)] + 6*(8 + x^2)*Cos[x/2]^2*PolyLog[2, I*E^((I/2)*x)] - (2
4*I)*x*Cos[x/2]^2*PolyLog[3, (-I)*E^((I/2)*x)] + (24*I)*x*Cos[x/2]^2*PolyL
og[3, I*E^((I/2)*x)] + 48*Cos[x/2]^2*PolyLog[4, (-I)*E^((I/2)*x)] - 48*Cos
[x/2]^2*PolyLog[4, I*E^((I/2)*x)] + I*x^3*Sin[x/2]))/(a*(1 + Cos[x]))^(3/2
)
```

3.180.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.62, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3800, 3042, 4674, 3042, 4669, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.180. $\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx$

$$\int \frac{x^3}{(a\cos(x)+a)^{3/2}}\,dx$$

$\downarrow$ 3042

$$\int \frac{x^3}{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$\downarrow$ 3800

$$\frac{\cos\left(\frac{x}{2}\right)\int x^3\sec^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a\cos(x)+a}}$$

$\downarrow$ 3042

$$\frac{\cos\left(\frac{x}{2}\right)\int x^3\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx}{2a\sqrt{a\cos(x)+a}}$$

$\downarrow$ 4674

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^3\sec\left(\frac{x}{2}\right)dx+12\int x\sec\left(\frac{x}{2}\right)dx+x^3\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-6x^2\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

$\downarrow$ 3042

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^3\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+12\int x\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+x^3\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-6x^2\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

$\downarrow$ 4669

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-6\int x^2\log\left(1-ie^{\frac{ix}{2}}\right)dx+6\int x^2\log\left(1+ie^{\frac{ix}{2}}\right)dx-4ix^3\arctan\left(e^{\frac{ix}{2}}\right)\right)+12\left(-2\int\log\left(1-ie^{\frac{ix}{2}}\right)d\right.\right.}{2a\sqrt{a\cos(x)+a}}$$

$\downarrow$ 2715

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-6\int x^2\log\left(1-ie^{\frac{ix}{2}}\right)dx+6\int x^2\log\left(1+ie^{\frac{ix}{2}}\right)dx-4ix^3\arctan\left(e^{\frac{ix}{2}}\right)\right)+12\left(4i\int e^{-\frac{ix}{2}}\log\left(1-ie^{\frac{i}{2}}\right.\right.\right.}{2a\sqrt{a\cos(x)+a}}$$

$\downarrow$ 2838

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-6\int x^2\log\left(1-ie^{\frac{ix}{2}}\right)dx+6\int x^2\log\left(1+ie^{\frac{ix}{2}}\right)dx-4ix^3\arctan\left(e^{\frac{ix}{2}}\right)\right)+12\left(-4ix\arctan\left(e^{\frac{ix}{2}}\right)+\right.\right.}{2a\sqrt{a\cos(x)+a}}$$

$\downarrow$ 3011

3.180. $\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx$

$$\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(6\left(2ix^2\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-4i\int x\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)dx\right)-6\left(2ix^2\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)-4i\int x\operatorname{Poly}\right.\right.\right.$$

$\downarrow$ 7163

$$\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(6\left(2ix^2\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-4i\left(2i\int\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)dx-2ix\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)\right)\right)-6\left(2ix^2\operatorname{Po}\right.\right.\right.$$

$\downarrow$ 2720

$$\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(6\left(2ix^2\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-4i\left(4\int e^{-\frac{ix}{2}}\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)de^{\frac{ix}{2}}-2ix\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)\right)\right)-6\left(2\right.\right.\right.$$

$\downarrow$ 7143

$$\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-4ix^3\arctan\left(e^{\frac{ix}{2}}\right)+6\left(2ix^2\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-4i\left(4\operatorname{PolyLog}\left(4,-ie^{\frac{ix}{2}}\right)-2ix\operatorname{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right.\right.\right.\right.\right.$$

input
```
Int[x^3/(a + a*Cos[x])^(3/2),x]
```

output
```
(Cos[x/2]*(12*((-4*I)*x*ArcTan[E^((I/2)*x)] + (4*I)*PolyLog[2, (-I)*E^((I/
2)*x)] - (4*I)*PolyLog[2, I*E^((I/2)*x)]) + ((-4*I)*x^3*ArcTan[E^((I/2)*x)
] + 6*((2*I)*x^2*PolyLog[2, (-I)*E^((I/2)*x)] - (4*I)*((-2*I)*x*PolyLog[3,
 (-I)*E^((I/2)*x)] + 4*PolyLog[4, (-I)*E^((I/2)*x)])) - 6*((2*I)*x^2*PolyL
og[2, I*E^((I/2)*x)] - (4*I)*((-2*I)*x*PolyLog[3, I*E^((I/2)*x)] + 4*PolyL
og[4, I*E^((I/2)*x)])))/2 - 6*x^2*Sec[x/2] + x^3*Sec[x/2]*Tan[x/2]))/(2*a*
Sqrt[a + a*Cos[x]])
```

3.180. $\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx$

3.180.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.180. $\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.180.4 Maple [F]

$$\int \frac{x^3}{(a+\cos(x)a)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a+cos(x)*a)^(3/2),x)
```

output
```
int(x^3/(a+cos(x)*a)^(3/2),x)
```

3.180.5 Fricas [F]

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cos(x) + a)*x^3/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

3.180.6 Sympy [F]

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{(a(\cos(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+a*cos(x))**(3/2),x)
```

output
```
Integral(x**3/(a*(cos(x) + 1))**(3/2), x)
```

3.180.7 Maxima [F]

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

output
```
4/9*((6*sqrt(2)*x^2*cos(3/2*x) - (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))
*cos(3*x)^3 + (6*sqrt(2)*x^2*sin(3/2*x) + (3*sqrt(2)*x^3 - 8*sqrt(2)*x)*co
s(3/2*x))*sin(3*x)^3 + 48*sqrt(2)*cos(2*x)^2*cos(3/2*x) + 48*sqrt(2)*cos(3
/2*x)*sin(2*x)^2 + ((2*(9*sqrt(2)*x^2 + 8*sqrt(2))*cos(3/2*x) - 3*(3*sqrt(
2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) + (18*sqrt(2)*x^2 + 2*(9*sqrt(2
)*x^2 + 8*sqrt(2))*cos(x) + 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(x))*cos(3/
2*x) + (3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*cos(3/2*x) + 2*(9*sqrt(2)*x^2 - 8*
sqrt(2))*sin(3/2*x))*sin(2*x) - (9*sqrt(2)*x^3 + 3*(3*sqrt(2)*x^3 - 8*sqrt
(2)*x)*cos(x) - 2*(9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) - 24*sqrt(2)*x)*sin(3
/2*x))*cos(3*x)^2 + 3*((2*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x) - 3*(3*sq
rt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(3*x) + 3*(2*(9*sqrt(2)*x^2 - 8*sq
rt(2))*cos(3/2*x) - 3*(3*sqrt(2)*x^3 - 8*sqrt(2)*x)*sin(3/2*x))*cos(2*x) +
 (18*sqrt(2)*x^2 + 6*(9*sqrt(2)*x^2 - 8*sqrt(2))*cos(x) + 9*(3*sqrt(2)*x^3
 - 8*sqrt(2)*x)*sin(x) - 16*sqrt(2))*cos(3/2*x) + 243*(sqrt(2)*a^2*cos(3*x
)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 + sqrt(2)*a^2*sin(
3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x)*sin(x) + 9*sqr
t(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2*(3*sqrt(2)*a^2*
cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) + 6*(3*sqrt(2)*a^2
*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x) + sqrt(2)*a^2*si
n(x))*sin(3*x))*integrate(1/9*(x^3*cos(4*x)*cos(3/2*x) + 4*x^3*cos(3*x)...
```

3.180.8 Giac [F]

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{(a\cos{(x)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(a*cos(x) + a)^(3/2), x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^3}{{(a+a\,\cos{(x)})}^{3/2}}\,dx$$

input
```
int(x^3/(a + a*cos(x))^(3/2),x)
```

output
```
int(x^3/(a + a*cos(x))^(3/2), x)
```

3.181 $\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx$

3.181.1 Optimal result

Integrand size = 14, antiderivative size = 257

$$\begin{aligned}\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx =& -\frac{2x}{a\sqrt{a+a\cos(x)}}-\frac{ix^2\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}\\&+\frac{4\text{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}}+\frac{2ix\cos\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}\\&-\frac{2ix\cos\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}-\frac{4\cos\left(\frac{x}{2}\right)\text{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}\\&+\frac{4\cos\left(\frac{x}{2}\right)\text{PolyLog}\left(3,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}}+\frac{x^2\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}\end{aligned}$$

output
```
-2*x/a/(a+a*cos(x))^(1/2)-I*x^2*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos
(x))^(1/2)+4*arctanh(sin(1/2*x))*cos(1/2*x)/a/(a+a*cos(x))^(1/2)+2*I*x*cos
(1/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-2*I*x*cos(1/2*x)*p
olylog(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-4*cos(1/2*x)*polylog(3,-I*ex
p(1/2*I*x))/a/(a+a*cos(x))^(1/2)+4*cos(1/2*x)*polylog(3,I*exp(1/2*I*x))/a/
(a+a*cos(x))^(1/2)+1/2*x^2*tan(1/2*x)/a/(a+a*cos(x))^(1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \frac{\cos\left(\frac{x}{2}\right)\left(-4x\cos\left(\frac{x}{2}\right)-2ix^2\arctan\left(e^{\frac{ix}{2}}\right)\cos^2\left(\frac{x}{2}\right)+8\text{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos^2\left(\frac{x}{2}\right)+\right.}{}$$

input
```
Integrate[x^2/(a + a*Cos[x])^(3/2),x]
```

output
```
(Cos[x/2]*(-4*x*Cos[x/2] - (2*I)*x^2*ArcTan[E^((I/2)*x)]*Cos[x/2]^2 + 8*Ar
cTanh[Sin[x/2]]*Cos[x/2]^2 + (4*I)*x*Cos[x/2]^2*PolyLog[2, (-I)*E^((I/2)*x
)] - (4*I)*x*Cos[x/2]^2*PolyLog[2, I*E^((I/2)*x)] - 8*Cos[x/2]^2*PolyLog[3
, (-I)*E^((I/2)*x)] + 8*Cos[x/2]^2*PolyLog[3, I*E^((I/2)*x)] + x^2*Sin[x/2
]))/(a*(1 + Cos[x]))^(3/2)
```

3.181.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.62, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3800, 3042, 4674, 3042, 4257, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a\cos(x)+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x^2}{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\frac{\cos\left(\frac{x}{2}\right)\int x^2\sec^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cos\left(\frac{x}{2}\right)\int x^2\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^2\sec\left(\frac{x}{2}\right)dx+4\int\sec\left(\frac{x}{2}\right)dx+x^2\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-4x\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^2\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+4\int\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+x^2\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-4x\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^2\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)dx+8\text{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)+x^2\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)-4x\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{4669}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-4\int x\log\left(1-ie^{\frac{ix}{2}}\right)dx+4\int x\log\left(1+ie^{\frac{ix}{2}}\right)dx-4ix^2\arctan\left(e^{\frac{ix}{2}}\right)\right)+8\text{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)+x^2\tan\right.}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(4\left(2ix\,\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-2i\int\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)dx\right)-4\left(2ix\,\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)-2i\int\text{PolyLog}\right.\right.\right.}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(4\left(2ix\,\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-4\int e^{-\frac{ix}{2}}\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)de^{\frac{ix}{2}}\right)-4\left(2ix\,\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)-4\int e^{-\frac{ix}{2}}\right.\right.\right.}{2a\sqrt{a\cos(x)}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-4ix^2\arctan\left(e^{\frac{ix}{2}}\right)+4\left(2ix\,\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-4\,\text{PolyLog}\left(3,-ie^{\frac{ix}{2}}\right)\right)-4\left(2ix\,\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)\right.\right.\right.}{2a\sqrt{a\cos(x)+a}}$$

input
```
Int[x^2/(a + a*Cos[x])^(3/2),x]
```

output
```
(Cos[x/2]*(8*ArcTanh[Sin[x/2]] + ((-4*I)*x^2*ArcTan[E^((I/2)*x)] + 4*((2*I
)*x*PolyLog[2, (-I)*E^((I/2)*x)] - 4*PolyLog[3, (-I)*E^((I/2)*x)]) - 4*((2
*I)*x*PolyLog[2, I*E^((I/2)*x)] - 4*PolyLog[3, I*E^((I/2)*x)]))/2 - 4*x*Se
c[x/2] + x^2*Sec[x/2]*Tan[x/2]))/(2*a*Sqrt[a + a*Cos[x]])
```

3.181. $\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx$

3.181.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.181. $\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.181.4 Maple [F]

$$\int \frac{x^2}{(a+\cos(x)\,a)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a+cos(x)*a)^(3/2),x)
```

output
```
int(x^2/(a+cos(x)*a)^(3/2),x)
```

3.181.5 Fricas [F]

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cos(x) + a)*x^2/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

3.181.6 Sympy [F]

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a(\cos(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+a*cos(x))**(3/2),x)
```

output
```
Integral(x**2/(a*(cos(x) + 1))**(3/2), x)
```

3.181.7 Maxima [F]

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

output
```
-1/9*(4*(3*sqrt(2)*x^2*sin(3/2*x) - 4*sqrt(2)*x*cos(3/2*x))*cos(3*x)^3 - 4
*(3*sqrt(2)*x^2*cos(3/2*x) + 4*sqrt(2)*x*sin(3/2*x))*sin(3*x)^3 + 96*sqrt(
2)*cos(2*x)^2*sin(3/2*x) + 96*sqrt(2)*sin(2*x)^2*sin(3/2*x) - 4*((12*sqrt(
2)*x*cos(3/2*x) - (9*sqrt(2)*x^2 + 8*sqrt(2))*sin(3/2*x))*cos(2*x) + (12*s
qrt(2)*x*cos(x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 12*sqrt(2)*x)*cos(3
/2*x) + (12*sqrt(2)*x*sin(3/2*x) + (9*sqrt(2)*x^2 - 8*sqrt(2))*cos(3/2*x))
*sin(2*x) - (9*sqrt(2)*x^2 - 12*sqrt(2)*x*sin(x) + (9*sqrt(2)*x^2 + 8*sqrt
(2))*cos(x))*sin(3/2*x))*cos(3*x)^2 - 12*((12*sqrt(2)*x*cos(3/2*x) - (9*sq
rt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(3*x) + 3*(12*sqrt(2)*x*cos(3/2*x) -
 (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(3/2*x))*cos(2*x) + 3*(12*sqrt(2)*x*cos(x)
 + (9*sqrt(2)*x^2 - 8*sqrt(2))*sin(x) + 4*sqrt(2)*x)*cos(3/2*x) + 243*(sqr
t(2)*a^2*cos(3*x)^2 + 9*sqrt(2)*a^2*cos(2*x)^2 + 9*sqrt(2)*a^2*cos(x)^2 +
sqrt(2)*a^2*sin(3*x)^2 + 9*sqrt(2)*a^2*sin(2*x)^2 + 18*sqrt(2)*a^2*sin(2*x
)*sin(x) + 9*sqrt(2)*a^2*sin(x)^2 + 6*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2 + 2
*(3*sqrt(2)*a^2*cos(2*x) + 3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(3*x) +
6*(3*sqrt(2)*a^2*cos(x) + sqrt(2)*a^2)*cos(2*x) + 6*(sqrt(2)*a^2*sin(2*x)
+ sqrt(2)*a^2*sin(x))*sin(3*x))*integrate(1/9*(x^2*cos(4*x)*cos(3/2*x) + 4
*x^2*cos(3*x)*cos(3/2*x) + 6*x^2*cos(2*x)*cos(3/2*x) + x^2*sin(4*x)*sin(3/
2*x) + 4*x^2*sin(3*x)*sin(3/2*x) + 6*x^2*sin(2*x)*sin(3/2*x) + 4*x^2*sin(3
/2*x)*sin(x) + (4*x^2*cos(x) + x^2)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^...
```

3.181.8 Giac [F]

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(a*cos(x) + a)^(3/2), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x^2}{(a+a\,\cos(x))^{3/2}}\,dx$$

input
```
int(x^2/(a + a*cos(x))^(3/2),x)
```

output
```
int(x^2/(a + a*cos(x))^(3/2), x)
```

3.182 $\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx$

3.182.1 Optimal result

Integrand size = 12, antiderivative size = 150

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = -\frac{1}{a\sqrt{a+a\cos(x)}} - \frac{ix\arctan\left(e^{\frac{ix}{2}}\right)\cos\left(\frac{x}{2}\right)}{a\sqrt{a+a\cos(x)}} + \frac{i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} - \frac{i\cos\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)}{a\sqrt{a+a\cos(x)}} + \frac{x\tan\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cos(x)}}$$

output
```
-1/a/(a+a*cos(x))^(1/2)-I*x*arctan(exp(1/2*I*x))*cos(1/2*x)/a/(a+a*cos(x))
^(1/2)+I*cos(1/2*x)*polylog(2,-I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)-I*cos(
1/2*x)*polylog(2,I*exp(1/2*I*x))/a/(a+a*cos(x))^(1/2)+1/2*x*tan(1/2*x)/a/(
a+a*cos(x))^(1/2)
```

3.182.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.10

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \frac{\sec\left(\frac{x}{2}\right)\left(-4\cos\left(\frac{x}{2}\right)+x\log\left(1-ie^{\frac{ix}{2}}\right)+x\cos(x)\log\left(1-ie^{\frac{ix}{2}}\right)-x\log\left(1+ie^{\frac{ix}{2}}\right.\right.}{}$$

input
```
Integrate[x/(a + a*Cos[x])^(3/2),x]
```

output
```
(Sec[x/2]*(-4*Cos[x/2] + x*Log[1 - I*E^((I/2)*x)] + x*Cos[x]*Log[1 - I*E^(
(I/2)*x)] - x*Log[1 + I*E^((I/2)*x)] - x*Cos[x]*Log[1 + I*E^((I/2)*x)] + (
2*I)*(1 + Cos[x])*PolyLog[2, (-I)*E^((I/2)*x)] - (2*I)*(1 + Cos[x])*PolyLo
g[2, I*E^((I/2)*x)] + 2*x*Sin[x/2]))/(4*a*Sqrt[a*(1 + Cos[x])])
```

3.182.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.69, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 4673, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a\cos(x)+a)^{3/2}}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{x}{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \ 3800$$

$$\frac{\cos\left(\frac{x}{2}\right)\int x\sec^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cos\left(\frac{x}{2}\right)\int x\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)^3dx}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \ 4673$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x\sec\left(\frac{x}{2}\right)dx-2\sec\left(\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x\csc\left(\frac{x}{2}+\frac{\pi}{2}\right)dx-2\sec\left(\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \ 4669$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-2\int\log\left(1-ie^{\frac{ix}{2}}\right)dx+2\int\log\left(1+ie^{\frac{ix}{2}}\right)dx-4ix\arctan\left(e^{\frac{ix}{2}}\right)\right)-2\sec\left(\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(4i\int e^{-\frac{ix}{2}}\log\left(1-ie^{\frac{ix}{2}}\right)de^{\frac{ix}{2}}-4i\int e^{-\frac{ix}{2}}\log\left(1+ie^{\frac{ix}{2}}\right)de^{\frac{ix}{2}}-4ix\arctan\left(e^{\frac{ix}{2}}\right)\right)-2\sec\left(\frac{x}{2}\right)+x\tan\left(\right.\right.}{2a\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\cos\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-4ix\arctan\left(e^{\frac{ix}{2}}\right)+4i\,\text{PolyLog}\left(2,-ie^{\frac{ix}{2}}\right)-4i\,\text{PolyLog}\left(2,ie^{\frac{ix}{2}}\right)\right)-2\sec\left(\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cos(x)+a}}$$

input
```
Int[x/(a + a*Cos[x])^(3/2),x]
```

output
```
(Cos[x/2]*(((-4*I)*x*ArcTan[E^((I/2)*x)] + (4*I)*PolyLog[2, (-I)*E^((I/2)*
x)] - (4*I)*PolyLog[2, I*E^((I/2)*x)])/2 - 2*Sec[x/2] + x*Sec[x/2]*Tan[x/2
]))/(2*a*Sqrt[a + a*Cos[x]])
```

3.182.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

3.182.4 Maple [F]

$$\int \frac{x}{(a+\cos(x)\,a)^{\frac{3}{2}}}dx$$

input
```
int(x/(a+cos(x)*a)^(3/2),x)
```

output
```
int(x/(a+cos(x)*a)^(3/2),x)
```

3.182.5 Fricas [F]

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx=\int \frac{x}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cos(x) + a)*x/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2), x)
```

3.182.6 Sympy [F]

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a(\cos(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*cos(x))**(3/2),x)
```

output
```
Integral(x/(a*(cos(x) + 1))**(3/2), x)
```

3.182.7 Maxima [F]

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

output
```
1/3*(8*x*cos(3/2*x)*sin(3*x)^3 - 8*x*cos(3*x)^3*sin(3/2*x) - 8*((3*x*sin(3
/2*x) + 2*cos(3/2*x))*cos(2*x) - (3*x*sin(x) - 2*cos(x))*cos(3/2*x) - (3*x
*cos(3/2*x) + 2*sin(3/2*x))*sin(2*x) + (3*x*cos(x) + 3*x - 2*sin(x))*sin(3
/2*x))*cos(3*x)^2 - 48*cos(2*x)^2*cos(3/2*x) - 24*((3*x*sin(3/2*x) - 2*cos
(3/2*x))*cos(3*x) + 3*(3*x*sin(3/2*x) - 2*cos(3/2*x))*cos(2*x) - (9*x*sin(
x) + 6*cos(x) + 2)*cos(3/2*x) - 27*(a^2*cos(3*x)^2 + 9*a^2*cos(2*x)^2 + 9*
a^2*cos(x)^2 + a^2*sin(3*x)^2 + 9*a^2*sin(2*x)^2 + 18*a^2*sin(2*x)*sin(x)
+ 9*a^2*sin(x)^2 + 6*a^2*cos(x) + a^2 + 2*(3*a^2*cos(2*x) + 3*a^2*cos(x) +
 a^2)*cos(3*x) + 6*(3*a^2*cos(x) + a^2)*cos(2*x) + 6*(a^2*sin(2*x) + a^2*s
in(x))*sin(3*x))*integrate(1/3*(x*cos(4*x)*cos(3/2*x) + 4*x*cos(3*x)*cos(3
/2*x) + 6*x*cos(2*x)*cos(3/2*x) + x*sin(4*x)*sin(3/2*x) + 4*x*sin(3*x)*sin
(3/2*x) + 6*x*sin(2*x)*sin(3/2*x) + 4*x*sin(3/2*x)*sin(x) + (4*x*cos(x) +
x)*cos(3/2*x))/(a^2*cos(4*x)^2 + 16*a^2*cos(3*x)^2 + 36*a^2*cos(2*x)^2 + 1
6*a^2*cos(x)^2 + a^2*sin(4*x)^2 + 16*a^2*sin(3*x)^2 + 36*a^2*sin(2*x)^2 +
48*a^2*sin(2*x)*sin(x) + 16*a^2*sin(x)^2 + 8*a^2*cos(x) + a^2 + 2*(4*a^2*c
os(3*x) + 6*a^2*cos(2*x) + 4*a^2*cos(x) + a^2)*cos(4*x) + 8*(6*a^2*cos(2*x
) + 4*a^2*cos(x) + a^2)*cos(3*x) + 12*(4*a^2*cos(x) + a^2)*cos(2*x) + 4*(2
*a^2*sin(3*x) + 3*a^2*sin(2*x) + 2*a^2*sin(x))*sin(4*x) + 16*(3*a^2*sin(2*
x) + 2*a^2*sin(x))*sin(3*x)), x) - (3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(3*x
) - 3*(3*x*cos(3/2*x) + 2*sin(3/2*x))*sin(2*x) + 3*(3*x*cos(x) + x - 2*...
```

3.182.8 Giac [F]

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a\cos(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(a*cos(x) + a)^(3/2), x)
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{x}{(a+a\,\cos(x))^{3/2}}\,dx$$

input
```
int(x/(a + a*cos(x))^(3/2),x)
```

output
```
int(x/(a + a*cos(x))^(3/2), x)
```

3.183 $\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$

3.183.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x(a+a\cos(x))^{3/2}},x\right)$$

output
```
Unintegrable(1/x/(a+a*cos(x))^(3/2),x)
```

3.183.2 Mathematica [N/A]

Not integrable

Time = 11.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx$$

input
```
Integrate[1/(x*(a + a*Cos[x])^(3/2)),x]
```

output
```
Integrate[1/(x*(a + a*Cos[x])^(3/2)), x]
```

3.183.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a\cos(x)+a)^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{x\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \; 3807$$

$$\int \frac{1}{x(a\cos(x)+a)^{3/2}}dx$$

input
```
Int[1/(x*(a + a*Cos[x])^(3/2)),x]
```

output
```
$Aborted
```

3.183.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.183.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+\cos \left(x\right) a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+cos(x)*a)^(3/2),x)
```

output
```
int(1/x/(a+cos(x)*a)^(3/2),x)
```

3.183.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx=\int \frac{1}{\left(a\cos\left(x\right)+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cos(x) + a)/(a^2*x*cos(x)^2 + 2*a^2*x*cos(x) + a^2*x), x)
```

3.183.6 Sympy [N/A]

Not integrable

Time = 9.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx=\int \frac{1}{x\left(a\left(\cos\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+a*cos(x))**(3/2),x)
```

output
```
Integral(1/(x*(a*(cos(x) + 1))**(3/2)), x)
```

3.183.7 Maxima [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{(a\cos(x)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cos(x) + a)^(3/2)*x), x)
```

3.183.8 Giac [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{(a\cos(x)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cos(x) + a)^(3/2)*x), x)
```

3.183.9 Mupad [N/A]

Not integrable

Time = 14.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cos(x))^{3/2}}\,dx = \int \frac{1}{x\,(a+a\,\cos(x))^{3/2}}\,dx$$

input
```
int(1/(x*(a + a*cos(x))^(3/2)),x)
```

output
```
int(1/(x*(a + a*cos(x))^(3/2)), x)
```

3.184 $\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$

3.184.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \text{Int}\left(\frac{\sqrt[3]{a+a\cos(c+dx)}}{x},x\right)$$

output
```
Unintegrable((a+a*cos(d*x+c))^(1/3)/x,x)
```

3.184.2 Mathematica [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx$$

input
```
Integrate[(a + a*Cos[c + d*x])^(1/3)/x,x]
```

output
```
Integrate[(a + a*Cos[c + d*x])^(1/3)/x, x]
```

3.184.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a\cos(c+dx)+a}}{x}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{a\sin\left(c+dx+\frac{\pi}{2}\right)+a}}{x}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{\sqrt[3]{a\cos(c+dx)+a}}{x}dx$$

input
```
Int[(a + a*Cos[c + d*x])^(1/3)/x,x]
```

output
```
$Aborted
```

3.184.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.184.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+\cos(dx+c)\,a)^{\frac{1}{3}}}{x}dx$$

input
```
int((a+cos(d*x+c)*a)^(1/3)/x,x)
```

output
```
int((a+cos(d*x+c)*a)^(1/3)/x,x)
```

3.184.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.184.6 Sympy [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a\,(\cos(c+dx)+1)}}{x}\,dx$$

input
```
integrate((a+a*cos(d*x+c))**(1/3)/x,x)
```

output
```
Integral((a*(cos(c + d*x) + 1))**(1/3)/x, x)
```

3.184.7 Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{(a\cos(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + a)^(1/3)/x, x)
```

3.184.8 Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{(a\cos(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+a*cos(d*x+c))^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + a)^(1/3)/x, x)
```

3.184.9 Mupad [N/A]

Not integrable

Time = 14.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cos(c+dx)}}{x}\,dx = \int \frac{(a+a\,\cos(c+d\,x))^{1/3}}{x}\,dx$$

input
```
int((a + a*cos(c + d*x))^(1/3)/x,x)
```

output
```
int((a + a*cos(c + d*x))^(1/3)/x, x)
```

3.185 $\int \frac{x^3}{a+b\cos(x)}\,dx$

3.185.1 Optimal result

Integrand size = 12, antiderivative size = 383

$$\begin{aligned}\int \frac{x^3}{a+b\cos(x)}\,dx = &-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &-\frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &-\frac{6ix\operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{6ix\operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\\ &+\frac{6\operatorname{PolyLog}\left(4,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{6\operatorname{PolyLog}\left(4,-\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

output
```
-I*x^3*ln(1+b*exp(I*x)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)+I*x^3*ln(1+b*e
xp(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)-3*x^2*polylog(2,-b*exp(I*x)/(
a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)+3*x^2*polylog(2,-b*exp(I*x)/(a+(a^2-b^
2)^(1/2)))/(a^2-b^2)^(1/2)-6*I*x*polylog(3,-b*exp(I*x)/(a-(a^2-b^2)^(1/2))
)/(a^2-b^2)^(1/2)+6*I*x*polylog(3,-b*exp(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^
2)^(1/2)+6*polylog(4,-b*exp(I*x)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)-6*po
lylog(4,-b*exp(I*x)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)
```

3.185.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \frac{-ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)+ix^3\log\left(1+\frac{be^{ix}}{a+\sqrt{a^2-b^2}}\right)-3x^2\operatorname{PolyLog}\left(2,\frac{be^{ix}}{-a+\sqrt{a^2-b^2}}\right)+3x^2\operatorname{PolyLog}\left(2,-\frac{}{a+}\right.}{}$$

input
```
Integrate[x^3/(a + b*Cos[x]),x]
```

output
```
((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])] + I*x^3*Log[1 + (b*E^
(I*x))/(a + Sqrt[a^2 - b^2])] - 3*x^2*PolyLog[2, (b*E^(I*x))/(-a + Sqrt[a^
2 - b^2])] + 3*x^2*PolyLog[2, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))] - (6*I
)*x*PolyLog[3, (b*E^(I*x))/(-a + Sqrt[a^2 - b^2])] + (6*I)*x*PolyLog[3, -(
(b*E^(I*x))/(a + Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(I*x))/(-a + Sqrt[
a^2 - b^2])] - 6*PolyLog[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])/Sqrt[a^
2 - b^2]
```

3.185.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3802, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^3}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3802}$$

$$2\int \frac{e^{ix}x^3}{2e^{ix}a+be^{2ix}+b}dx$$

$$\Big\downarrow \text{2694}$$

$$2\left(\frac{b\int \frac{e^{ix}x^3}{2\left(a+be^{ix}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{ix}x^3}{2\left(a+be^{ix}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 27

$$2\left(\frac{b\int \frac{e^{ix}x^3}{a+be^{ix}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{ix}x^3}{a+be^{ix}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 2620

$$2\left(\frac{b\left(\frac{3i\int x^2\log\left(\frac{e^{ix}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{b}-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{b}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{3i\int x^2\log\left(\frac{e^{ix}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{b}-\frac{ix^3\log\left(1+\frac{be^{ix}}{\sqrt{a^2-b^2}+a}\right)}{b}\right)}{2\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 3011

$$2\left(\frac{b\left(\frac{3i\left(ix^2\operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)-2i\int x\operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)dx\right)}{b}-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{b}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{3i\left(ix^2\operatorname{PolyLog}\left(2,-\frac{}{a+}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 7163

$$2\left(\frac{b\left(\frac{3i\left(ix^2\operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)dx-ix\operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)\right)\right)}{b}-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{b}\right)}{2\sqrt{a^2-b^2}}\right.$$

$\downarrow$ 2720

$$2\left(\frac{b\left(\frac{3i\left(ix^2\operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)-2i\left(\int e^{-ix}\operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)de^{ix}-ix\operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)\right)\right)}{b}-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b}}\right.}{b}\right.}{2\sqrt{a^2-b^2}}\right.$$

$\downarrow$ 7143

3.185. $\int \frac{x^3}{a+b\cos(x)}\,dx$

$$2\left(\frac{b\left(\frac{3i\left(ix^2\operatorname{PolyLog}\left(2,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)-2i\left(\operatorname{PolyLog}\left(4,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)-ix\operatorname{PolyLog}\left(3,-\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)\right)\right)}{b}-\frac{ix^3\log\left(1+\frac{be^{ix}}{a-\sqrt{a^2-b^2}}\right)}{b}\right)}{2\sqrt{a^2-b^2}}-\cdots\right.$$

input
```
Int[x^3/(a + b*Cos[x]),x]
```

output
```
2*((b*(((-I)*x^3*Log[1 + (b*E^(I*x))/(a - Sqrt[a^2 - b^2])])/b + ((3*I)*(I
*x^2*PolyLog[2, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))] - (2*I)*((-I)*x*Poly
Log[3, -((b*E^(I*x))/(a - Sqrt[a^2 - b^2]))] + PolyLog[4, -((b*E^(I*x))/(a
 - Sqrt[a^2 - b^2]))])))/b))/(2*Sqrt[a^2 - b^2]) - (b*(((-I)*x^3*Log[1 + (
b*E^(I*x))/(a + Sqrt[a^2 - b^2])])/b + ((3*I)*(I*x^2*PolyLog[2, -((b*E^(I*
x))/(a + Sqrt[a^2 - b^2]))] - (2*I)*((-I)*x*PolyLog[3, -((b*E^(I*x))/(a +
Sqrt[a^2 - b^2]))] + PolyLog[4, -((b*E^(I*x))/(a + Sqrt[a^2 - b^2]))])))/b
))/(2*Sqrt[a^2 - b^2]))
```

3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.185. $\int \frac{x^3}{a+b\cos(x)}\,dx$

3.185.4 Maple [F]

$$\int \frac{x^3}{a+\cos(x)\,b}dx$$

input
```
int(x^3/(a+cos(x)*b),x)
```

output
```
int(x^3/(a+cos(x)*b),x)
```

3.185.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1030 vs. $2(317) = 634$.

Time = 0.36 (sec) , antiderivative size = 1030, normalized size of antiderivative = 2.69

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
1/2*(-I*b*x^3*sqrt((a^2 - b^2)/b^2)*log((a*cos(x) + I*a*sin(x) + (b*cos(x)
 + I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b) + I*b*x^3*sqrt((a^2 - b^2)/b^
2)*log((a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b
^2) + b)/b) + I*b*x^3*sqrt((a^2 - b^2)/b^2)*log((a*cos(x) - I*a*sin(x) + (
b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b) - I*b*x^3*sqrt((a^2 -
 b^2)/b^2)*log((a*cos(x) - I*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt((a^2
- b^2)/b^2) + b)/b) - 3*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(x) + I*a
*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 3*b*
x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*
sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*b*x^2*sqrt((a^2 - b^2)/b^2)*
dilog(-(a*cos(x) - I*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b
^2) + b)/b + 1) + 3*b*x^2*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(x) - I*a*sin
(x) - (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 6*I*b*x*
sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x) + I*a*sin(x) + (b*cos(x) + I*b
*sin(x))*sqrt((a^2 - b^2)/b^2))/b) + 6*I*b*x*sqrt((a^2 - b^2)/b^2)*polylog
(3, -(a*cos(x) + I*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt((a^2 - b^2)/b^2
))/b) + 6*I*b*x*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cos(x) - I*a*sin(x) +
 (b*cos(x) - I*b*sin(x))*sqrt((a^2 - b^2)/b^2))/b) - 6*I*b*x*sqrt((a^2 - b
^2)/b^2)*polylog(3, -(a*cos(x) - I*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt
((a^2 - b^2)/b^2))/b) + 6*b*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cos(x)...
```

3.185.6 Sympy [F]

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \int \frac{x^3}{a+b\cos{(x)}}\,dx$$

input
```
integrate(x**3/(a+b*cos(x)),x)
```

output
```
Integral(x**3/(a + b*cos(x)), x)
```

3.185.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.185.8 Giac [F]

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \int \frac{x^3}{b\cos{(x)}+a}\,dx$$

input
```
integrate(x^3/(a+b*cos(x)),x, algorithm="giac")
```

output
```
integrate(x^3/(b*cos(x) + a), x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\cos(x)}\,dx = \int \frac{x^3}{a+b\,\cos\left(x\right)}\,dx$$

input
```
int(x^3/(a + b*cos(x)),x)
```

output
```
int(x^3/(a + b*cos(x)), x)
```

3.186 $\int \frac{x^2}{a+b\cos(c+dx)}\,dx$

3.186.1 Optimal result

Integrand size = 16, antiderivative size = 329

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx = -\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}+\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d} - \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}+\frac{2x\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2} - \frac{2i\,\text{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3}+\frac{2i\,\text{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^3}$$

output
```
-I*x^2*ln(1+b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)+I*x^2*
ln(1+b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)-2*x*polylog(2
,-b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)+2*x*polylog(2,
-b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)-2*I*polylog(3,-
b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2)+2*I*polylog(3,-b
*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d^3/(a^2-b^2)^(1/2)
```

3.186.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx = \frac{-2dx\,\text{PolyLog}\left(2,\frac{be^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)-i\left(d^2x^2\log\left(1-\frac{be^{i(c+dx)}}{-a+\sqrt{a^2-b^2}}\right)-d^2x^2\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)+2idx\,\text{PolyLog}\right.}{\sqrt{a^2-b^2}d^3}$$

input
```
Integrate[x^2/(a + b*Cos[c + d*x]),x]
```

output
```
(-2*d*x*PolyLog[2, (b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - I*(d^2*x^
2*Log[1 - (b*E^(I*(c + d*x)))/(-a + Sqrt[a^2 - b^2])] - d^2*x^2*Log[1 + (b
*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2])] + (2*I)*d*x*PolyLog[2, -((b*E^(I*
(c + d*x)))/(a + Sqrt[a^2 - b^2]))] + 2*PolyLog[3, (b*E^(I*(c + d*x)))/(-a
 + Sqrt[a^2 - b^2])] - 2*PolyLog[3, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 -
b^2]))]))/(Sqrt[a^2 - b^2]*d^3)
```

3.186.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3802, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\cos(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{x^2}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3802$$

$$2\int \frac{e^{i(c+dx)}x^2}{2e^{i(c+dx)}a+be^{2i(c+dx)}+b}dx$$

$$\Big\downarrow \; 2694$$

$$2\left(\frac{b\int \frac{e^{i(c+dx)}x^2}{2\left(a+be^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{i(c+dx)}x^2}{2\left(a+be^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 27

$$2\left(\frac{b\int \frac{e^{i(c+dx)}x^2}{a+be^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{i(c+dx)}x^2}{a+be^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 2620

$$2\left(\frac{b\left(\frac{2i\int x\log\left(\frac{e^{i(c+dx)}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{2i\int x\log\left(\frac{e^{i(c+dx)}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 3011

$$2\left(\frac{b\left(\frac{2i\left(\frac{ix\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{i\int \text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{bd}-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{2i\left(\frac{ix\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}\cdots\right)}{\cdots}\right)}{\cdots}\right.$$

$\downarrow$ 2720

$$2\left(\frac{b\left(\frac{2i\left(\frac{ix\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-i(c+dx)}\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)de^{i(c+dx)}}{d^2}\right)}{bd}-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{2i\left(\frac{ix\,\text{PolyLog}(\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right.$$

$\downarrow$ 7143

3.186. $\int \frac{x^2}{a+b\cos(c+dx)}\,dx$

$$2\left(\frac{b\left(\frac{2i\left(\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\mathrm{PolyLog}\left(3,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{bd}-\frac{ix^2\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{2i\left(\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\mathrm{Poly}}{}\right.}{bd}\right.}{}\right.$$

input
```
Int[x^2/(a + b*Cos[c + d*x]),x]
```

output
```
2*((b*(((-I)*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d)
 + ((2*I)*((I*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))])/
d - PolyLog[3, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))]/d^2))/(b*d)))
/(2*Sqrt[a^2 - b^2]) - (b*(((-I)*x^2*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt
[a^2 - b^2])])/(b*d) + ((2*I)*((I*x*PolyLog[2, -((b*E^(I*(c + d*x)))/(a +
Sqrt[a^2 - b^2]))])/d - PolyLog[3, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b
^2]))]/d^2))/(b*d)))/(2*Sqrt[a^2 - b^2]))
```

3.186.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.186.4 Maple [F]

$$\int \frac{x^2}{a+\cos(dx+c)\,b}dx$$

input
```
int(x^2/(a+cos(d*x+c)*b),x)
```

output
```
int(x^2/(a+cos(d*x+c)*b),x)
```

3.186.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(283) = 566$.

Time = 0.43 (sec) , antiderivative size = 1263, normalized size of antiderivative = 3.84

$$\int \frac{x^2}{a + b\cos(c + dx)}\, dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x +
 c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1
) - 2*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c
) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1)
+ 2*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c)
+ (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) -
2*b*d*x*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c) -
(b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - I*
b*c^2*sqrt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*
b*sqrt((a^2 - b^2)/b^2) + 2*a) + I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(2*b*cos
(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - I*b*c^
2*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b*s
qrt((a^2 - b^2)/b^2) - 2*a) + I*b*c^2*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d
*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) + (I*b*d^2
*x^2 - I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x +
c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + (
-I*b*d^2*x^2 + I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*si
n(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b
)/b) + (-I*b*d^2*x^2 + I*b*c^2)*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c)
- I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^...
```

3.186.6 Sympy [F]

$$\int \frac{x^2}{a + b\cos(c + dx)}\, dx = \int \frac{x^2}{a + b\cos(c + dx)}\, dx$$

input
```
integrate(x**2/(a+b*cos(d*x+c)),x)
```

output
```
Integral(x**2/(a + b*cos(c + d*x)), x)
```

3.186.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + b\cos(c + dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.186.8 Giac [F]

$$\int \frac{x^2}{a + b\cos(c + dx)}\,dx = \int \frac{x^2}{b\cos(dx + c) + a}\,dx$$

input
```
integrate(x^2/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2/(b*cos(d*x + c) + a), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\cos(c + dx)}\,dx = \int \frac{x^2}{a + b\cos(c + d\,x)}\,dx$$

input
```
int(x^2/(a + b*cos(c + d*x)),x)
```

output
```
int(x^2/(a + b*cos(c + d*x)), x)
```

3.187 $\int \frac{x}{a+b\cos(c+dx)}\,dx$

3.187.1 Optimal result

Integrand size = 14, antiderivative size = 214

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = -\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}+\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}-\frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d^2}$$

output
```
-I*x*ln(1+b*exp(I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)+I*x*ln(1
+b*exp(I*(d*x+c))/(a+(a^2-b^2)^(1/2)))/d/(a^2-b^2)^(1/2)-polylog(2,-b*exp(
I*(d*x+c))/(a-(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)+polylog(2,-b*exp(I*(d*
x+c))/(a+(a^2-b^2)^(1/2)))/d^2/(a^2-b^2)^(1/2)
```

3.187.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 756 vs. 2(214) = 428.

Time = 0.94 (sec) , antiderivative size = 756, normalized size of antiderivative = 3.53

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = \frac{2(c+dx)\text{arctanh}\left(\frac{(a+b)\cot\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)-2\left(c+\arccos\left(-\frac{a}{b}\right)\right)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)+\left(\arccos\left(-\frac{a}{b}\right)-\right.}{}$$

input
```
Integrate[x/(a + b*Cos[c + d*x]),x]
```

output
```
(2*(c + d*x)*ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]] - 2*(c +
 ArcCos[-(a/b)])*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]] + (
ArcCos[-(a/b)] - (2*I)*ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2 + b^2]
] + (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*Log[Sqrt[
-a^2 + b^2]/(Sqrt[2]*Sqrt[b]*E^((I/2)*(c + d*x))*Sqrt[a + b*Cos[c + d*x]])
] + (ArcCos[-(a/b)] + (2*I)*(ArcTanh[((a + b)*Cot[(c + d*x)/2])/Sqrt[-a^2
+ b^2]] - ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]))*Log[(Sqr
t[-a^2 + b^2]*E^((I/2)*(c + d*x)))/(Sqrt[2]*Sqrt[b]*Sqrt[a + b*Cos[c + d*x
]])] - (ArcCos[-(a/b)] - (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a
^2 + b^2]])*Log[((a + b)*(-a + b - I*Sqrt[-a^2 + b^2])*(1 + I*Tan[(c + d*x
)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2]))] - (ArcCos[-(a/b)]
+ (2*I)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*Log[((a + b
)*(I*a - I*b + Sqrt[-a^2 + b^2])*(I + Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[
-a^2 + b^2]*Tan[(c + d*x)/2]))] + I*(PolyLog[2, ((a - I*Sqrt[-a^2 + b^2])*
(a + b - Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*
Tan[(c + d*x)/2]))] - PolyLog[2, ((a + I*Sqrt[-a^2 + b^2])*(a + b - Sqrt[-
a^2 + b^2]*Tan[(c + d*x)/2]))/(b*(a + b + Sqrt[-a^2 + b^2]*Tan[(c + d*x)/2
]))]))/(Sqrt[-a^2 + b^2]*d^2)
```

3.187.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3802, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\cos(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3802}$$

$$2\int \frac{e^{i(c+dx)}x}{2e^{i(c+dx)}a+be^{2i(c+dx)}+b}dx$$

$$\Big\downarrow \text{2694}$$

$$2\left(\frac{b\int \frac{e^{i(c+dx)}x}{2\left(a+be^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{i(c+dx)}x}{2\left(a+be^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 27

$$2\left(\frac{b\int \frac{e^{i(c+dx)}x}{a+be^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{i(c+dx)}x}{a+be^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 2620

$$2\left(\frac{b\left(\frac{i\int \log\left(\frac{e^{i(c+dx)}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{i\int \log\left(\frac{e^{i(c+dx)}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 2715

$$2\left(\frac{b\left(\frac{\int e^{-i(c+dx)}\log\left(\frac{e^{i(c+dx)}b}{a-\sqrt{a^2-b^2}}+1\right)de^{i(c+dx)}}{bd^2}-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{\int e^{-i(c+dx)}\log\left(\frac{e^{i(c+dx)}b}{a+\sqrt{a^2-b^2}}+1\right)de^{i(c+dx)}}{bd^2}-\frac{ix\log\left(1+\cdots\right.}{\cdots}\right.}{2\sqrt{a^2-b^2}}\right.$$

$\downarrow$ 2838

$$2\left(\frac{b\left(-\frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(-\frac{\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{ix\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\right)$$

input
```
Int[x/(a + b*Cos[c + d*x]),x]
```

output
```
2*((b*(((-I)*x*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) -
 PolyLog[2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))]/(b*d^2)))/(2*Sqr
t[a^2 - b^2]) - (b*(((-I)*x*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(b*d) - PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))]/(b*
d^2)))/(2*Sqrt[a^2 - b^2]))
```

3.187. $\int \frac{x}{a+b\cos(c+dx)}\,dx$

3.187.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.187.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(188) = 376$.

Time = 0.84 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.93

method	result
risch	$-\frac{i \ln\left(\frac{-b\,e^{i(dx+c)}+\sqrt{a^2-b^2}-a}{\sqrt{a^2-b^2}-a}\right) x}{d\sqrt{a^2-b^2}}+\frac{i \ln\left(\frac{b\,e^{i(dx+c)}+\sqrt{a^2-b^2}+a}{\sqrt{a^2-b^2}+a}\right) x}{d\sqrt{a^2-b^2}}-\frac{i \ln\left(\frac{-b\,e^{i(dx+c)}+\sqrt{a^2-b^2}-a}{\sqrt{a^2-b^2}-a}\right) c}{d^2\sqrt{a^2-b^2}}+\frac{i \ln\left(\frac{b\,e^{i(dx+c)}+\sqrt{a^2-}\cdots}{\sqrt{a^2-b^2}+a}\right.}{d^2\sqrt{a^2-b^2}}$

input
```
int(x/(a+cos(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
-I/d/(a^2-b^2)^(1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/((a^2-b^2)^(
1/2)-a))*x+I/d/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/((a
^2-b^2)^(1/2)+a))*x-I/d^2/(a^2-b^2)^(1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^
(1/2)-a)/((a^2-b^2)^(1/2)-a))*c+I/d^2/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))
+(a^2-b^2)^(1/2)+a)/((a^2-b^2)^(1/2)+a))*c-1/d^2/(a^2-b^2)^(1/2)*dilog((-b
*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/((a^2-b^2)^(1/2)-a))+1/d^2/(a^2-b^2)^(1
/2)*dilog((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/((a^2-b^2)^(1/2)+a))+2*I/d^
2*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))
```

3.187.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(184) = 368$.

Time = 0.42 (sec) , antiderivative size = 915, normalized size of antiderivative = 4.28

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = \frac{-i\,bc\sqrt{\frac{a^2-b^2}{b^2}}\log\left(2\,b\cos\left(dx+c\right)+2i\,b\sin\left(dx+c\right)+2\,b\sqrt{\frac{a^2-b^2}{b^2}}+2\,a\right)+i\,bc\sqrt{\frac{a^2-b^2}{b^2}}\log\left(2\,b\cos\left(dx+c\right.\right.\cdots}{\cdots}$$

input
```
integrate(x/(a+b*cos(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(-I*b*c*sqrt((a^2 - b^2)/b^2)*log(2*b*cos(d*x + c) + 2*I*b*sin(d*x + c
) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + I*b*c*sqrt((a^2 - b^2)/b^2)*log(2*b
*cos(d*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - I*
b*c*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d*x + c) + 2*I*b*sin(d*x + c) + 2*b
*sqrt((a^2 - b^2)/b^2) - 2*a) + I*b*c*sqrt((a^2 - b^2)/b^2)*log(-2*b*cos(d
*x + c) - 2*I*b*sin(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) - 2*a) - b*sqrt((
a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x + c
) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + b*sqrt((a^2 - b^
2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*b*
sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - b*sqrt((a^2 - b^2)/b^2)*
dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x
+ c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + b*sqrt((a^2 - b^2)/b^2)*dilog(-(
a*cos(d*x + c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sq
rt((a^2 - b^2)/b^2) + b)/b + 1) - (I*b*d*x + I*b*c)*sqrt((a^2 - b^2)/b^2)*
log((a*cos(d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c
))*sqrt((a^2 - b^2)/b^2) + b)/b) - (-I*b*d*x - I*b*c)*sqrt((a^2 - b^2)/b^2
)*log((a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*b*sin(d*x +
 c))*sqrt((a^2 - b^2)/b^2) + b)/b) - (-I*b*d*x - I*b*c)*sqrt((a^2 - b^2)/b
^2)*log((a*cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c) - I*b*sin(d*x
 + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - (I*b*d*x + I*b*c)*sqrt((a^2 - b^...
```

3.187.6 Sympy [F]

$$\int \frac{x}{a + b\cos(c + dx)}\, dx = \int \frac{x}{a + b\cos{(c + dx)}}\, dx$$

input
```
integrate(x/(a+b*cos(d*x+c)),x)
```

output
```
Integral(x/(a + b*cos(c + d*x)), x)
```

3.187.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+b*cos(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.187.8 Giac [F]

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = \int \frac{x}{b\cos(dx+c)+a}\,dx$$

input
```
integrate(x/(a+b*cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x/(b*cos(d*x + c) + a), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\cos(c+dx)}\,dx = \int \frac{x}{a+b\cos(c+d\,x)}\,dx$$

input
```
int(x/(a + b*cos(c + d*x)),x)
```

output
```
int(x/(a + b*cos(c + d*x)), x)
```

3.188 $\int \frac{1}{x(a+b\cos(x))}\,dx$

3.188.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \text{Int}\left(\frac{1}{x(a+b\cos(x))},x\right)$$

output
```
Unintegrable(1/x/(a+b*cos(x)),x)
```

3.188.2 Mathematica [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{x(a+b\cos(x))}\,dx$$

input
```
Integrate[1/(x*(a + b*Cos[x])),x]
```

output
```
Integrate[1/(x*(a + b*Cos[x])), x]
```

3.188.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\cos(x))}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x\left(a+b\sin\left(x+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x(a+b\cos(x))}dx$$

input
```
Int[1/(x*(a + b*Cos[x])),x]
```

output
```
$Aborted
```

3.188.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

3.188.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+\cos \left(x\right) b\right)}dx$$

input
```
int(1/x/(a+cos(x)*b),x)
```

output
```
int(1/x/(a+cos(x)*b),x)
```

3.188.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{\left(b\cos\left(x\right)+a\right)x}\,dx$$

input
```
integrate(1/x/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*cos(x) + a*x), x)
```

3.188.6 Sympy [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{x\left(a+b\cos\left(x\right)\right)}\,dx$$

input
```
integrate(1/x/(a+b*cos(x)),x)
```

output
```
Integral(1/(x*(a + b*cos(x))), x)
```

3.188.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{(b\cos{(x)}+a)x}\,dx$$

input
```
integrate(1/x/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(x) + a)*x), x)
```

3.188.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{(b\cos{(x)}+a)x}\,dx$$

input
```
integrate(1/x/(a+b*cos(x)),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(x) + a)*x), x)
```

3.188.9 Mupad [N/A]

Not integrable

Time = 14.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x(a+b\cos(x))}\,dx = \int \frac{1}{x\,\left(a+b\,\cos{(x)}\right)}\,dx$$

input
```
int(1/(x*(a + b*cos(x))),x)
```

output
```
int(1/(x*(a + b*cos(x))), x)
```

3.189 $\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx$

3.189.1 Optimal result

Integrand size = 18, antiderivative size = 296

$$\begin{aligned}\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = &-\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} \\ &+\frac{ia(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} \\ &-\frac{f\log(a+b\cos(c+dx))}{(a^2-b^2)\,d^2}-\frac{af\operatorname{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2} \\ &+\frac{af\operatorname{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d^2}-\frac{b(e+fx)\sin(c+dx)}{(a^2-b^2)\,d(a+b\cos(c+dx))}\end{aligned}$$

output
```
-f*ln(a+b*cos(d*x+c))/(a^2-b^2)/d^2-I*a*(f*x+e)*ln(1+b*exp(I*(d*x+c))/(a-(
a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d+I*a*(f*x+e)*ln(1+b*exp(I*(d*x+c))/(a+(a
^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/d-a*f*polylog(2,-b*exp(I*(d*x+c))/(a-(a^2-
b^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2+a*f*polylog(2,-b*exp(I*(d*x+c))/(a+(a^2-b
^2)^(1/2)))/(a^2-b^2)^(3/2)/d^2-b*(f*x+e)*sin(d*x+c)/(a^2-b^2)/d/(a+b*cos(
d*x+c))
```

3.189.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 933 vs. $2(296) = 592$.

Time = 10.62 (sec) , antiderivative size = 933, normalized size of antiderivative = 3.15

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \frac{-bde\sin(c+dx)+bcf\sin(c+dx)-bf(c+dx)\sin(c+dx)}{(a-b)(a+b)d^2(a+b\cos(c+dx))} + \frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{2a(de-cf)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+f\log\left(\sec^2\left(\frac{1}{2}(c+dx)\right)\right)-f\log\left((a+b\cos(c+dx)\right.\right.}{}$$

input
```
Integrate[(e + f*x)/(a + b*Cos[c + d*x])^2,x]
```

output
```
(-(b*d*e*Sin[c + d*x]) + b*c*f*Sin[c + d*x] - b*f*(c + d*x)*Sin[c + d*x])/
((a - b)*(a + b)*d^2*(a + b*Cos[c + d*x])) + (Cos[(c + d*x)/2]^2*((2*a*(d*
e - c*f)*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]) + f*Log[Sec[(c + d*x)/2]^2] - f*Log[(a + b*Cos[c + d*x])*Sec[
(c + d*x)/2]^2] - (I*a*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(Sqrt[a + b] - S
qrt[-a + b]*Tan[(c + d*x)/2])/(I*Sqrt[-a + b] + Sqrt[a + b])] + PolyLog[2,
 (Sqrt[-a + b]*(1 - I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] - I*Sqrt[a + b])]))
/(Sqrt[-a + b]*Sqrt[a + b]) + (I*a*f*(Log[1 - I*Tan[(c + d*x)/2]]*Log[(I*(
Sqrt[a + b] + Sqrt[-a + b]*Tan[(c + d*x)/2]))/(Sqrt[-a + b] + I*Sqrt[a + b
])] + PolyLog[2, (Sqrt[-a + b]*(1 - I*Tan[(c + d*x)/2]))/(Sqrt[-a + b] + I
*Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]) - (I*a*f*(Log[1 + I*Tan[(c + d
*x)/2]]*Log[(Sqrt[a + b] + Sqrt[-a + b]*Tan[(c + d*x)/2])/(I*Sqrt[-a + b]
+ Sqrt[a + b])] + PolyLog[2, (Sqrt[-a + b]*(1 + I*Tan[(c + d*x)/2]))/(Sqrt
[-a + b] - I*Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]) + (I*a*f*(Log[1 +
I*Tan[(c + d*x)/2]]*Log[(I*(Sqrt[a + b] - Sqrt[-a + b]*Tan[(c + d*x)/2]))/
(Sqrt[-a + b] + I*Sqrt[a + b])] + PolyLog[2, (Sqrt[-a + b]*(1 + I*Tan[(c +
 d*x)/2]))/(Sqrt[-a + b] + I*Sqrt[a + b])]))/(Sqrt[-a + b]*Sqrt[a + b]))*(
a*d*e + a*d*f*x + b*f*Sin[c + d*x])*(Sqrt[a + b] - Sqrt[-a + b]*Tan[(c + d
*x)/2])*(Sqrt[a + b] + Sqrt[-a + b]*Tan[(c + d*x)/2]))/((a^2 - b^2)*d^2*(a
 + b*Cos[c + d*x])*(a*(d*e - c*f + I*f*Log[1 - I*Tan[(c + d*x)/2]] - I*...
```

3.189.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3805, 25, 3042, 3147, 16, 3802, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{e+fx}{\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{e+fx}{a+b\cos(c+dx)}dx}{a^2-b^2}-\frac{bf\int -\frac{\sin(c+dx)}{a+b\cos(c+dx)}dx}{d\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\int \frac{e+fx}{a+b\cos(c+dx)}dx}{a^2-b^2}+\frac{bf\int \frac{\sin(c+dx)}{a+b\cos(c+dx)}dx}{d\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{e+fx}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{bf\int \frac{\cos\left(c+dx-\frac{\pi}{2}\right)}{a-b\sin\left(c+dx-\frac{\pi}{2}\right)}dx}{d\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3147}$$

$$-\frac{f\int \frac{1}{a+b\cos(c+dx)}d(b\cos(c+dx))}{d^2\left(a^2-b^2\right)}+\frac{a\int \frac{e+fx}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\int \frac{e+fx}{a+b\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{3802}$$

$$\frac{2a\int \frac{e^{i(c+dx)}(e+fx)}{2e^{i(c+dx)}a+be^{2i(c+dx)}+b}dx}{a^2-b^2}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{b\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a+be^{i(c+dx)}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{i(c+dx)}(e+fx)}{2\left(a+be^{i(c+dx)}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 27

$$\frac{2a\left(\frac{b\int \frac{e^{i(c+dx)}(e+fx)}{a+be^{i(c+dx)}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{i(c+dx)}(e+fx)}{a+be^{i(c+dx)}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 2620

$$\frac{2a\left(\frac{b\left(\frac{if\int \log\left(\frac{e^{i(c+dx)}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{i(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{if\int \log\left(\frac{e^{i(c+dx)}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{i(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 2715

$$\frac{2a\left(\frac{b\left(\frac{f\int e^{-i(c+dx)}\log\left(\frac{e^{i(c+dx)}b}{a-\sqrt{a^2-b^2}}+1\right)de^{i(c+dx)}}{bd^2}-\frac{i(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{f\int e^{-i(c+dx)}\log\left(\frac{e^{i(c+dx)}b}{a+\sqrt{a^2-b^2}}+1\right)de^{i(c+dx)}}{bd^2}-\frac{i(e+fx)\log\left(\cdots\right.}{\cdots}\right.}{2\sqrt{a^2-b^2}}\cdots}{a^2-b^2}$$
$$\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

$\downarrow$ 2838

$$\frac{2a\left(\frac{b\left(-\frac{f\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{i(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{a-\sqrt{a^2-b^2}}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(-\frac{f\,\text{PolyLog}\left(2,-\frac{be^{i(c+dx)}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{i(e+fx)\log\left(1+\frac{be^{i(c+dx)}}{\sqrt{a^2-b^2}+a}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{f\log(a+b\cos(c+dx))}{d^2\left(a^2-b^2\right)}-\frac{b(e+fx)\sin(c+dx)}{d\left(a^2-b^2\right)(a+b\cos(c+dx))}$$

3.189. $\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx$

input
```
Int[(e + f*x)/(a + b*Cos[c + d*x])^2,x]
```

output
```
-((f*Log[a + b*Cos[c + d*x]])/((a^2 - b^2)*d^2)) + (2*a*((b*(((-I)*(e + f*
x)*Log[1 + (b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2])])/(b*d) - (f*PolyLog[
2, -((b*E^(I*(c + d*x)))/(a - Sqrt[a^2 - b^2]))])/(b*d^2)))/(2*Sqrt[a^2 -
b^2]) - (b*(((-I)*(e + f*x)*Log[1 + (b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^
2])])/(b*d) - (f*PolyLog[2, -((b*E^(I*(c + d*x)))/(a + Sqrt[a^2 - b^2]))])
/(b*d^2)))/(2*Sqrt[a^2 - b^2])))/(a^2 - b^2) - (b*(e + f*x)*Sin[c + d*x])/
((a^2 - b^2)*d*(a + b*Cos[c + d*x]))
```

3.189.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

3.189.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 673 vs. 2(270) = 540.

Time = 3.34 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.28

method	result
risch	$\frac{2i(fx+e)\left(a\,\mathrm{e}^{i(dx+c)}+b\right)}{d(-a^2+b^2)\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b\right)}-\frac{2f\ln\left(\mathrm{e}^{i(dx+c)}\right)}{(-a^2+b^2)d^2}+\frac{f\ln\left(b\,\mathrm{e}^{2i(dx+c)}+2a\,\mathrm{e}^{i(dx+c)}+b\right)}{(-a^2+b^2)d^2}+\frac{2iae\arctan\left(\frac{2b\,\mathrm{e}^{i(dx+c)}+2a}{2\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{\frac{3}{2}}d}$

input
```
int((f*x+e)/(a+cos(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*(f*x+e)*(a*exp(I*(d*x+c))+b)/d/(-a^2+b^2)/(b*exp(2*I*(d*x+c))+2*a*exp(
I*(d*x+c))+b)-2/(-a^2+b^2)/d^2*f*ln(exp(I*(d*x+c)))+1/(-a^2+b^2)/d^2*f*ln(
b*exp(2*I*(d*x+c))+2*a*exp(I*(d*x+c))+b)+2*I/(-a^2+b^2)^(3/2)/d*a*e*arctan
(1/2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))+I/(-a^2+b^2)/d*f*a/(a^2-b^
2)^(1/2)*ln((-b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)-a)/((a^2-b^2)^(1/2)-a))*x-I
/(-a^2+b^2)/d*f*a/(a^2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/
((a^2-b^2)^(1/2)+a))*x+I/(-a^2+b^2)/d^2*f*a/(a^2-b^2)^(1/2)*ln((-b*exp(I*(
d*x+c))+(a^2-b^2)^(1/2)-a)/((a^2-b^2)^(1/2)-a))*c-I/(-a^2+b^2)/d^2*f*a/(a^
2-b^2)^(1/2)*ln((b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/((a^2-b^2)^(1/2)+a))*
c+1/(-a^2+b^2)/d^2*f*a/(a^2-b^2)^(1/2)*dilog((-b*exp(I*(d*x+c))+(a^2-b^2)^
(1/2)-a)/((a^2-b^2)^(1/2)-a))-1/(-a^2+b^2)/d^2*f*a/(a^2-b^2)^(1/2)*dilog((
b*exp(I*(d*x+c))+(a^2-b^2)^(1/2)+a)/((a^2-b^2)^(1/2)+a))-2*I/(-a^2+b^2)^(3
/2)/d^2*a*f*c*arctan(1/2*(2*b*exp(I*(d*x+c))+2*a)/(-a^2+b^2)^(1/2))
```

3.189.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(266) = 532$.

Time = 0.46 (sec) , antiderivative size = 1482, normalized size of antiderivative = 5.01

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*((a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos
(d*x + c) + I*a*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a
^2 - b^2)/b^2) + b)/b + 1) - (a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 -
b^2)/b^2)*dilog(-(a*cos(d*x + c) + I*a*sin(d*x + c) - (b*cos(d*x + c) + I*
b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (a*b^2*f*cos(d*x + c)
+ a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x + c) - I*a*sin(d*x + c)
 + (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) -
 (a*b^2*f*cos(d*x + c) + a^2*b*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cos(d*x
+ c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b + 1) - (-I*a^2*b*d*f*x - I*a^2*b*c*f + (-I*a*b^2*d*f*x -
I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a
*sin(d*x + c) + (b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2)
+ b)/b) - (I*a^2*b*d*f*x + I*a^2*b*c*f + (I*a*b^2*d*f*x + I*a*b^2*c*f)*cos
(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cos(d*x + c) + I*a*sin(d*x + c) -
(b*cos(d*x + c) + I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - (I*a^2
*b*d*f*x + I*a^2*b*c*f + (I*a*b^2*d*f*x + I*a*b^2*c*f)*cos(d*x + c))*sqrt(
(a^2 - b^2)/b^2)*log((a*cos(d*x + c) - I*a*sin(d*x + c) + (b*cos(d*x + c)
- I*b*sin(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - (-I*a^2*b*d*f*x - I*a^
2*b*c*f + (-I*a*b^2*d*f*x - I*a*b^2*c*f)*cos(d*x + c))*sqrt((a^2 - b^2)/b^
2)*log((a*cos(d*x + c) - I*a*sin(d*x + c) - (b*cos(d*x + c) - I*b*sin(d...
```

3.189.6 Sympy [F(-1)]

Timed out.

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(a+b*cos(d*x+c))**2,x)
```

output
```
Timed out
```

3.189.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.189.8 Giac [F]

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \int \frac{fx+e}{(b\cos(dx+c)+a)^2}\,dx$$

input
```
integrate((f*x+e)/(a+b*cos(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((f*x + e)/(b*cos(d*x + c) + a)^2, x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(a+b\cos(c+dx))^2}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/(a + b*cos(c + d*x))^2,x)
```

output
```
\text{Hanged}
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```